第三届地下和空间粒子物理与宇宙物理前沿问题研讨会

2024年5月7-11日，西昌

Beyond mean-field description of nuclear weak interaction processes

牛一斐

兰州大学

Outline

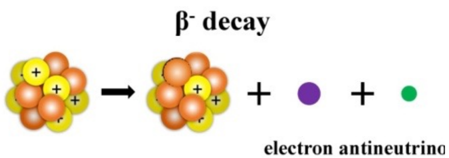
- **Nuclear weak-interaction processes**
- **Nuclear β decay**
- **Electron capture in stellar environment**
- **Nuclear double β decay**
- **Summary and Perspective**

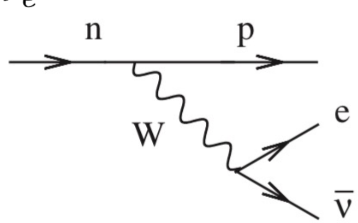
Nuclear weak-interaction processes: interdisciplinary study

- **β-decay**

Nuclear Physics

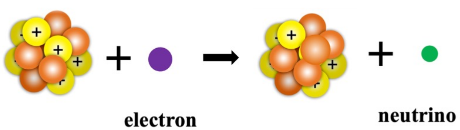
$$X_{(Z,N)} \rightarrow X^*_{(Z+1,N-1)} + e^- + \bar{\nu}_e$$

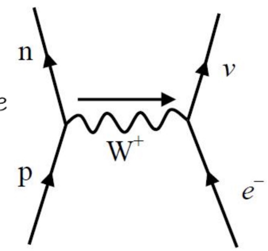




- **Electron capture**

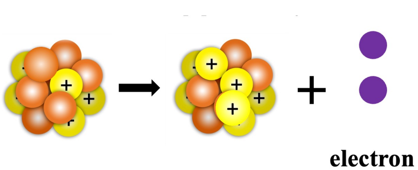
$$X_{(Z,N)} + e^- \rightarrow X^*_{(Z-1,N+1)} + \nu_e$$

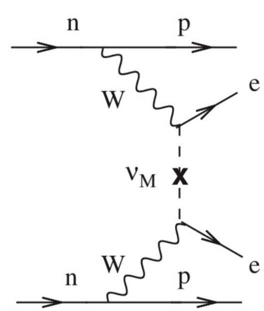




- **0νββ-decay**

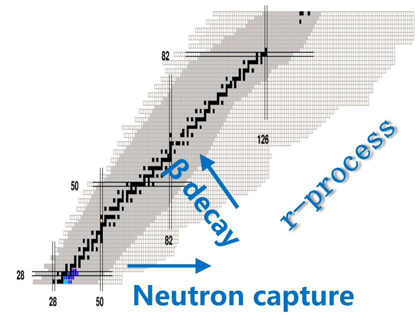
second order effect





Avignone, eta al., RMP 80, 481(2008)

