Recent status of theoretical studies for neutrinoless double beta decay

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第二届地下和空间粒子物理与宇宙物理前沿问题研讨会

## Outline

- Background
- Underlying new physics
- Mechanism at nucleon level
- NMEs from nuclear many-body calculations
- Attempts to measure the NME
- Conclusions and Outlook


## Background

## Background



- Nuclear pairing induces double beta decay
- Neutrinoless double beta decay reveals Majorana nature of neutrino


## Background

- A systematic way of describing neutrinoless double beta decay has been developed recently

Cirigliano 18'

## BSM-Models

## Ov $\beta \beta$ operators

## NMEs



- Advantages: better controlled errors and hierarchies

New Physics

## Mechanisms

- Different new physics models with broken lepton-number conservation could lead to this decay mode
- L-R symmetric models with see-saw

Mohapatra 81'

- R-parity violating SUSY

Hirsh 95'

- Extra dimension model

Dienes 99'

## New Physics

## Cirigliano 18'

## EFT is a useful tool to provide more complete description

- Dim-5 operator $\epsilon_{k l} \epsilon_{m n}\left(L_{k}^{T} \mathcal{C}^{(5)} C L_{m}\right) H_{l} H_{n}$
- Dim-7 operators

| Class 5 | $\psi^{4} D$ |
| :---: | :---: |
| $\mathcal{O}_{L L \bar{d} u D}^{(1)}$ | $\epsilon_{i j}\left(\bar{d} \gamma_{\mu} u\right)\left(L_{i}^{T} C\left(D^{\mu} L\right)_{j}\right)$ |
| Class 6 | $\psi^{4} H$ |
| $\mathcal{O}_{L L \bar{e} H}$ | $\epsilon_{i j} \epsilon_{m n}\left(\bar{e} L_{i}\right)\left(L_{j}^{T} C L_{m}\right) H_{n}$ |
| $\mathcal{O}_{L L Q \bar{d} H}^{(1)}$ | $\epsilon_{i j} \epsilon_{m n}\left(\bar{d} L_{i}\right)\left(Q_{j}^{T} C L_{m}\right) H_{n}$ |
| $\mathcal{O}_{L L Q \bar{d} H}^{(2)}$ | $\epsilon_{i m} \epsilon_{j n}\left(\bar{d} L_{i}\right)\left(Q_{j}^{T} C L_{m}\right) H_{n}$ |
| $\mathcal{O}_{L L \bar{Q} u H}$ | $\epsilon_{i j}\left(\bar{Q}_{m} u\right)\left(L_{m}^{T} C L_{i}\right) H_{j}$ |
| $\mathcal{O}_{\text {Leu } \bar{d} H}$ | $\epsilon_{i j}\left(L_{i}^{T} C \gamma_{\mu} e\right)\left(\bar{d} \gamma^{\mu} u\right) H_{j}$ |

- Dim-3 operator: mass
- Dim-6 operators

$$
\begin{aligned}
& \frac{2 G_{F}}{\sqrt{2}}\left(C_{\mathrm{VL}, i j}^{(6)} \bar{u}_{L} \gamma^{\mu} d_{L} \bar{e}_{R, i} \gamma_{\mu} C \bar{\nu}_{L, j}^{T}+C_{\mathrm{VR}, i j}^{(6)} \bar{u}_{R} \gamma^{\mu} d_{R} \bar{e}_{R, i} \gamma_{\mu} C \bar{\nu}_{L, j}^{T}\right. \\
& \quad+C_{\mathrm{SR}, i j}^{(6)} \bar{u}_{L} d_{R} \bar{e}_{L, i} C \bar{\nu}_{L, j}^{T}+C_{\mathrm{SL}, i j}^{(6)} \bar{u}_{R} d_{L} \bar{e}_{L, i} C \bar{\nu}_{L, j}^{T} \\
& \left.\quad+C_{\mathrm{T}, i j}^{(6)} \bar{u}_{L} \sigma^{\mu \nu} d_{R} \bar{e}_{L, i} \sigma_{\mu \nu} C \bar{\nu}_{L, j}^{T}\right)+ \text { h.c. }
\end{aligned}
$$

