What do experimental data tell us?

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April 11, 2023
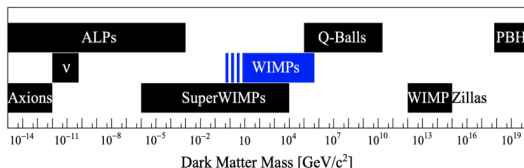
Conjectures on the natural realization of WIMP DM in economic SUSY; Studies initiated in 2016.
Supersymmetry is still a healthy theory!

Section I

Key points of this report

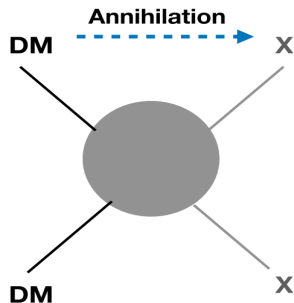# Point 1: WIMP DM– Thermal Freeze-out

X:SM or non-SM particle

- DM starts with thermal distribution;
- Relic abundance is determined by **freeze-out mechanism**;
- DM has an electroweak-scale coupling (WIMP miracle); Consider DM DM → X X:

$$\langle \sigma v \rangle \sim \frac{g^4}{m_{\text{DM}}^2} \sim 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \Rightarrow g \sim \sqrt{\frac{m_{\text{DM}}}{10\text{TeV}}},$$
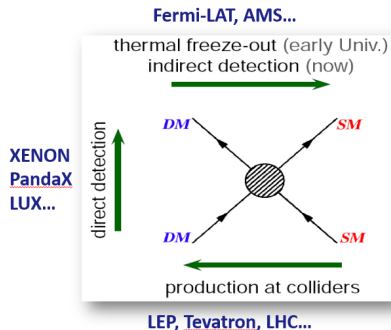$$g \sim 0.1 \text{ for } m_{\text{DM}} = 100 \text{ GeV};$$

- Mass bound: N_{eff} from CMB and unitary
$$5\text{MeV} \lesssim m_{\text{DM}} \lesssim 110\text{TeV}.$$

Point 1: WIMP DM– Paradox Properties

Fermi-LAT, AMS...

thermal freeze-out (early Univ.)
indirect detection (now)

DM → SM

direct detection

DM → SM

production at colliders

LEP, Tevatron, LHC...

No new physics signal !

arXiv:2207.03764

FIG. 5

FIG. 7

Simple WIMP DM Theory:

DM mass: $m_{\rm DM} \sim 100$ GeV

Relic density: $\langle \sigma v \rangle \simeq 10^{-26} {\rm cm}^3 s^{-1}$

SI scattering: $\sigma_{\tilde{\chi}-N}^{\rm SI} \sim 10^{-45} {\rm cm}^2$,

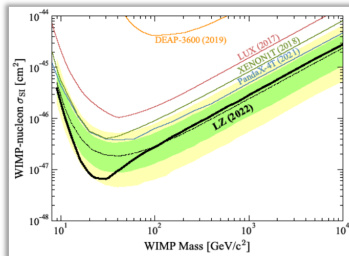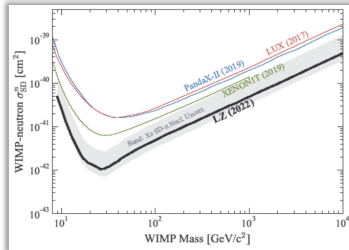**SD scattering:** $\sigma_{\tilde{\chi}-N}^{\rm SD} \sim 10^{-39} {\rm cm}^2$.

Solutions: Go beyond minimal realizations of WIMP miracle.

DM EFTs	Examples	DM Abundance	$\tilde{\chi} - N$ Scattering	Remarks
SM+DM	SM+S_{real}	h/Z funnels	Suppressed by feeble interactions	Increasingly Fine-tuned: $\Delta > 150$.
		Weak/contact interactions	$\sigma_{\rm SI} \gtrsim 10^{-45}{\rm cm}^2$ and/or $\sigma_{\rm SD} \gtrsim 10^{-39}{\rm cm}^2$	Experimentally excluded.
			Suppressed by cancellation	Symmetry! (Very rare case)
SM+DM+X	MSSM with Light Gauginos	Coannihilation partner and Mediator	Suppressed by feeble interactions	Fine-tuning: $\Delta > 30$; Tight LHC constraints.
SM+DM+XY Y = X or Y ≠ X	GNMSSM ISS-NMSSM	May form secluded DM sector	Naturally suppressed by feeble interactions	No tuning. Three portal to SM: Higgs, Neutrino, Gauge.

Why is the dark matter still called WIMP?
Weak interactions in the DM sector to predict proper Ωh^2, feeble connections between SM and DM sectors to suppress ...

At least two directions to build models:

- Naturally realize EWSB: **MSSM → Z_3-NMSSM → General NMSSM**.
- Generate neutrino mass: **Type-I NMSSM → ISS-NMSSM → B-L NMSSM**.
- New symmetry may be needed to reduce the number of input parameters! A well-motivated example is R-symmetry at GUT or electroweak scale.

Point 2: Statistics - Bayesian Theorem

Bayesian theorem: $\Theta = (\Theta_1, \Theta_2, \cdots)$ is theoretical input parameters.

$$\mathrm{P}(\boldsymbol{\Theta} \mid \mathbf{D}, H) \equiv \frac{\mathrm{P}(\mathbf{D} \mid \boldsymbol{\Theta}, H)\, \mathrm{P}(\boldsymbol{\Theta} \mid H)}{\mathrm{P}(\mathbf{D} \mid H)} \Longrightarrow P(\boldsymbol{\Theta}) \equiv \frac{\mathcal{L}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta})}{\mathcal{Z}}$$

- $\mathrm{P}(\mathbf{D} \mid \boldsymbol{\Theta}, H) \equiv \mathcal{L}(\boldsymbol{\Theta})$: Likelihood function.
  The preference of experimental results to parameter point
  $p = \{\Theta\}$, e.g., Gaussian distribution:
  $$\mathcal{L} = e^{-\frac{[\mathcal{O}_{th}(\Theta) - \mathcal{O}_{exp}]^2}{2\sigma^2}}.$$
  $\mathcal{O}_{th}(\Theta)$: theoretical prediction, $\mathcal{O}_{exp}$: experimental measurement,
  σ: total uncertainty.

- $\mathrm{P}(\boldsymbol{\Theta} \mid H) \equiv \pi(\boldsymbol{\Theta})$: Prior probability Density Function.
  Is there any physical motivation in selecting $\pi(\boldsymbol{\Theta})$?
  1. It may be better to choose physical quantities, such as particle masses and couplings, as inputs. Their reasonable ranges defined!
  2. It may be better to assume undetermined physical inputs to be flatly distributed in their reasonable ranges.
  3. Underlying physics becomes more apparent!