Astrophysics



- Origin of heavy elements in the universe
- Supernova simulation

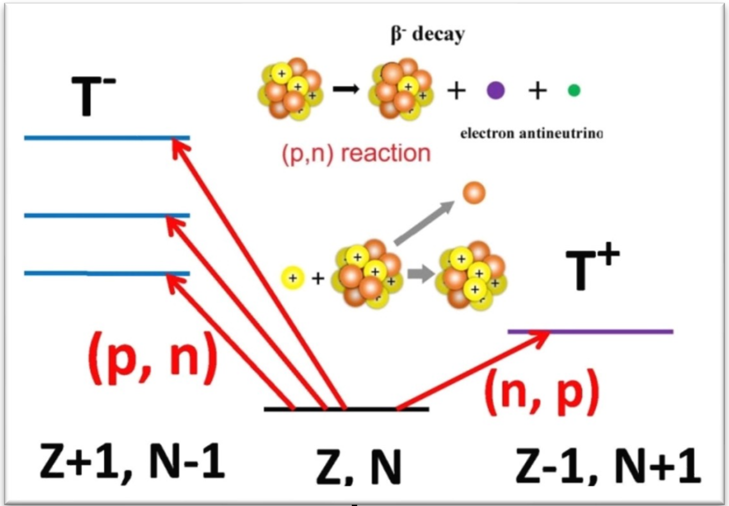
Particle Physics

- Neutrino

  Majorana or Dirac nature?
- Neutrino Mass
- Lepton number conservation

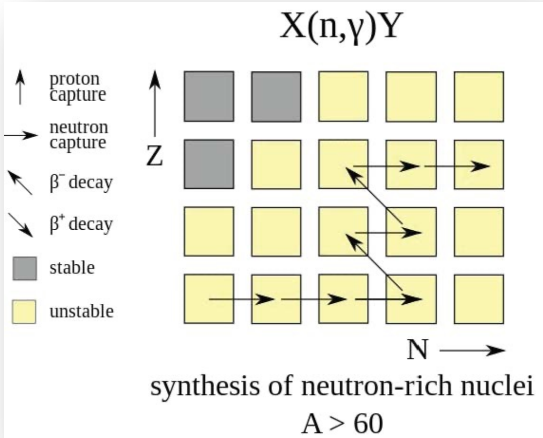
Nuclear weak-interaction process and charge-exchange transitions

Charge-exchange transition



Gamow Teller (GT)
Spin-Dipole (SD)
…

β-Decay



R-Process

Electron Capture
Neutrino-nucleus reaction

Supernova

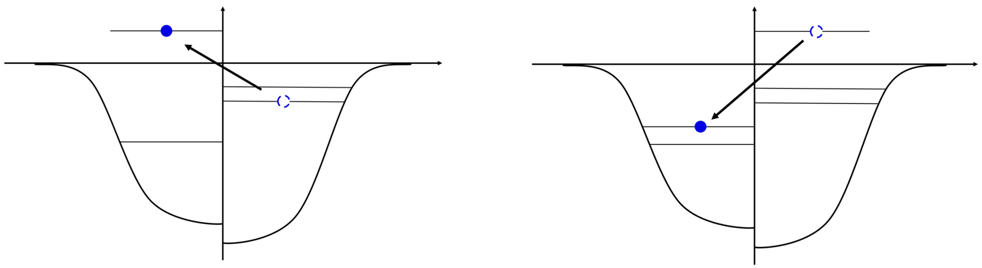
Random Phase Approximation (RPA) Model

- **proton-neutron RPA:**

widely used model for the description of charge-exchange transitions

✓ **The RPA excited state is generated by**

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi} a_i^\dagger a_m$$



Full 1p1h configuration space ⇒ almost whole nuclear chart

✓ **RPA: magic nuclei**

✓ **Quasiparticle RPA (QRPA): open-shell nuclei**

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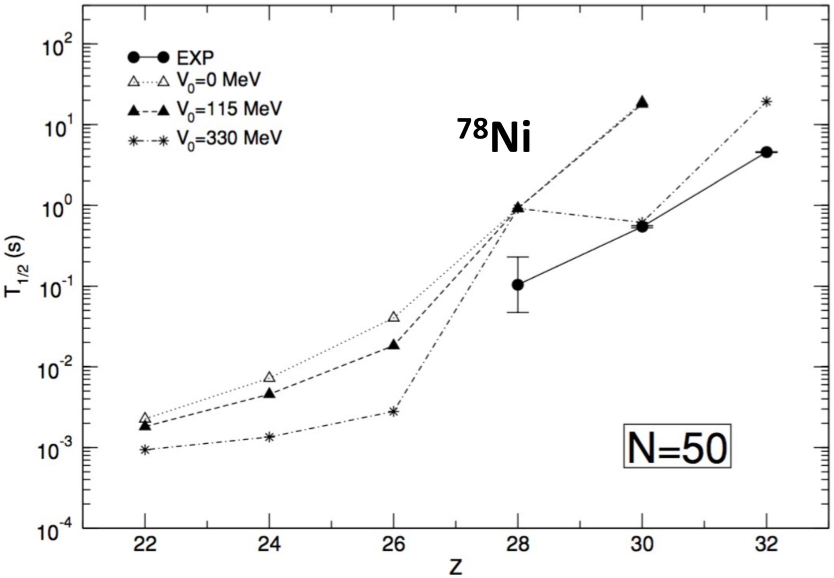
β-decay Half-life Is a Hard Problem