- Dim-7 operators
$\frac{2 G_{F}}{\sqrt{2} v}\left(C_{\mathrm{VL}, i j}^{(7)} \bar{u}_{L} \gamma^{\mu} d_{L} \bar{e}_{L, i} C i \overleftrightarrow{\partial}_{\mu} \bar{\nu}_{L, j}^{T}+C_{\mathrm{VR}, i j}^{(7)} \bar{u}_{R} \gamma^{\mu} d_{R} \bar{e}_{L, i} C i \not{\partial} \overleftrightarrow{\partial}^{\mu} \bar{\nu}_{L, j}^{T}\right)+$ h.c.
- Dim-9 operator

$$
\frac{1}{v^{5}} \sum_{i}\left[\left(C_{i \mathrm{R}}^{(9)} \bar{e}_{R} C \bar{e}_{R}^{T}+C_{i \mathrm{~L}}^{(9)} \bar{e}_{L} C \bar{e}_{L}^{T}\right) O_{i}+C_{i}^{(9)} \bar{e} \gamma_{\mu} \gamma_{5} C \bar{e}^{T} O_{i}^{\mu}\right]_{8}
$$

Matching at nucleon level

## Operators for free nucleons

## Cirigliano 18'

$$
\bar{N} \tau^{+}\left[\frac{l_{\mu}+r_{\mu}}{2} J_{V}^{\mu}+\frac{l_{\mu}-r_{\mu}}{2} J_{A}^{\mu}-s J_{S}+i p J_{P}+t_{R \mu \nu} J_{T}^{\mu \nu}\right] N
$$



- Matching following xEFT technique for dim-6,7 operator

$$
\begin{aligned}
J_{V}^{\mu} & =g_{V}\left(\mathbf{q}^{2}\right)\left(v^{\mu}+\frac{p^{\mu}+p^{\prime \mu}}{2 m_{N}}\right)+\frac{i g_{M}\left(\mathbf{q}^{2}\right)}{m_{N}} \varepsilon^{\mu \nu \alpha \beta} v_{\alpha} S_{\beta} q_{\nu}, & s+i p & =-\frac{2 G_{F}}{\sqrt{2}}\left[C_{\mathrm{SL}}^{(6)}\left(\tau^{+}\right) \bar{e}_{L} C \bar{\nu}_{L}^{T}+C_{\mathrm{SR}}^{(6) *}\left(\tau^{-}\right) \nu_{L}^{T} C e_{L}\right], \\
J_{A}^{\mu} & =-g_{A}\left(\mathbf{q}^{2}\right)\left(2 S^{\mu}-\frac{v^{\mu}}{2 m_{N}} 2 S \cdot\left(p+p^{\prime}\right)\right)+\frac{g_{P}\left(\mathbf{q}^{2}\right)}{2 m_{N}} 2 q^{\mu} S \cdot q, & s-i p & =-\frac{2 G_{F}}{\sqrt{2}}\left[C_{\mathrm{SR}}^{(6)}\left(\tau^{+}\right) \bar{e}_{L} C \bar{\nu}_{L}^{T}+C_{\mathrm{SL}}^{(6) *}\left(\tau^{-}\right) \nu_{L}^{T} C e_{L}\right], \\
J_{S} & =g_{S}\left(\mathbf{q}^{2}\right), & l_{\mu} & =\frac{2 G_{F}}{\sqrt{2} v}\left(\tau^{+}\right)\left[-2 v V_{u d} \bar{e}_{L} \gamma_{\mu} \nu_{L}+v C_{\mathrm{VL}}^{(6)} \bar{e}_{R} \gamma_{\mu} C \bar{\nu}_{L}^{T}+C_{\mathrm{VL}}^{(7)} \bar{e}_{L} C i \not \overleftrightarrow{\partial}_{\mu} \bar{\nu}_{L}^{T}\right]+\text { h.c. } \\
J_{P} & =B \frac{g_{P}\left(\mathbf{q}^{2}\right)}{m_{N}} S \cdot q, & r_{\mu} & =\frac{2 G_{F}}{\sqrt{2} v}\left(\tau^{+}\right)\left[v C_{\mathrm{VR}}^{(6)} \bar{e}_{R} \gamma_{\mu} C \bar{\nu}_{L}^{T}+C_{\mathrm{VR}}^{(7)} \bar{e}_{L} C i \overleftrightarrow{\partial} \overleftrightarrow{\partial}_{\mu} \bar{\nu}_{L}^{T}\right]+\text { h.c. }, \\
J_{T}^{\mu \nu} & =-2 g_{T}\left(\mathbf{q}^{2}\right) \varepsilon^{\mu \nu \alpha \beta}\left(v_{\alpha}+\frac{p_{\alpha}+p_{\alpha}^{\prime}}{2 m_{N}}\right) S_{\beta}-i \frac{g_{T}^{\prime}\left(\mathbf{q}^{2}\right)}{2 m_{N}}\left(v^{\mu} q^{\nu}-v^{\nu} q^{\mu}\right) . & t_{R}^{\mu \mu} & =\frac{2 G_{F}}{\sqrt{2}}\left(\tau^{+}\right) C_{T}^{(6)} \bar{e}_{L^{\prime}} \sigma^{\mu \nu} C \bar{\nu}_{L}^{T},
\end{aligned}
$$