Point 2: Statistics-Bayesian Theorem

Bayesian theorem: $\Theta = (\Theta_1, \Theta_2, \cdots)$ is theoretical input parameters.

$$P(\boldsymbol{\Theta} \mid \mathbf{D}, H) \equiv \frac{P(\mathbf{D} \mid \boldsymbol{\Theta}, H) \, P(\boldsymbol{\Theta} \mid H)}{P(\mathbf{D} \mid H)} \Longrightarrow P(\boldsymbol{\Theta}) \equiv \frac{\mathcal{L}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta})}{\mathcal{Z}}$$

- $P(\mathbf{D} \mid H) \equiv \mathcal{Z}$: Bayesian evidence, normalization factor.
  Averaged likelihood, reflecting theory's capability to keep consistent with the data. Small $\mathcal{Z} \Longrightarrow$ The theory is fine tuned!

  $$\mathcal{Z} = \int \mathcal{L}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta}) d^D \boldsymbol{\Theta}.$$

  - Depend on $\mathcal{L}$, Θ, π, and the integrated parameter space!
  - **Application: MC sampling of the parameter space.**

- $P(\boldsymbol{\Theta} \mid \mathbf{D}, H) \equiv P(\boldsymbol{\Theta})$ – Posterior probability distribution function.
  The state of our knowledge about the parameters Θ given the experimental data D, or alternatively speaking, the updated prior PDF after considering the impact of the experimental data.
  One can infer from $P(\Theta)$ the underlying physics of the model.

Point 3: Global Fits – MultiNest Algorithm

GF: Map expt measurments into theory's parameter space.

- Characteristics of the parameter space in a new physics theory: High dimensional, highly degenerated likelihood, isolated physical parameter island, inefficient for random and Markov chain scan.

- **MultiNest algorithm** is well adaptive to explore such a situation by comprehensively scanning the parameter space.
  - Use *nlive* samples to decide iso-likelihood contour in each iteration; provide comprehensive information of the space;
  - The results are statistically significant.

- Explore the **properties** of acquired parameter points, such as the prediction on various experimental measurements, theoretical fine-tuning, and vacuum stability, etc..
  Rich material for researchers to improve understanding!

- Reveal **global characteristics** of the theory by **statistics**: **Some fundamental physical mechanisms can be inferred.**

- Provide intuitive **understanding** by analytic **formulae**.

Rich experimental data have been accumulated !

- Precision electroweak data;
- Heavy flavor data;
- Neutrino experiments;
- Higgs property measurement.
- **Dark matter search experiments;**
- **LHC search for supersymmetry;**
- **Optional: Muon anomalous magnetic moment, W mass.**

Global fit combines all the data to analyze theories.
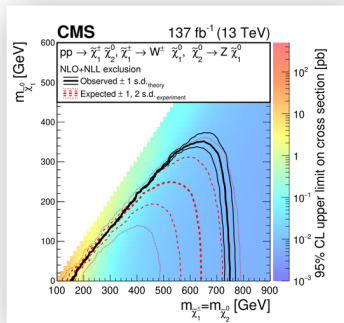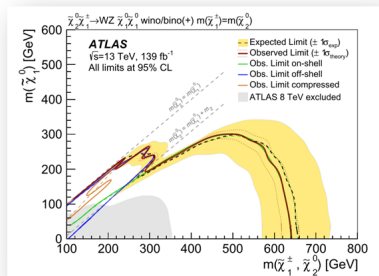
Latest LHC searches for Tri- and Bi-lepton signals.

1. Simplified model for a specified process.
2. Invalid for a specific theory: complex decay chain, multiple production processes, and various signals to be analyzed.
3. Elaborated Monte Carlo simulations are necessary.

Point 3: Global Fits – Bayesian Statistics

Marginal posterior PDFs: reflecting the preference to specific regions of one or more parameters.

$$1D : P\left(\Theta_A\right) = \int P(\Theta)d\Theta_1 d\Theta_2 \cdots d\Theta_{A-1} d\Theta_{A+1} \cdots \cdots$$

$$2D : P\left(\Theta_A, \Theta_B\right) = \int P(\Theta)d\Theta_1 d\Theta_2 \cdots d\Theta_{A-1} d\Theta_{A+1} \cdots d\Theta_{B-1} d\Theta_{B+1} \cdots$$

Credible Regions: most preferred parameter regions by data; it depends on both likelihood function and phase space.

$$1D : \int_{\Theta_{A_1}}^{\Theta_{A_2}} P\left(\Theta_A\right) d\Theta_A = 1 - \alpha$$

$$2D : \int_{P(\Theta_A, \Theta_B) \geq p_{\text{crit}}} P\left(\Theta_A, \Theta_B\right) d\Theta_A d\Theta_B = 1 - \alpha$$

$$1\sigma : \alpha = 0.317, \qquad 2\sigma : \alpha = 0.055.$$

Point 3: Global Fits – Frequencist Statistics

Profile Likelihood: parameter's capability to explain the data.

$$1D : \mathcal{L}(\Theta_A) = \max_{\Theta_1, \cdots, \Theta_{A-1}, \Theta_{A+1}, \cdots} \mathcal{L}(\Theta),$$

$$2D : \mathcal{L}(\Theta_A, \Theta_B) = \max_{\Theta_1, \cdots, \Theta_{A-1}, \Theta_{A+1}, \cdots, \Theta_{B-1}, \Theta_{B+1}, \cdots} \mathcal{L}(\Theta)$$

Confidence Intervals: most favored regions to explain the data; it depends only on the likelihood function.

$$1D : \left\{ \chi^2(\Theta_A) - \chi^2_{Best} \right\} \leq F^{-1}_{\chi^2_1}(1 - \alpha),$$

$$2D : \left\{ \chi^2(\Theta_A, \Theta_B) - \chi^2_{Best} \right\} \leq F^{-1}_{\chi^2_2}(1 - \alpha)$$

$\chi^2(\Theta_A) \equiv -2 \log \mathcal{L}(\Theta_A), \quad \chi^2(\Theta_A, \Theta_B) \equiv -2 \log \mathcal{L}(\Theta_A, \Theta_B);$

χ^2_{Best}: the χ^2 value for the best point;

$F^{-1}_{\chi^2_n}$: the inverse cdf for a chi-squared distribution with n dof:

1σ $(\alpha = 0.317)$: $F^{-1}_{\chi^2_1} = 1.00, \quad F^{-1}_{\chi^2_2} = 2.30;$

2σ $(\alpha = 0.046)$: $F^{-1}_{\chi^2_1} = 4.00, \quad F^{-1}_{\chi^2_2} = 6.18.$

Point 3: Global Fits – Technical Support

Results are based on global fits of supersymmetric theories.