- **RHB+QRPA**

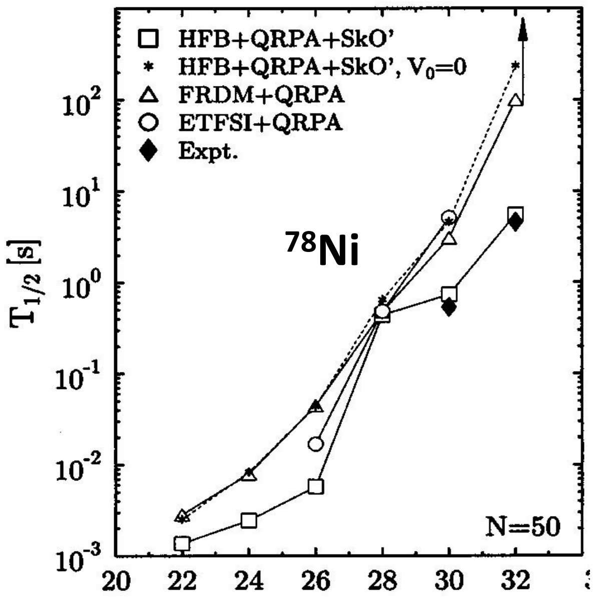
Self-consistent QRPA model based on relativistic density functional



Niksic, et al., PRC 71, 014308 (2005)

- **Skyrme HFB+QRPA**

Self-consistent QRPA model based on non-relativistic density functional

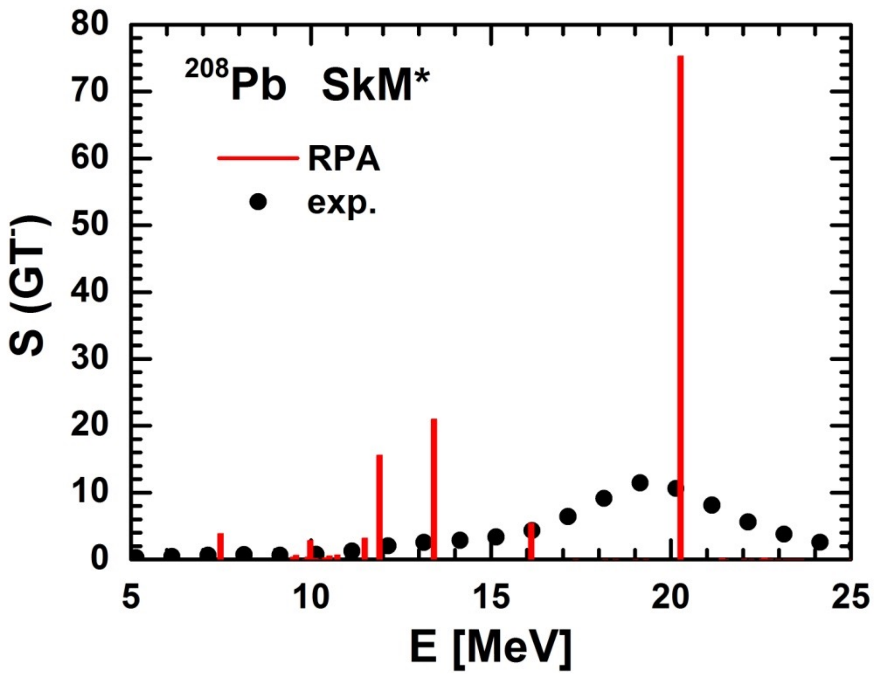


Engel, et al., PRC 60, 014302 (1999)

- **Half-lives are overestimated**
- **Due to the nuclear structure part – Gamow-Teller transition**

Limitations of (Q)RPA Description

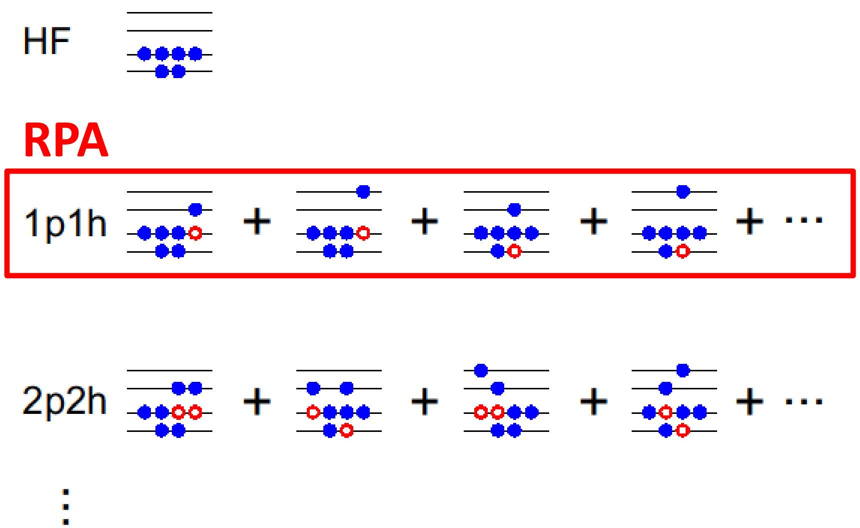
- **(Q)RPA cannot describe the spreading width**