- Lepton parts are treated as external currents


## Operators for free nucleons

- For dim-9 operator

$$
\begin{aligned}
& \frac{F_{0}^{4}}{4}\left[\frac{5}{3} g_{1}^{\pi \pi} C_{1 \mathrm{~L}}^{(9)} L_{21}^{\mu} L_{21 \mu}+\left(g_{2}^{\pi \pi} C_{2 \mathrm{~L}}^{(9)}+g_{3}^{\pi \pi} C_{3 \mathrm{~L}}^{(9)}\right) \operatorname{Tr}\left(U \tau^{+} U \tau^{+}\right)\right. \\
& \left.+\left(g_{4}^{\pi \pi} C_{4 \mathrm{~L}}^{(9)}+g_{5}^{\pi \pi} C_{5 \mathrm{~L}}^{(9)}\right) \operatorname{Tr}\left(U \tau^{+} U^{\dagger} \tau^{+}\right)\right] \frac{\bar{e}_{L} C \bar{e}_{L}^{T}}{v^{5}}+(L \leftrightarrow R)
\end{aligned}
$$

$$
g_{A} g_{1}^{\pi N} C_{1 \mathrm{~L}}^{(9)} F_{0}^{2}\left[\bar{N} S^{\mu} u^{\dagger} \tau^{+} u N \operatorname{Tr}\left(u_{\mu} u^{\dagger} \tau^{+} u\right)\right] \frac{\bar{e}_{L} C \bar{e}_{L}^{T}}{v^{5}}
$$


$g_{1}^{N N} C_{1 \mathrm{~L}}^{(9)}\left(\bar{N} u^{\dagger} \tau^{+} u N\right)\left(\bar{N} u^{\dagger} \tau^{+} u N\right) \frac{\bar{e}_{L} C \bar{e}_{L}^{T}}{v^{5}}$
$+\left(g_{2}^{N N} C_{2 \mathrm{~L}}^{(9)}+g_{3}^{N N} C_{3 \mathrm{~L}}^{(9)}\right)\left(\bar{N} u^{\dagger} \tau^{+} u^{\dagger} N\right)\left(\bar{N} u^{\dagger} \tau^{+} u^{\dagger} N\right) \frac{\bar{e}_{L} C \bar{e}_{L}^{T}}{v^{5}}$
$+\left(g_{4}^{N N} C_{4 \mathrm{~L}}^{(9)}+g_{5}^{N N} C_{5 \mathrm{~L}}^{(9)}\right)\left(\bar{N} u^{\dagger} \tau^{+} u N\right)\left(\bar{N} u \tau^{+} u^{\dagger} N\right) \frac{\bar{e}_{L} C \bar{e}_{L}^{T}}{v^{5}}+(L \leftrightarrow R)$