1. Specially designed clusters.
2. SARAH suite for calculation.
   - Model building: SARAH-4.14.3;
   - Spectrum generator: SPheno-4.0.4;
   - DM physics calculator: MicrOMEGAs-5.0.4;
   - Higgs physics calculator: HiggsSingal-2.2.3, HiggsBounds-5.3.2;
   - Flavor physics calculator: FlavorKit;
   - MC simulation: MadGraph_aMC@NLO, PYTHIA8, and Delphes;
   - LHC SUSY search: SModelS-2.2.1, CheckMATE-2.0.37.
3. Scan strategy: parallel MultiNest algorithm.
   High performance:
   Simultaneous computation of more than 10^6 processes.
4. Members of the developers for the package CheckMATE.
   Reproduce more than 40 experimental analyses.

Table 1: Experimental analyses of the electroweakino production processes.

Scenario	Final State	Name
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to W Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell(n \geq 2) + nj(n \geq 0) + \mathrm{E_T^{miss}}$	CMS-SUS-20-001$(137 fb^{-1})$ ATLAS-2106-01676$(139 fb^{-1})$ CMS-SUS-17-004$(35.9 fb^{-1})$ CMS-SUS-16-039$(35.9 fb^{-1})$ ATLAS-1803-02762$(36.1 fb^{-1})$ ATLAS-1806-02293$(36.1 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to \ell\bar{\nu}\ell\bar{\ell}$	$n\ell(n = 3) + \mathrm{E_T^{miss}}$	CMS-SUS-16-039$(35.9 fb^{-1})$ ATLAS-1803-02762$(36.1 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to \tilde{\tau}\nu\ell\bar{\ell}$	$2\ell + 1\tau + \mathrm{E_T^{miss}}$	CMS-SUS-16-039$(35.9 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to \tilde{\tau}\nu\tilde{\tau}\tau$	$3\tau + \mathrm{E_T^{miss}}$	CMS-SUS-16-039$(35.9 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to W h \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell(n \geq 1) + nb(n \geq 0) + nj(n \geq 0) + \mathrm{E_T^{miss}}$	ATLAS-1909-09226$(139 fb^{-1})$ CMS-SUS-17-004$(35.9 fb^{-1})$ CMS-SUS-16-039$(35.9 fb^{-1})$ ATLAS-1812-09432$(36.1 fb^{-1})$ CMS-SUS-16-034$(35.9 fb^{-1})$ CMS-SUS-16-045$(35.9 fb^{-1})$
$\tilde{\chi}_1^\mp \tilde{\chi}_1^\pm \to W W \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + \mathrm{E_T^{miss}}$	ATLAS-1908-08215$(139 fb^{-1})$ CMS-SUS-17-010$(35.9 fb^{-1})$
$\tilde{\chi}_1^\mp \tilde{\chi}_1^\pm \to 2\tilde{\ell}\nu(\tilde{\nu}\ell)$	$2\ell + \mathrm{E_T^{miss}}$	ATLAS-1908-08215$(139 fb^{-1})$ CMS-SUS-17-010$(35.9 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\mp \to h/Z W \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to \gamma/Z\tilde{G}$ $\tilde{\chi}_1^\mp \tilde{\chi}_1^\mp \to W W \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to \gamma/Z\tilde{G}$	$2\gamma + n\ell(n \geq 0) + nb(n \geq 0) + nj(n \geq 0) + \mathrm{E_T^{miss}}$	ATLAS-1802-03158$(36.1 fb^{-1})$
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to Z W \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to h/Z\tilde{G}$ $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \to W W \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to h/Z\tilde{G}$ $\tilde{\chi}_2^0 \tilde{\chi}_1^0 \to Z \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to h/Z\tilde{G}$ $\tilde{\chi}_1^\mp \tilde{\chi}_1^0 \to W \tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \to h/Z\tilde{G}$	$n\ell(n \geq 4) + \mathrm{E_T^{miss}}$	ATLAS-2103-11684$(139 fb^{-1})$
$\tilde{\chi}^{0,\pm} \tilde{\chi}^{0,\mp} \to \tilde{\chi}^0 \tilde{\chi}^0 + \chi_{soft} \to Z Z / H \tilde{G}\tilde{G}$	$n\ell(n \geq 2) + nb(n \geq 0) + nj(n \geq 0) + \mathrm{E_T^{miss}}$	CMS-SUS-16-039$(35.9 fb^{-1})$ CMS-SUS-17-004$(35.9 fb^{-1})$

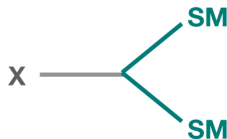# Section II

Six Mechanisms to Suppress
DM-Nucleon Scattering

• 1. Very small coupling:

  • 1.1 Secluded dark matter (dark sector)
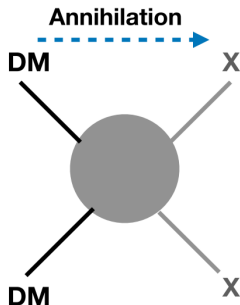
Proposed in 0711.4866.
Three types of portals:
Higgs portal;
Gauge portal;
Neutrino portal

**Dark mediator
with very small coupling to SM**

This mechanism can be realized in non-minimal SUSY!

- **2. Suppressed scattering cross-section:**

  - **By velocity or momentum transfer**

Case for Fermionic DM
Kumar & Marfatia:1305.1611 (PRD)

	Name	Interaction Structure	$\sigma_{\rm SI}$ suppression	$\sigma_{\rm SD}$ suppression	s-wave?
Scalar	F1	$\bar{X}X\bar{q}q$	1	$q^2v^{\perp2}$ (SM)	No
	F2	$\bar{X}\gamma^5X\bar{q}q$	q^2 (DM)	$q^2v^{\perp2}$ (SM); q^2 (DM)	Yes
Pseudoscalar	F3	$\bar{X}X\bar{q}\gamma^5q$	0	q^2 (SM)	No
	F4	$\bar{X}\gamma^5X\bar{q}\gamma^5q$	0	q^2 (SM); q^2 (DM)	Yes
Vector	F5	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (vanishes for Majorana X)	1	$q^2v^{\perp2}$ (SM) q^2 (SM); q^2 or $v^{\perp2}$ (DM)	Yes
Anapole	F6	$\bar{X}\gamma^\mu\gamma^5X\bar{q}\gamma_\mu q$	$v^{\perp2}$ (SM or DM)	q^2 (SM)	No
	F7	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5q$ (vanishes for Majorana X)	$q^2v^{\perp2}$ (SM); q^2 (DM)	$v^{\perp2}$ (SM) $v^{\perp2}$ or q^2 (DM)	Yes
	F8	$\bar{X}\gamma^\mu\gamma^5X\bar{q}\gamma_\mu\gamma^5q$	$q^2v^{\perp2}$ (SM)	1	$\propto m_f^2/m_X^2$
	F9	$\bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM); q^2 or $v^{\perp2}$ (DM) $q^2v^{\perp2}$ (SM)	1	Yes
	F10	$\bar{X}\sigma^{\mu\nu}\gamma^5X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM)	$v^{\perp2}$ (SM) q^2 or $v^{\perp2}$ (DM)	Yes