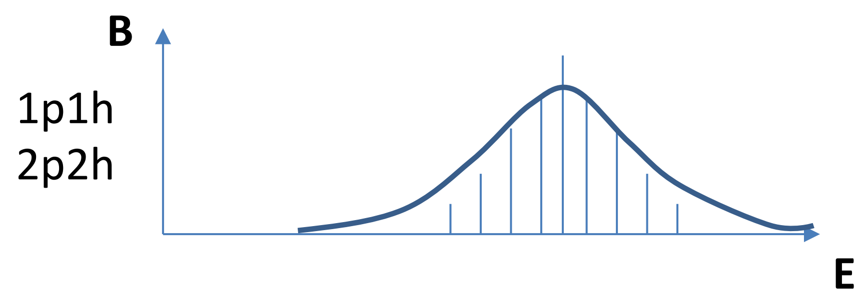


- **Spreading Width (Damping Width)**

  energy and angular momentum of coherent vibrations
  → more complicated states of 2p-2h, 3p-3h, … character

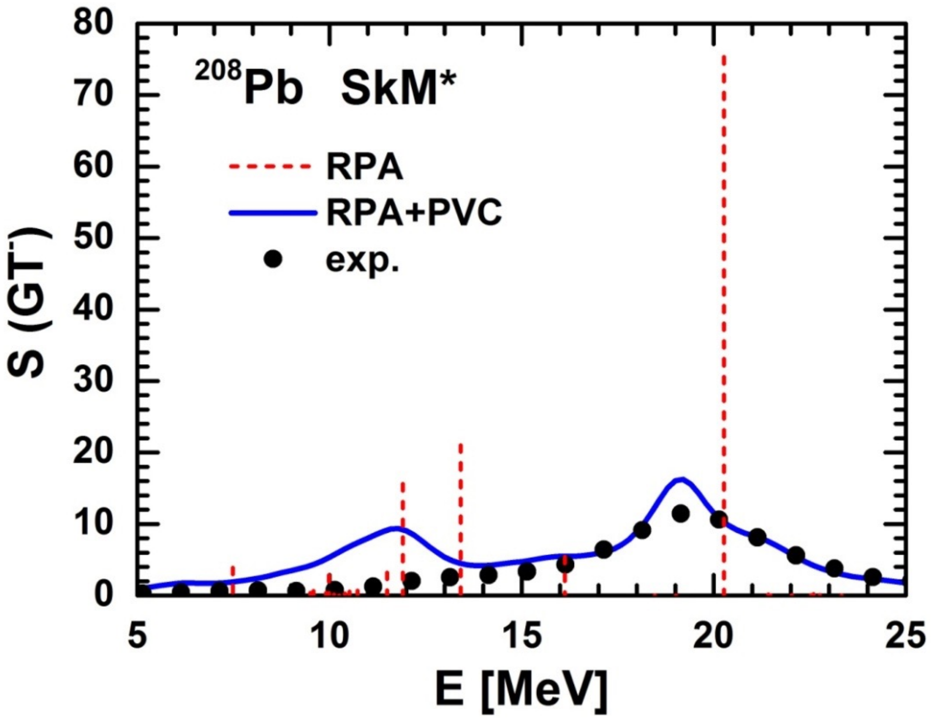
- **Correlations beyond RPA**



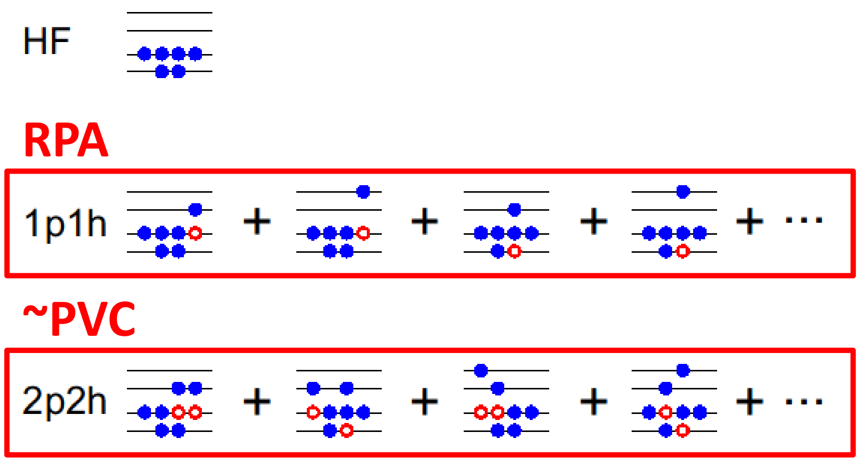