## Expressions for nuclei

## Decay rate

## Cirigliano 18'



- The master formula for decay width ( $0^{+}->0^{+}$):

$$
\begin{aligned}
\left(T_{1 / 2}^{0 \nu}\right)^{-1}=g_{A}^{4}\left\{G_{01}\right. & \left(\left|\mathcal{A}_{\nu}\right|^{2}+\left|\mathcal{A}_{R}\right|^{2}\right)-2\left(G_{01}-G_{04}\right) \operatorname{Re} \mathcal{A}_{\nu}^{*} \mathcal{A}_{R}+4 G_{02}\left|\mathcal{A}_{E}\right|^{2} \\
& +2 G_{04}\left[\left|\mathcal{A}_{m_{e}}\right|^{2}+\operatorname{Re}\left(\mathcal{A}_{m_{e}}^{*}\left(\mathcal{A}_{\nu}+\mathcal{A}_{R}\right)\right)\right] \\
& -2 G_{03} \operatorname{Re}\left[\left(\mathcal{A}_{\nu}+\mathcal{A}_{R}\right) \mathcal{A}_{E}^{*}+2 \mathcal{A}_{m_{e}} \mathcal{A}_{E}^{*}\right] \\
& \left.+G_{09}\left|\mathcal{A}_{M}\right|^{2}+G_{06} \operatorname{Re}\left[\left(\mathcal{A}_{\nu}-\mathcal{A}_{R}\right) \mathcal{A}_{M}^{*}\right]\right\}
\end{aligned}
$$

## Decay rate

$$
\begin{aligned}
& \mathcal{A}_{\nu}=\frac{m_{\beta \beta}}{m_{e}} \mathcal{M}_{\nu}^{(3)}+\frac{m_{N}}{m_{e}} \mathcal{M}_{\nu}^{(6)}+\frac{m_{N}^{2}}{m_{e} v} \mathcal{M}_{\nu}^{(9)} \quad \mathcal{A}_{M}=\frac{m_{N}}{m_{e}} \mathcal{M}_{M}^{(6)}+\frac{m_{N}^{2}}{m_{e} v} \mathcal{M}_{M}^{(9)} \\
& \mathcal{A}_{E}=\mathcal{M}_{E, L}^{(6)}+\mathcal{M}_{E, R}^{(6)} \quad \mathcal{A}_{m_{e}}=\mathcal{M}_{m_{e}, L}^{(6)}+\mathcal{M}_{m_{e}, R}^{(6)} \quad \mathcal{A}_{R}=\frac{m_{N}^{2}}{m_{e} v} \mathcal{M}_{R}^{(9)}
\end{aligned}
$$

- M's here are the combinations of NMEs, for the neutrino mass mechanism, we have $\mathrm{M}_{\mathrm{F}}, \mathrm{M}_{\mathrm{G}}$ and $\mathrm{M}_{T}$

$$
\begin{aligned}
\mathcal{M}_{\nu}^{(3)}= & -V_{u d}^{2}\left(-\frac{1}{g_{A}^{2}} M_{F}+\mathcal{M}_{G T}+\mathcal{M}_{T}+2 \frac{m_{\pi}^{2} g_{\nu}^{N N}}{g_{A}^{2}} M_{F, s d}\right), \quad \mathcal{M}_{R}^{(9)}=\left.\mathcal{M}_{\nu}^{(9)}\right|_{L \rightarrow R} \\
\mathcal{M}_{\nu}^{(9)}= & -\frac{1}{2 m_{N}^{2}} C_{\pi \pi \mathrm{L}}^{(9)}\left(\frac{1}{2} M_{G T, s d}^{A P}+M_{G T, s d}^{P P}+\frac{1}{2} M_{T, s d}^{A P}+M_{T, s d}^{P P}\right) \\
& +\frac{m_{\pi}^{2}}{2 m_{N}^{2}} C_{\pi N \mathrm{~L}}^{(9)}\left(M_{G T, s d}^{A P}+M_{T, s d}^{A P}\right)-\frac{2}{g_{A}^{2}} \frac{m_{\pi}^{2}}{m_{N}^{2}} C_{N N \mathrm{~L}}^{(9)} M_{F, s d}
\end{aligned}
$$