Not easy to build specific models! Let alone in supersymmetric

- ## 3. Coannihilation mechanism

Annihilation

Y   X

DM   X
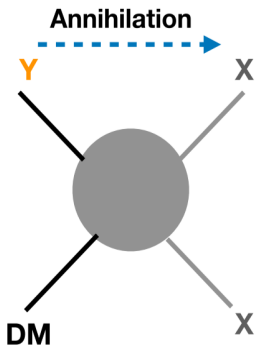
- Y has a close mass with DM

  - Y is not populated today due to decay

  - Charged Y: near degenerate spectrum of SUSY, AMSB; EW multiplet DM (2n+1, 0) ($\delta m \sim 166$ MeV)
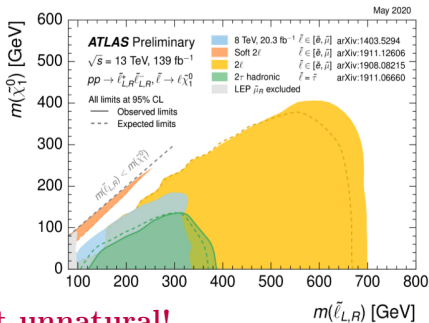
Easily realized in SUSY, but unnatural!

- ## 4. Resonant annihilation

  - $2m_{\rm DM} \approx m_X$

Scalar DM (s) with a Higgs portal coupling

$$\Delta \mathcal{L}_s = -\frac{1}{2} m_s^2 s^2 - \frac{1}{4} \lambda_s s^4 - \frac{1}{4} \lambda_{Hss} \phi^\dagger \phi s^2$$

+ 2 diagrams to hh

Scalar Higgs Portal
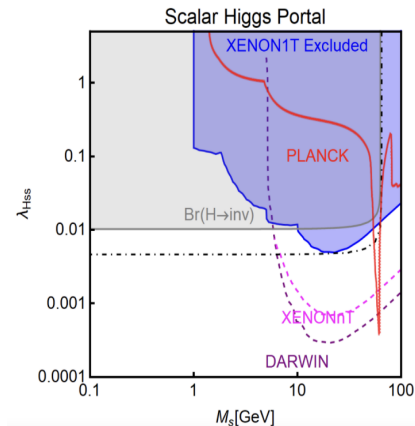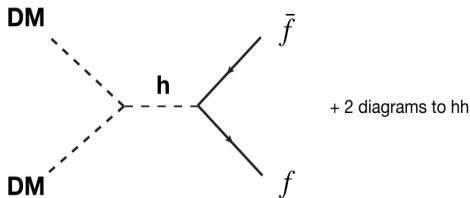
Arcadi et al: 2101.02507

See also WL Guo, LY Wu et al 2010; B Li, YF Zhou 2015

Easily realized in SUSY, but needs severe fine-tuning!

Six Mechanisms to Suppress DM-Nucleon Scattering

- ## 5. Cancellation effect in scattering cross-section

  - ### SM Higgs - Dark scalar mediator cancellation

    Gross, Lebedev1, Toma: 1708.02253 (PRL)

$$V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_{HS}|H|^2|S|^2 + \frac{\lambda_S}{2}|S|^4$$

$$V_{\text{soft}} = -\frac{\mu_S'^2}{4}S^2 + \text{h.c.} \qquad \text{symmetry}: S \leftrightarrow S^*$$

$$S = (v_s + s + i\chi)/\sqrt{2} \qquad \text{Pseudoscalar DM}$$

CP-even scalar mixing (s, h) $\rightarrow$ (h_1, h_2)

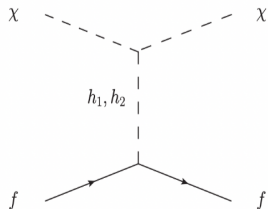$$\mathcal{L} \supset -(h_1\cos\theta + h_2\sin\theta)\sum_f \frac{m_f}{v}\bar{f}f \qquad \mathscr{L} \supset \frac{\chi^2}{2v_s}\left(m_{h_1}^2\sin\theta h_1 - m_{h_2}^2\cos\theta h_2\right)$$

$$\mathcal{A}_{dd}(t) \propto \sin\theta\cos\theta\left(\frac{m_{h_2}^2}{t - m_{h_2}^2} - \frac{m_{h_1}^2}{t - m_{h_1}^2}\right) \simeq \sin\theta\cos\theta \frac{t(m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2 m_{h_2}^2} \simeq 0$$

See JL, XP Wang and F Yu 1704.00730 (JHEP), for cancellation between A' - Z boson in kinetic mixing dark photon model

The amplitude is suppressed by q²

Can not be realized in SUSY!

Six Mechanisms to Suppress DM-Nucleon Scattering

- **6. Leptophilic models**

  - Only couples to electrons, couples to nucleons at 1-loop

    - For light DM, e-DM recoils can have stringent limits (e.g. XENON1T, PANDAX, CDEX)

    - For heavy DM, neuclus-DM recoils wins over e-DM recoil
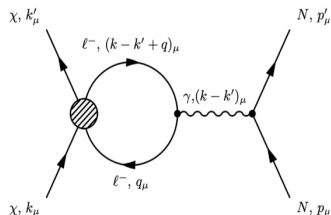
$$R^{\text{WAS}} : R^{\text{WES}} : R^{\text{WNS}} \sim \epsilon_{\text{WAS}} : \epsilon_{\text{WES}} \frac{m_e}{m_N} : \left(\frac{\alpha_{\text{em}} Z}{\pi} \right)^2 \sim 10^{-17} : 10^{-10} : 1$$

WAS = e kicked out

WES = e to higher energy level

WNS = nucleus recoil

The probability to find a high p electron
in the wave function is highly suppressed!

Kopp et al: 0907.3159 (PRD)

Realized the SM extensionos with $L_{\mu-\tau}$, $B-L$, or left-right symmetry, and their supersymmetric versions!

Indirect Detection from DM Annihilation

- observable quantity:
  CMB photon, photon from dwarf galaxies, and positron from cosmic ray, etc.
- $S \propto \sum_i \langle \sigma v \rangle_{0,i} \epsilon_i$,

  $\langle \sigma v \rangle_{0,i}$: annihilation rate for DM DM $\to e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^-, t\bar{t}, \cdots$
  at present day;

  ϵ_i: efficiency translating annihilation products into signal.