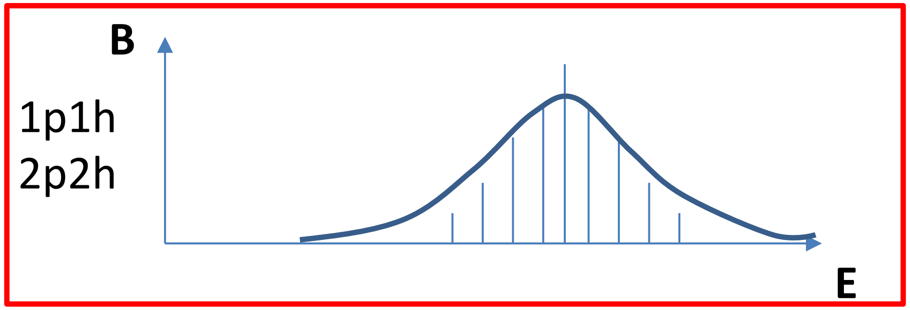
Solution: RPA + PVC

- **Particle Vibration Coupling (PVC) effect**



- **Correlations beyond RPA**





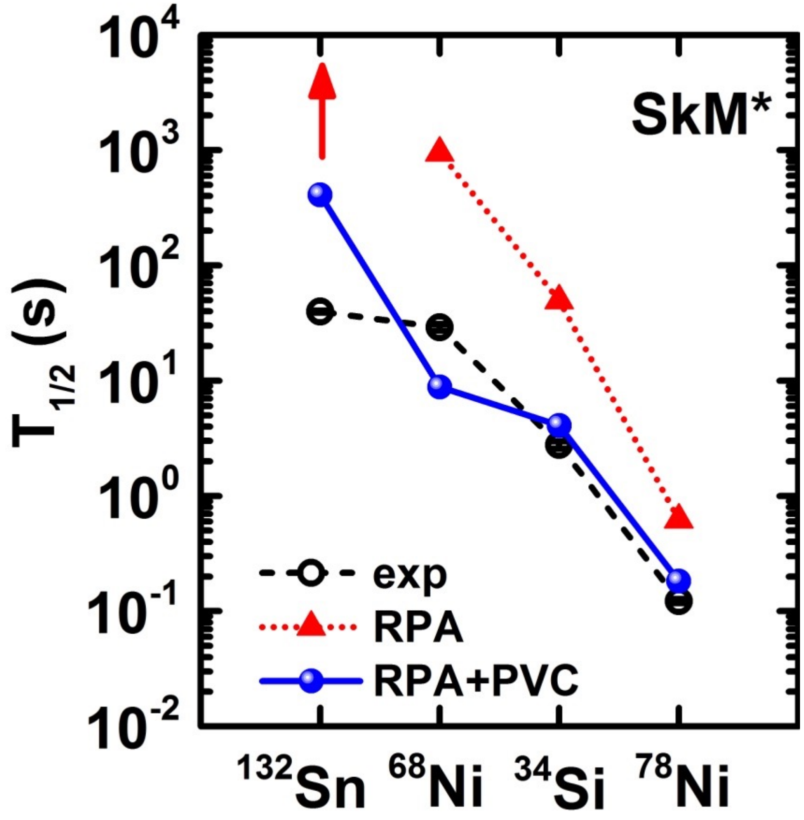
✓ **Develop a spreading width**

✓ **Reproduce resonance lineshape**