- Many approximations are used here


## Decay rate

- Mf, Mgt and Mt are the long range Fermi, Gamow-Teller and tensor parts we are familiar with

$$
\mathcal{M}_{G T}=M_{G T}^{A A}+M_{G T}^{A P}+M_{G T}^{P P}+M_{G T}^{M M} \quad \mathcal{M}_{T}=M_{T}^{A P}+M_{T}^{P P}+M_{T}^{M M}
$$

- Where

$$
M_{I}^{K}=\langle f| \frac{2 R}{\pi} \int h_{I}^{K}(q) j_{I}(q r) \frac{q d q}{q+E_{N}} \widehat{O}_{I}|i\rangle
$$

- Short range NMEs are similar $M_{I, s d}^{K}=\langle f| \frac{2 R}{\pi} \int h_{I}^{K}(q) j_{I}(q r) \frac{q^{2} d q}{q+E_{N}} \mathcal{O}_{I}|i\rangle$
- All these M's can then be expressed in 15 NMEs
$M_{F} \quad M_{G T}^{A A} \quad M_{G T}^{A P} \quad M_{G T}^{P P} \quad M_{G T}^{M M} \quad M_{T}^{A A} \quad M_{T}^{A P} \quad M_{T}^{P P} \quad M_{T}^{M M}$
$M_{F, s d} \quad M_{G T, s d}^{A A} \quad M_{G T, s d}^{A P} \quad M_{G T, s d}^{P P} \quad M_{T, s d}^{A P} \quad M_{T, s d}^{P P}$


## Decay rate

- A comparison with LR symmetric model in traditional treatment where left- and right-handed neutrino are treated equally (short range mechanism neglected)

$$
\begin{aligned}
{\left[T_{1 / 2}^{0 \nu}\right]^{-1} } & =g_{A}^{4}\left|M_{G T}\right|^{2}\left\{C_{m m}\left(\frac{\left|m_{\beta \beta}\right|}{m_{e}}\right)^{2}+C_{m \lambda} \frac{\left|m_{\beta \beta}\right|}{m_{e}}\langle\lambda\rangle \cos \psi_{1}\right. \\
& \left.+C_{m \eta} \frac{\left|m_{\beta \beta}\right|}{m_{e}}\langle\eta\rangle \cos \psi_{2}+C_{\lambda \lambda}\langle\lambda\rangle^{2}+C_{\eta \eta}\langle\eta\rangle^{2}+C_{\lambda \eta}\langle\lambda\rangle\langle\eta\rangle \cos \left(\psi_{1}-\psi_{2}\right)\right\}
\end{aligned}
$$

- Where

$$
\begin{aligned}
C_{m m}= & \left(1-\chi_{F}+\chi_{T}\right)^{2} G_{01}, & C_{\eta \eta}= & \chi_{2+}^{2} G_{02}+\frac{1}{9} \chi_{1-}^{2} G_{011}-\frac{2}{9} \chi_{1-} \chi_{2+} G_{010}+\chi_{P}^{2} G_{08} \\
C_{m \lambda}= & -\left(1-\chi_{F}+\chi_{T}\right)\left[\chi_{2-} G_{03}-\chi_{1+} G_{04}\right], & & -\chi_{P} \chi_{R} G_{07}+\chi_{R}^{2} G_{09}, \\
C_{m \eta}= & \left(1-\chi_{F}+\chi_{T}\right)\left[\chi_{2+} G_{03}-\chi_{1-} G_{04}\right. & C_{\lambda \eta}= & -2\left[\chi_{2-} \chi_{2+} G_{02}-\frac{1}{9}\left(\chi_{1+} \chi_{2+}+\chi_{2-} \chi_{1-}\right) G_{010}\right. \\
& \left.-\chi_{P} G_{05}+\chi_{R} G_{06}\right], & & \left.+\frac{1}{9} \chi_{1+} \chi_{1-} G_{011}\right] . \\
C_{\lambda \lambda}= & \chi_{2-}^{2} G_{02}+\frac{1}{9} \chi_{1+}^{2} G_{011}-\frac{2}{9} \chi_{1+} \chi_{2-} G_{010}, & &
\end{aligned}
$$