Note:$\langle \sigma v \rangle_F$ may differ significantly from $\langle \sigma v \rangle_{0,i}$

$\sigma v \sim \sigma_s + \sigma_p v^2 + \sigma_d v^4 + \dots$ (s-, p-, and d-wave contribution)
- Freeze-out: $v^2 \sim 0.25$
- CMB: $v^2 \sim eV/m_{DM} \sim 10^{-5}$
- Today: $v \sim 10^{-3} c$

ϵ_i **may differ greatly for different annihilation final state!**

Indirect Detection from DM Annihilation
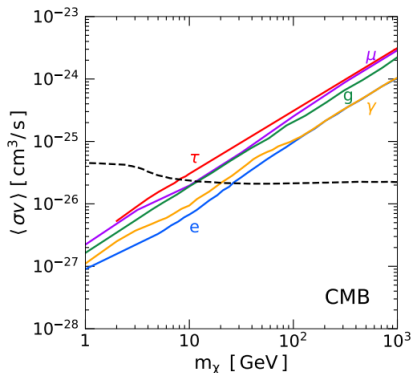
Figure 1: Planck CMB limits at 95% C.L. for DM annihilation 100% to individual channels.
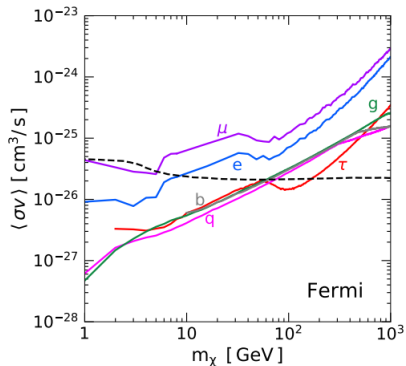
Figure 2: Fermi-LAT limits at 95% C.L. for DM annihilation 100% to individual channels.

Depending on final state, CMB limits are powerful for light DM, while Fermi-LAT limits are effective for $m_{DM} \lesssim 100$ GeV.
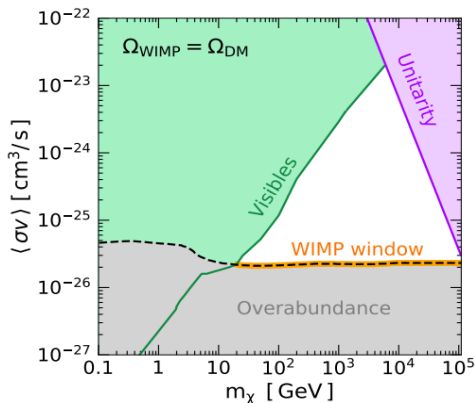
Figure 3: Bounds on the generic thermal WIMP window (s-wave $2 \to 2$ annihilation, standard cosmological history), assuming WIMP DM is 100% of the DM. Shown is the conservative bound calculated from the data of CMB, Fermi-LAT and AMS-02 (Visibles), and the unitarity bound. The remaining WIMP window is the orange line, and the white space is unprobed. Thermal relic cross section is the dashed line.

Conclusion about WIMP DM

1. As far as WIMP DM itself is concerned, it can fit experiments very well.

2. WIMP DM can be easily embedded into renormalizable theories. In this case, DM physics usually entangles with Higgs physics, sparticle physics, and sometimes neutrino physics. Global fit is necessary.

3. In economic WIMP DM theories, DM physics are usually in tension with various experiments, and consequently, the theories become unnatural.

4. What is the most economic and natural (supersymmetric) WIMP DM theory?

Section III

Appeal of Bino-dominated DM is losing!

MSSM: Gauge Group and Superpotential

- Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}$	$\lambda_{\tilde{B}}$	B	U(1)	g_1	hypercharge
$\hat{W}$	$\lambda_{\tilde{W}}$	W	SU(2)	g_2	left
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_3	color

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(U(1) \otimes SU(2) \otimes SU(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$

- Superpotential: μ-the only dimensional parameter. μ-problem!

$$W_{\text{MSSM}} = \mu \hat{H}_u \hat{H}_d - Y_d \hat{d}\hat{q}\hat{H}_d - Y_e \hat{e}\hat{l}\hat{H}_d + Y_u \hat{u}\hat{q}\hat{H}_u$$

MSSM: DM-Nucleon Scattering

Neutralino mass matrix:

$$
m_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u \\
0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\
-\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu \\
\frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu & 0
\end{pmatrix}
$$

Replace M_1 by $m_{\tilde{\chi}_1^0}$ as an input parameter:

$$
N_{i,j} = \frac{1}{\sqrt{C_i}} \begin{pmatrix}
\left(\mu^2 - m_{\tilde{\chi}_i^0}^2\right)\left(M_2 - m_{\tilde{\chi}_i^0}\right) - M_Z^2 c_W^2 \left(m_{\tilde{\chi}_i^0} + 2\mu s_\beta c_\beta\right) \\
-M_Z^2 s_W c_W \left(m_{\tilde{\chi}_i^0} + 2\mu s_\beta c_\beta\right) \\
\left(M_2 - m_{\tilde{\chi}_i^0}\right)\left(m_{\tilde{\chi}_i^0} c_\beta + \mu s_\beta\right) M_Z s_W \\
-\left(M_2 - m_{\tilde{\chi}_i^0}\right)\left(m_{\tilde{\chi}_i^0} s_\beta + \mu c_\beta\right) M_Z s_W
\end{pmatrix}_j
$$

C_i: Normalization factor

$$
\begin{aligned}
C_i =& M_Z^2 c_W^2 \left(m_{\tilde{\chi}_i^0} + 2\mu s_\beta c_\beta\right) \left[M_Z^2 \left(m_{\tilde{\chi}_i^0} + 2\mu s_\beta c_\beta\right) + 2\left(\mu^2 - m_{\tilde{\chi}_i^0}^2\right)\left(m_{\tilde{\chi}_i^0} - M_2\right)\right] \\
&+ \left(m_{\tilde{\chi}_i^0} - M_2\right)^2 \left\{ M_Z^2 s_W^2 \left[\left(m_{\tilde{\chi}_i^0}^2 + \mu^2\right) + 4\mu m_{\tilde{\chi}_i^0} s_\beta c_\beta\right] + \left(m_{\tilde{\chi}_i^0}^2 - \mu^2\right)^2 \right\}
\end{aligned}
$$

Why is the Appeal of Bino-dominated DM Losing?

- MSSM: Full expression complicated; $\mu/m_{\tilde{\chi}_1^0}$ is Higgsino/DM mass.

$$\sigma_{\tilde{\chi}_1^0-N}^{\mathrm{SI}} \simeq 5 \times 10^{-45} \ \mathrm{cm}^2 \left(\frac{C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h}}{0.1}\right)^2 \left(\frac{m_h}{125\mathrm{GeV}}\right)^2$$

$$\sigma_{\tilde{\chi}_1^0-N}^{\mathrm{SD}} \simeq 10^{-39} \ \mathrm{cm}^2 \left(\frac{C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z}}{0.1}\right)^2$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h} \simeq e \tan\theta_W \frac{m_Z}{\mu\left(1 - m_{\tilde{\chi}_1^0}^2/\mu^2\right)} \left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right)$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z} \simeq \frac{e \tan\theta_W \cos 2\beta}{2} \frac{m_Z^2}{\mu^2 - m_{\tilde{\chi}_1^0}^2}$$

- **Conservative bounds on Higgsino mass:**
  LZ Experiment: $\mu \gtrsim 380$ GeV,   **LZ + LHC + a_μ:** $\mu \gtrsim 500$ GeV.
- **Higgsino mass is related with electroweak symmetry breaking!**
  $$m_Z^2 = 2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)/(\tan^2\beta - 1) - \mathbf{2\mu^2}.$$

A tuning of 1% in EWSB. The situation will become worse if no DM is found.