## Decay

- The rich structures for these NMEs are presented

$$
\chi_{1 \pm}=\chi_{q G T}-6 \chi_{q T} \pm 3 \chi_{q F}, \quad \chi_{2 \pm}=\chi_{G T \omega}+\chi_{T \omega} \pm \chi_{F \omega}-\frac{1}{9} \chi_{1 \mp} .
$$

- These are terms from the helicity exchange terms in neutrino propagator

$$
\begin{aligned}
& M_{\omega F, \omega G T, \omega T}=\sum\left\langle A_{f}\left\|h_{\omega F, \omega G T, \omega T}\left(r_{-}\right) \mathcal{O}_{F, G T, T}\right\| A_{i}\right\rangle \\
& M_{q F, q G T, q T}=\sum\left\langle A_{f}\left\|h_{q F, q G T, q T}\left(r_{-}\right) \mathcal{O}_{F, G T, T}\right\| A_{i}\right\rangle
\end{aligned}
$$

- And also time-space components and recoil terms

$$
\begin{aligned}
M_{P} & =\sum i\left\langle A_{f}\left\|h_{P}\left(r_{-}\right) \tau_{r}^{+} \tau_{s}^{+} \frac{\left(\mathbf{r}_{-} \times \mathbf{r}_{+}\right)}{R^{2}} \cdot \vec{\sigma}_{r}\right\| A_{i}\right\rangle \\
M_{R} & =\sum\left\langle A_{f}\left\|\left[h_{R G}\left(r_{-}\right) \mathcal{O}_{G T}+h_{R T}\left(r_{-}\right) \mathcal{O}_{T}\right]\right\| A_{i}\right\rangle
\end{aligned}
$$

## Decay rate

- A comparison with LR symmetric model in traditional treatment where left- and right-handed neutrino are treated equally

$$
\begin{aligned}
{\left[T_{1 / 2}^{0 \nu}\right]^{-1} } & =g_{A}^{4}\left|M_{G T}\right|^{2}\left\{C_{m m}\left(\frac{\left|m_{\beta \beta}\right|}{m_{e}}\right)^{2}+C_{m \lambda} \frac{\left|m_{\beta \beta}\right|}{m_{e}}\langle\lambda\rangle \cos \psi_{1}\right. \\
& \left.+C_{m \eta} \frac{\left|m_{\beta \beta}\right|}{m_{e}}\langle\eta\rangle \cos \psi_{2}+C_{\lambda \lambda}\langle\lambda\rangle^{2}+C_{\eta \eta}\langle\eta\rangle^{2}+C_{\lambda \eta}\langle\lambda\rangle\langle\eta\rangle \cos \left(\psi_{1}-\psi_{2}\right)\right\}
\end{aligned}
$$

- A direct comparison with SMEFT haven't been done

$$
\begin{aligned}
\left(T_{1 / 2}^{0 \nu}\right)^{-1}=g_{A}^{4}\left\{G_{01}\right. & \left(\left|\mathcal{A}_{\nu}\right|^{2}+\left|\mathcal{A}_{R}\right|^{2}\right)-2\left(G_{01}-G_{04}\right) \operatorname{Re} \mathcal{A}_{\nu}^{*} \mathcal{A}_{R}+4 G_{02}\left|\mathcal{A}_{E}\right|^{2} \\
& +2 G_{04}\left[\left|\mathcal{A}_{m_{e}}\right|^{2}+\operatorname{Re}\left(\mathcal{A}_{m_{e}}^{*}\left(\mathcal{A}_{\nu}+\mathcal{A}_{R}\right)\right)\right] \\
& -2 G_{03} \operatorname{Re}\left[\left(\mathcal{A}_{\nu}+\mathcal{A}_{R}\right) \mathcal{A}_{E}^{*}+2 \mathcal{A}_{m_{e}} \mathcal{A}_{E}^{*}\right] \\
& \left.+G_{09}\left|\mathcal{A}_{M}\right|^{2}+G_{06} \operatorname{Re}\left[\left(\mathcal{A}_{\nu}-\mathcal{A}_{R}\right) \mathcal{A}_{M}^{*}\right]\right\}
\end{aligned}
$$

NME

## Cirigliano 17', Hyvarinen15', Barea 15', Horoi 18'

- NME correspondence in different references

| NMEs | Ref. $[76,84,85]$ | Ref. $[83]$ | Ref. $[32]$ |
| :---: | :---: | :---: | :---: |
| $M_{F}$ | $M_{F}$ | $M_{F}$ | $M_{F, F \omega, F q}$ |
| $M_{G T}^{A A}$ | $M_{G T}^{A A}$ | $M_{G T}^{A A}$ | $M_{G T \omega, G T q}$ |
| $M_{G T}^{A P}$ | $M_{G T}^{A P}$ | $M_{G T}^{A P}$ | $4 \frac{m_{e}}{B} M_{G T \pi \nu}+\frac{1}{3} M_{G T 2 \pi}$ |
| $M_{G T}^{P P}$ | $M_{G T}^{P P}$ | $M_{G T}^{P P}$ | $-\frac{1}{6} M_{G T 2 \pi}$ |
| $M_{G T}^{M M}$ | $r_{M}^{2} M_{G T}^{M M}$ | $M_{G T}^{M M}$ | $r_{M} \frac{g_{M}}{2 g_{A} g_{V} R_{A} m_{N}} M_{R}=\frac{g_{M}^{2}}{6 g_{A}^{2} R_{A} m_{N}} M_{G T^{\prime}}$ |
| $M_{T}^{A A}$ | $\boldsymbol{x}$ | $\boldsymbol{x}$ | $\boldsymbol{x}$ |
| $M_{T}^{A P}$ | $M_{T}^{A P}$ | $M_{T}^{A P}$ | $4 \frac{m_{e}}{B} M_{T \pi \nu}+\frac{1}{3} M_{T 2 \pi}$ |
| $M_{T}^{P P}$ | $M_{T}^{P P}$ | $M_{T}^{P P}$ | $-\frac{1}{6} M_{T 2 \pi}$ |
| $M_{T}^{M M}$ | $r_{M}^{2} M_{T}^{M M}$ | $M_{T}^{M M}$ | $-\frac{g_{M}^{2}}{12 g_{A}^{2} R_{A} m_{N}} M_{T}^{\prime}$ |
| $M_{F, s d}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{F, s d}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{F, s d}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{F N}=\frac{m_{N}}{R_{A} m_{\pi}^{2}} M_{F}^{\prime}$ |
| $M_{G T, s d}^{A A}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{A A}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{A A}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T N}=\frac{m_{N}}{R_{A} m_{\pi}^{2}} M_{G T}^{\prime}$ |
| $M_{G T, s d}^{A P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{A P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{A P}$ | $\frac{2}{3} M_{G T 1 \pi}$ |
| $M_{G T, s d}^{P P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{P P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{G T, s d}^{P P}$ | $\frac{1}{6}\left(M_{G T 2 \pi}-2 M_{G T 1 \pi}\right)$ |
| $M_{T, s d}^{A P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{T, s d}^{A P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{T, s d}^{A P}$ | $\frac{2}{3} M_{T 1 \pi}$ |
| $M_{T, s d}^{P P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{T, s d}^{P P}$ | $\frac{m_{e} m_{N}}{m_{\pi}^{2}} M_{T, s d}^{P P}$ | $\frac{1}{6}\left(M_{T 2 \pi}-2 M_{T 1 \pi}\right)$ |

## New Contributions

Cirigliano 18'

- The requirement of renormalizablility of $x$ EFT

- A contact operator need to be prompted to LO to cancel
 the divergence

- The operator is actually with a simple form

$$
V_{\nu, C T}=-2 g_{\nu}^{\mathrm{NN}} \tau^{(1)+} \tau^{(2)+}
$$

- The LEC needs to be determined

NMEs

## Approaches

- Modern nuclear structure calculations relay on our understanding of nuclear force and many-body correlations
- For the nuclear force used in many-body approaches:
- Realistic nuclear force - derived from bare nucleon force and softened by certain methods (ab initio)

IMSRG, ab initio CC, VSIMSRG+NSM, ...