MSSM: Dominant Annihilation Channels

Preferred DM annihilation channels:

Co-annihilating with a Wino-dominated NLSP:

1. Constraints on LHC search for SUSY is relatively weak.
2. Tri-lepton signal for compressed sprectrum: $|m_{\tilde{\chi}_1^0}| > 210$ GeV.

Co-annihilating with Sletpon NLSP:

1. The Slepton may be right-handed or left-handed.
2. LHC cosntraints are very strong!

Disfavored annihilation channels:

Co-annihilating with a Higgsino-dominated NLSP:

1. DM direct detection experiments prefer an excessively large $|\mu|$.
2. Unable to explain the muon g-2 anomaly since $|\mu|$ is large.

h/Z funnel: tight LHC constraints, finely tuned!

Section IV

Why can't Z_3-NMSSM
explain DM experiments naturally?

Z_3-NMSSM: Motivations and Superpotential

- Field content and gauge group

SF	Spin 0	Spin $\frac{1}{2}$	Generations	(U(1) $\otimes$ SU(2) $\otimes$ SU(3))
$\hat{q}$	$\tilde{q}$	q	3	$\left(\frac{1}{6}, \mathbf{2}, \mathbf{3}\right)$
$\hat{l}$	$\tilde{l}$	l	3	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$\left(\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$\left(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$\left(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{\mathbf{s}}$	$\mathbf{S}$	$\tilde{S}$	$\mathbf{1}$	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential — an ad hoc Z_3 discrete symmetry

$$W_{\text{NMSSM}} = W_{\text{Yukawa}} + \lambda \hat{S}\hat{H}_u\hat{H}_d + \frac{1}{3}\kappa \hat{S}^3$$

Try to solve μ-problem and little hierarchy problem.

- DM may be Bino- or Singlino-dominated. <u>For Bino-dom. case:</u>
  DM physics is the same as that of MSSM since $\lambda \lesssim 0.3$.
  LZ Experiment: $\mu \gtrsim 380$ GeV,   Higgs Data: $\lambda\mu \lesssim 100$ GeV.

Singlino-dominated DM:

- Neutralino mass matrix — diagonalized by a rotation matrix N

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_1 v_u}{\sqrt{2}} & 0 \\ & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda v_u \\ & & & 0 & -\lambda v_d \\ & & & & \frac{2\kappa}{\lambda}\mu \end{pmatrix}$$

- DM mass and its couplings are approximated by: $\mu \equiv \frac{\lambda}{\sqrt{2}} v_s$

$$m_{\tilde{\chi}_1^0} \approx \frac{2\kappa}{\lambda}\mu + \frac{\lambda^2 v^2}{\mu^2}(\mu \sin 2\beta - \frac{2\kappa}{\lambda}\mu) \simeq \frac{2\kappa}{\lambda}\mu, \qquad N_{15} \simeq 1,$$

$$\frac{N_{13}}{N_{15}} = \frac{\lambda v}{\sqrt{2}\mu} \frac{(m_{\tilde{\chi}_1^0}/\mu)\sin\beta - \cos\beta}{1 - \left(m_{\tilde{\chi}_1^0}/\mu\right)^2}, \qquad \frac{N_{14}}{N_{15}} = \frac{\lambda v}{\sqrt{2}\mu} \frac{(m_{\tilde{\chi}_1^0}/\mu)\cos\beta - \sin\beta}{1 - \left(m_{\tilde{\chi}_1^0}/\mu\right)^2},$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_i} \simeq \frac{\sqrt{2}\mu}{v} \left(\frac{\lambda v}{\mu}\right)^2 \frac{V_{h_i}^{SM}(m_{\tilde{\chi}_1^0}/\mu - \sin 2\beta)}{1 - (m_{\tilde{\chi}_1^0}/\mu)^2} + \dots,$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z} \simeq \frac{m_Z}{\sqrt{2}v} \left(\frac{\lambda v}{\mu}\right)^2 \frac{\cos 2\beta}{1 - (m_{\tilde{\chi}_1^0}/\mu)^2},$$

Singlino-dominated DM:

- DM properties are described by **four** independent parameters:

$$\tan\beta, \quad \lambda, \quad \mu, \quad m_{\tilde{\chi}_1^0} \text{ or } \kappa, \quad \text{and } 2|\kappa/\lambda| < 1.$$

- DM-Nucleon Scattering in the alignment limit:

$$\sigma_{\tilde{\chi}_1^0-N}^{\text{SI}} \simeq 5\times 10^{-45}\text{cm}^2 \times \left(\frac{\mathcal{A}}{0.1}\right)^2, \quad \sigma_{\tilde{\chi}_1^0-N}^{\text{SD}} \simeq 10^{-39} \text{ cm}^2 \left(\frac{C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z}}{0.1}\right)^2,$$

$$\mathcal{A} \simeq \left(\frac{125\text{GeV}}{m_h}\right)^2 V_h^{\text{SM}} C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h} + \left(\frac{125\text{GeV}}{m_{h_s}}\right)^2 V_{h_s}^{\text{SM}} C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_s}$$

$$\simeq \sqrt{2}\left(\frac{125\text{GeV}}{m_h}\right)^2 \lambda\frac{\lambda v}{\mu} \frac{(m_{\tilde{\chi}_1^0}/\mu - \sin 2\beta)}{1-(m_{\tilde{\chi}_1^0}/\mu)^2},$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z} \simeq \frac{m_Z}{\sqrt{2}v}\left(\frac{\lambda v}{\mu}\right)^2 \frac{\cos 2\beta}{1-(m_{\tilde{\chi}_1^0}/\mu)^2}.$$

LZ Experiment: $\lambda \lesssim 0.1$, **DM-$\tilde{H}$ coannihilation to obtain proper abudance.**
Bayesian evidence is heavily suppressed $\rightarrow$ A fine-tuning theory!

Z_3-NMSSM: Dominant Annihilation Channels

Conditions to obtain the measured DM abundance:

❶ $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to t\bar{t}$: s-channel exchange of Z and Higgs bosons.

$$|C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 G^0}| = \frac{\sqrt{2} m_{\tilde{\chi}_1^0}}{v} \left(\frac{\lambda v}{\mu_{eff}} \right)^2 \frac{\cos 2\beta}{1 - (m_{\tilde{\chi}_1^0}/\mu_{eff})^2} \simeq 0.1.$$

❷ $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to h_s A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

$$|C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_s}| = |C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A_s}| = -\sqrt{2}\kappa \simeq 0.2 \times \left(\frac{m_{\tilde{\chi}_1^0}}{300 \text{ GeV}} \right)^{1/2}.$$

❸ $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to h A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

$$\lambda^3 \sin 2\beta \simeq \left(\frac{\mu_{eff}}{700 \text{ GeV}} \right)^2.$$

$\lambda > 0.3$ is preferred to predict the measured abundance for traditional annihilation mechanisms.

Section V

Advantages of Singlino-dominated DM in GNMSSM

GNMSSM: Motivations and Superpotential

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\text{U}(1) \otimes \text{SU}(2) \otimes \text{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$\left(\frac{1}{6}, \mathbf{2}, \mathbf{3}\right)$
$\hat{l}$	$\tilde{l}$	l	3	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$\left(\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$\left(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$\left(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential — no ad hoc symmetry!

$$W_{\text{GNMSSM}} = W_{\text{Y}} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3 + \mu \hat{H}_u \cdot \hat{H}_d + \frac{1}{2} \mu' \hat{S}^2 + \xi \hat{S}$$

1. Free from domain wall and tadpole problems in Z_3-NMSSM.

2. Z_3-violating terms originate from unified theories with a Z_4^n or Z_8^n sym..

3. The $\xi \hat{S}$ term can be eliminated by field redefinitions.

Singlino-dominated DM:

- Neutralino mass matrix: $\mu_{eff} \equiv \frac{\lambda}{\sqrt{2}} v_s$, $\mu_{tot} \equiv \mu + \mu_{eff}$.

$$m_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu_{tot} & -\frac{1}{\sqrt{2}}v_u \lambda \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu_{tot} & 0 & -\frac{1}{\sqrt{2}}v_d \lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}}v_u \lambda & -\frac{1}{\sqrt{2}}v_d \lambda & \mathbf{m_N} \end{pmatrix}$$

Mass and couplings of the singlino-dominated DM are given by:

$$m_{\tilde{\chi}_1^0} \simeq m_N + \frac{1}{2}\frac{\lambda^2 v^2 (m_{\tilde{\chi}_1^0} - \mu_{tot}\sin 2\beta)}{m_{\tilde{\chi}_1^0}^2 - \mu_{tot}^2} \simeq \mathbf{m_N}, \quad \mathbf{m_N} \equiv \frac{2\kappa}{\lambda}\mu_{\mathbf{eff}} + \mu',$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_i} = C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_i}^{Z_3-\text{NMSSM}}|_{\mu \to \mu_{tot}}, \quad C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z} = C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z}^{Z_3-\text{NMSSM}}|_{\mu \to \mu_{tot}}.$$

- DM properties are described by **five** independent parameters:

  $$\tan\beta, \ \lambda, \ \kappa, \ \mu_{\mathbf{tot}}, \ and \ m_{\tilde{\chi}_1^0}. \qquad \mu_{\mathbf{tot}} : \textbf{Higgsino mass}.$$

  Note:
  Different from Z_3-NMSSM, $m_{\tilde{\chi}_1^0}$, λ, and κ are not correlated!
- In the limit $\lambda \to 0$, matrix decomposition: $5 \times 5 = 4 \oplus 1$, **decoupled!**

GNMSSM: Higgs Sector

Soft-breaking terms:

$$-\mathcal{L}_{soft} = \left[\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3} A_\kappa \kappa S^3 + m_3^2 H_u \cdot H_d + \frac{1}{2} {m'_S}^2 S^2 + h.c. \right]$$
$$+ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2.$$

CP-odd Higgs mass matrix in bases $(A_{\mathrm{NSM}}, \mathrm{Im}(S))$**:**

$$\mathcal{M}_{P,11}^2 = \frac{2\left[\mu_{eff}(\lambda A_\lambda + \kappa \mu_{eff} + \lambda \mu') + \lambda m_3^2 \right]}{\lambda \sin 2\beta} \equiv \mathbf{m_A^2},$$

$$\mathcal{M}_{P,22}^2 = \frac{(\lambda A_\lambda + 4\kappa \mu_{eff} + \lambda \mu') \sin 2\beta}{4\mu_{eff}} \lambda v^2 - \frac{\kappa \mu_{eff}}{\lambda}(3A_\kappa + \mu') - \frac{\mu}{2\mu_{eff}} \lambda^2 v^2 - 2{m'_S}^2$$

$$\mathcal{M}_{P,12}^2 = \frac{v}{\sqrt{2}}(\lambda A_\lambda - 2\kappa \mu_{eff} - \lambda \mu') \equiv \frac{\mathbf{\lambda v}}{\sqrt{\mathbf{2}}}(\mathbf{A_\lambda - m_N}).$$

- In the limit $\lambda \to 0$, matrix decomposition: $2 \times 2 = 1 \oplus 1$, **singlet decoupled**!
- m_A: hevay doublet mass scale, $\quad m_B \equiv \sqrt{\mathcal{M}_{P,22}^2}$: CP-odd singlet Higgs mass.

$$m_3^2 = \frac{\lambda \mathbf{m_A^2} \sin 2\beta - 2\kappa \mu_{\mathrm{eff}}^2 - 2\lambda \mu_{\mathrm{eff}} \mu' - 2\lambda \mu_{\mathrm{eff}} A_\lambda}{2\lambda}$$

$$m_S'^2 = -\frac{1}{2}\left[\mathbf{m_B^2} + \frac{\mu}{2\mu_{\mathrm{eff}}} \lambda^2 v^2 + \frac{\kappa \mu_{\mathrm{eff}}}{\lambda}(3A_\kappa + \mu') - \frac{(\lambda A_\lambda + 4\kappa \mu_{\mathrm{eff}} + \lambda \mu') \sin 2\beta}{4\mu_{\mathrm{eff}}} \lambda v^2 \right]$$

GNMSSM: Higgs Sector

CP-even Higgs mass matrix in bases $(H_{\text{NSM}}, H_{\text{SM}}, \text{Re}[S])$**:**

$$\mathcal{M}^2_{S,11} = m_A^2 + \frac{1}{2}(2m_Z^2 - \lambda^2 v^2)\sin^2 2\beta,$$

$$\mathcal{M}^2_{S,12} = -\frac{1}{4}(2m_Z^2 - \lambda^2 v^2)\sin 4\beta,$$

$$\mathcal{M}^2_{S,13} = -\frac{1}{\sqrt{2}}(\lambda A_\lambda + 2\kappa\mu_{eff} + \lambda\mu')v\cos 2\beta \equiv -\frac{\lambda}{\sqrt{2}}(A_\lambda + m_N)v\cos 2\beta,$$

$$\mathcal{M}^2_{S,22} = m_Z^2 \cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta,$$

$$\mathcal{M}^2_{S,23} = \frac{v}{\sqrt{2}}\left[2\lambda(\mu_{eff} + \mu) - (\lambda A_\lambda + 2\kappa\mu_{eff} + \lambda\mu')\sin 2\beta\right],$$

$$\equiv \frac{\lambda \mathbf{v}}{\sqrt{2}}\left[2\mu_{\mathbf{tot}} - (\mathbf{A}_\lambda + \mathbf{m_N})\sin 2\beta\right],$$

$$\mathcal{M}^2_{S,33} = \frac{\lambda(A_\lambda + \mu')\sin 2\beta}{4\mu_{eff}}\lambda v^2 + \frac{\mu_{eff}}{\lambda}(\kappa A_\kappa + \frac{4\kappa^2\mu_{eff}}{\lambda} + 3\kappa\mu') - \frac{\mu}{2\mu_{eff}}\lambda^2 v^2,$$