- Phenomenological force - starting with certain symmetries and the parameters are fitted by nuclear properties


## Approaches

- Most traditional methods used in double beta decay calculations are based on phenomenological forces
- Shell Model (configuration interaction)
- DFT based on relativistic and non-relativistic mean-field
- GCM based on DFT
- QRPA based on DFT or phenomenological mean-field
- Geometric models without explicit inclusions of nuclear forces: $\mathrm{pSU}(3)$, IBM etc.


## Results

- The light neutrino mass mechanism has been in last decade well investigated although the new LO terms haven't been included
- It is impossible to give a complete list
- NSM: renormalization of operator; larger model space Caurier 12', Horoi 13', Menendez 14', Iwata 16', Menendez 18', Coraggio 20'
- QRPA: isospin symmetry restoration

Mustonen 13', Simkovic13', Hyvarinen 15', Fang 18' , Lv22'

- IBM: ISR

Barea 13', Barea15'

- PHFB

Sahu 15', Rath 19', Wang 21'

- DFT(+GCM): relativistic

Vaquero 13', Song14', Yao 15', Song17', Jiao 17', Wang 23'

## Results

- Compared to light neutrino mass mechanism, there are less on heavy neutrino mass
- SM: renormalization of operator; larger model space Horoi 13', Menendez 18'
- QRPA: isospin symmetry restoration

Hyvarinen 15', Fang 18'

- IBM: ISR

Barea15'

- PHFB

Rath 19'

- DFT+GCM: relativity

Song17'

## Results

- Deviations from different methods

Agostini 22'


- Different mechanisms have different deviation
- Originating from various sources


## Results

- Comparative studies between SM and EDF

Menendez 14'


- They come out with the conclusion, SM and EDF are similar at some level when seniority is 0 for SM and only spherical shape are assumed for EDF


## Results

$$
\begin{gathered}
M^{0 \nu}=[3.0(3)][1.2(2)][0.97(3)][1.12(7)]=3.9(8) \\
M^{0 N}=[155(10)][1.65(25)][0.80(20)][1.13(13)]=232(80)
\end{gathered}
$$



- comparative studies between SM and QRPA and estimations of errors


## Results

Yao 15', Hinohara 14'


- Experience from DFT+GCM


## Results

Agostini 22'


- Converged results are expected

Measuring NME

NME from experiments

- Are there any observables which can be related to the NMEs?
- Early attempts are to relate the Fermi NME with double Fermi transition or coulomb excitations

$$
M_{F}^{0 v} \approx-\frac{2}{e^{2}} \bar{\omega}_{\mathrm{IAS}}\left\langle 0_{f}\right| \hat{T}^{-}|\mathrm{IAS}\rangle\langle\mathrm{IAS}| \hat{T}^{-}\left|0_{i}\right\rangle
$$

## Rodin 09'

IAS


NME from experiments

- The idea of EM transitions from DIAS to ground states has been formulated with shell model recently

Romeo 21'


## NME from experiments

- Above results has a similar nucleon pair structure as double beta decay

Rebeiro 20'



- Two nucleon removal amplitude constrained with charge changing ( $\mathrm{p}, \mathrm{t}$ ) reactions


## NME from experiments

- Recently, the measurement of DGT for determinations of double beta decay matrix elements are proposed


- What they found in shell model calculations,


## NME from experiments

## Yao 22', Jokiniemi 23'



- Some claim a strong correlation between DGT or $2 v \beta \beta$ and $0 v \beta \beta$, while others doubt


## Conclusion

- New formalism of double beta decay based on SMEFT frame has been proposed
- But deviations are still presented
- Deviations among traditional many-body approaches are large and we are trying to understand the reason
- There are also efforts of constraining the NMEs from experiment side


## Outlook

- A more complete expression is needed for non-standard mechanisms
- With more powerful HPCs, we are confident that the calculations will be more precise
- More measurements will help determine the NMEs


## Thanks for your attention