- In the limit $\lambda \to 0$, matrix decomposition: $3 \times 3 = 2 \oplus 1$, **singlet decoupled!**
- $m_C \equiv \sqrt{\mathcal{M}^2_{S,33}}$: CP-even singlet Higgs mass.

$$A_\kappa = \frac{\mathbf{m_C^2} + \frac{\mu}{2\mu_{\text{eff}}}\lambda^2 v^2 - \frac{\lambda(A_\lambda + \mu')sin2\beta}{4\mu_{\text{eff}}}\lambda v^2 - \frac{4\kappa^2}{\lambda^2}\mu_{\text{eff}}^2 - \frac{3\kappa}{\lambda}\mu_{\text{eff}}\mu'}{\frac{\mu_{\text{eff}}}{\lambda}\kappa}$$

GNMSSM: Input Parameters

Input parameters in the original Lagrangian:

- Soft-breaking masses: $m_{H_u}^2$, $m_{H_d}^2$, and m_S^2;
- Yukawa couplings in Higgs sector: λ and κ;
- Soft-breaking trilinear coefficients A_λ and A_κ;
- Bilinear mass parameters μ and μ', and their soft-breaking parameters m_3^2 and $m_S'^2$.

Physical inputs: λ, κ, $\tan\beta$, v_s, $m_{H^\pm}$, m_{h_s}, m_{A_s}, $m_{\tilde{\chi}_1^0}$, **and** μ_{tot}.

- Vacuum expectation values: v_u, v_d, v_s;
- Yukawa couplings in Higgs sector: λ and κ;
- Electroweakino masses: $m_{\tilde{\chi}_1^0} \simeq m_N$, and Higgsino mass μ_{tot};
- Higgs boson masses: $m_{H^\pm}^2 \simeq m_A^2$, $m_{A_s} \simeq m_B$, and $m_{h_s} \simeq m_C$;
- Soft-breaking trilinear coefficients A_λ, **which is an insensitive parameter for all observables.**

GNMSSM: Key Features

Important applications of the Singlino-dominated DM:

- **Singlet-dominated particles form a secluded DM sector:**
  Measured DM abundance is generated by
  - s-**wave process** $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to h_s A_s$, **occuring by**
    s-**channel exchange of Higgs bosons and**
    t-**channel exchange of neutralinos**:
    $$|C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_s}| = |C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A_s}| = \sqrt{2}\kappa \simeq 0.2 \times \left(\frac{m_{\tilde{\chi}_1^0}}{300 \text{ GeV}}\right)^{1/2};$$
  - p-**wave process** $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to h_s h_s, A_s A_s$, **via adjusting** κ;
  - h_s/A_s-**funnels, via adjusting** m_{h_s}/m_{A_s}.

- **DM-nucleon scatterings suppressed by** λ^4:
  Current LZ experiment requires $\lambda \lesssim 0.1$. **Future DD expt. will further suppress** λ, **but not affect GNMSSM phenomenology.**

- **Bayesian analyses:**
  DM is primarily preferred to be Singlino-dominated.

- **The simplest SUSY framework to realize secluded DM sector.**

GNMSSM: Other Distinct Features

Characteristics:

1. **Free from the domain wall and tadpole problems**;
2. **More stable vacuum than the MSSM**;

$$V_{\min}^{\text{des}} = \cdots - \frac{\kappa^2}{\lambda^4}\mu_{eff}^4 - \frac{1}{3}\frac{\kappa A_\kappa}{\lambda^3}\mu_{eff}^3.$$

3. **Significant alleviation of the LHC constraints**.
   Heavy sparticles prefer to decay into NLSP or NNLSP first. Their decay chains are thus lengthened and their decay products become more complex.

4. **Every EW parameter takes natural values**.
   Considering LZ + LHC + a_μ,
   Z_3-NMSSM: $m_{\tilde{\chi}_1^0} \gtrsim 260\text{GeV}, \ \mu \gtrsim 550\text{GeV}, \ v_s \gtrsim 2 \text{ TeV}$;
   GNMSSM: $\ m_{\tilde{\chi}_1^0} \gtrsim 100\text{GeV}, \ \mu_{tot} \gtrsim 200\text{GeV}, \ v_s < 1 \text{ TeV}$.

5. **Bayesian evidence is much larger than that of Z_3-NMSSM**.

Possible problems:

- What's the origin of the S field?
- Why is λ small? Is there other reason than the Higgs data for it?
- Which value A_λ is preferred?
- What's the origin of neutrino mass?

Solution:

R-symmetry: the largest subgroup of automorphism group of supersymmetry algebra which commutes with Lorenz group.

R-symmetry + Seesaw mechanism!

Secluded DM sector: $\hat{S}$ and $\hat{\nu}_R$ form ..., Higgs or Neutrino Portal. $\tilde{S}\tilde{S} \to \nu_R \bar{\nu}_R$ or $\tilde{\nu}_R \tilde{\nu}_R \to SS$.

Conclusions

1. Experimental data provides many hints to fundamental physics.

2. Global fit deepens greatly our understanding of new physics.

3. **The DM relic density prefers DM to participate in weak interactions, while the results of the DM direct detection experiments indicate that the interaction between DM and the nucleus is feeble at most.**
   Many DM theories fail to naturally explain this fact!

4. **WIMP crisis just means that the simplest realizations of the WIMP miracle are facing challenges** $\longrightarrow$
   More elaborate theories are encouraged:
   Secluded DM theories are most favored to account for this phenomenon.

Conclusions

1. Economic supersymmetric theories are facing increasingly strong experimental restrictions.
   More complex theories were constructed by us to alleviate the constraints.

2. **Occam razor was incorrectly applied to the NMSSM**. Specifically, the Z_3-NMSSM is too restricted to exhibit all the essential characteristics of the NMSSM.

3. **It is time to explore the phenomenology of GNMSSM**, which is one of the simplest supersymmetric theories to explain naturally current experiments.
   - **Signlino DM is primarily preferred by Bayesian statistics!**
   - **The GNMSSM has many distinct theoretical advantages!**

4. **Seemingly independent problems may have common physical origins!**
   Go forward to explore them with creative ideas and more sophisticated techniques.

thanks!