Y. F. Niu, G. Colo, and E. Vigezzi, **PRC** 90, 054328 (2014)

β-Decay Half-Lives in Magic Nuclei

- **Improved description of β-decay half-lives**

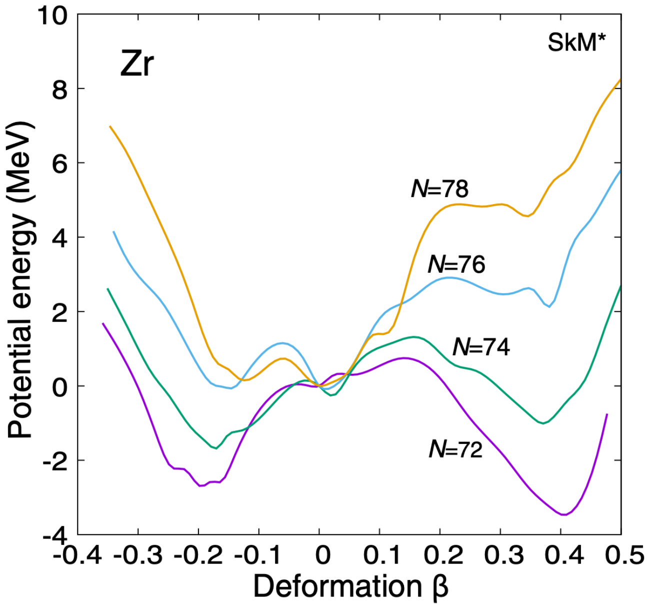


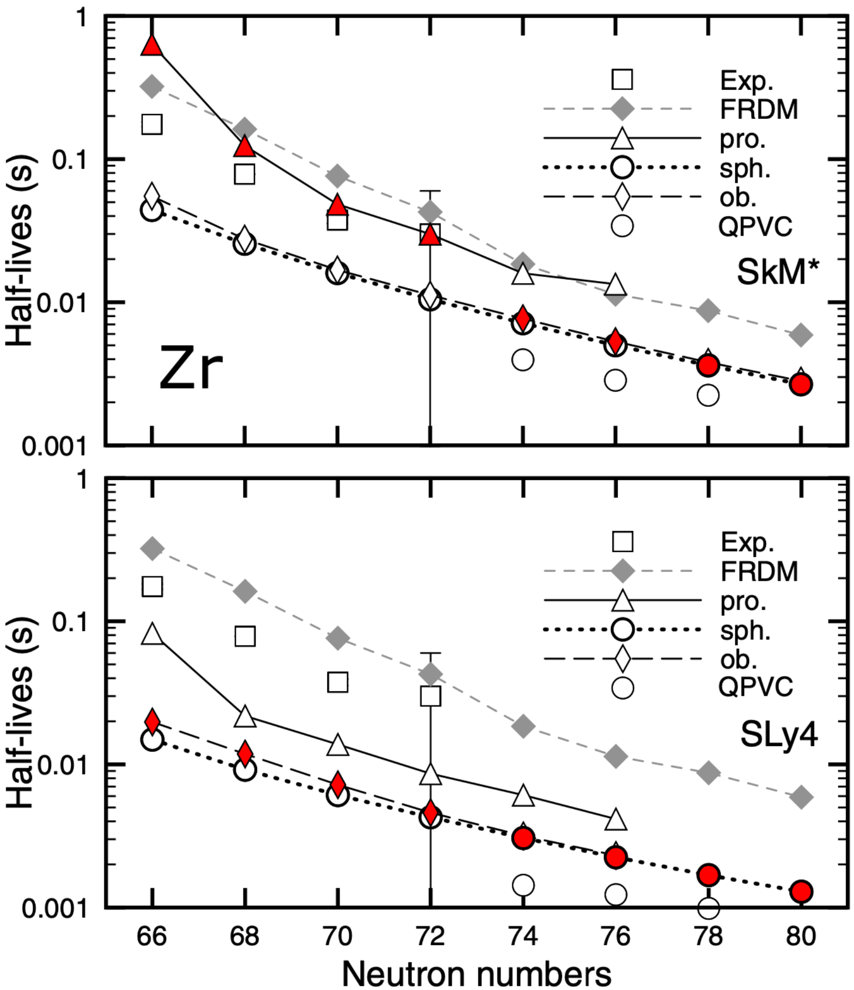
✓ **Reduce half-lives systematically** ✓ **Reproduce β-decay half-lives**

Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PRL** 114, 142501 (2015)

Exp: G. Audi, F. G. Kondev, M. Wang, W. J. Huang, and S. Naimi, CPC 41, 030001 (2017)

β-Decay Half-Lives as indicator of shape-phase transition





➢ A sudden shortening of β-decay half-lives is found at the nuclear shape changes

K. Yoshida, Y. F. Niu and F. Minato, Phys. Rev. C **108**, 034305 (2023)

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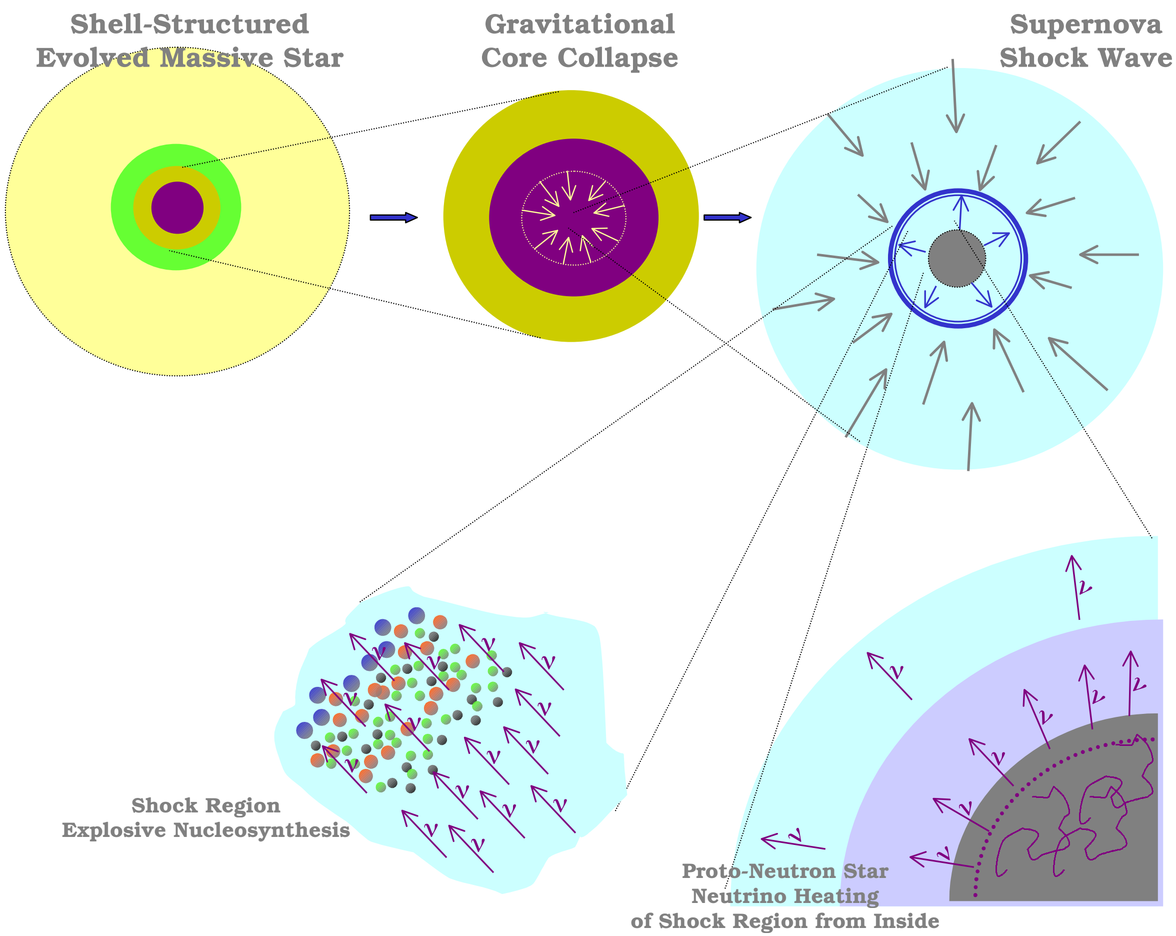
Electron capture in core-collapse supernova

Collapse of a massive star and a supernova explosion



Electron capture (EC) on nucleus

$${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}X^{*}_{N+1} + \nu_{e}$$

↓

change Y_e
change entropy

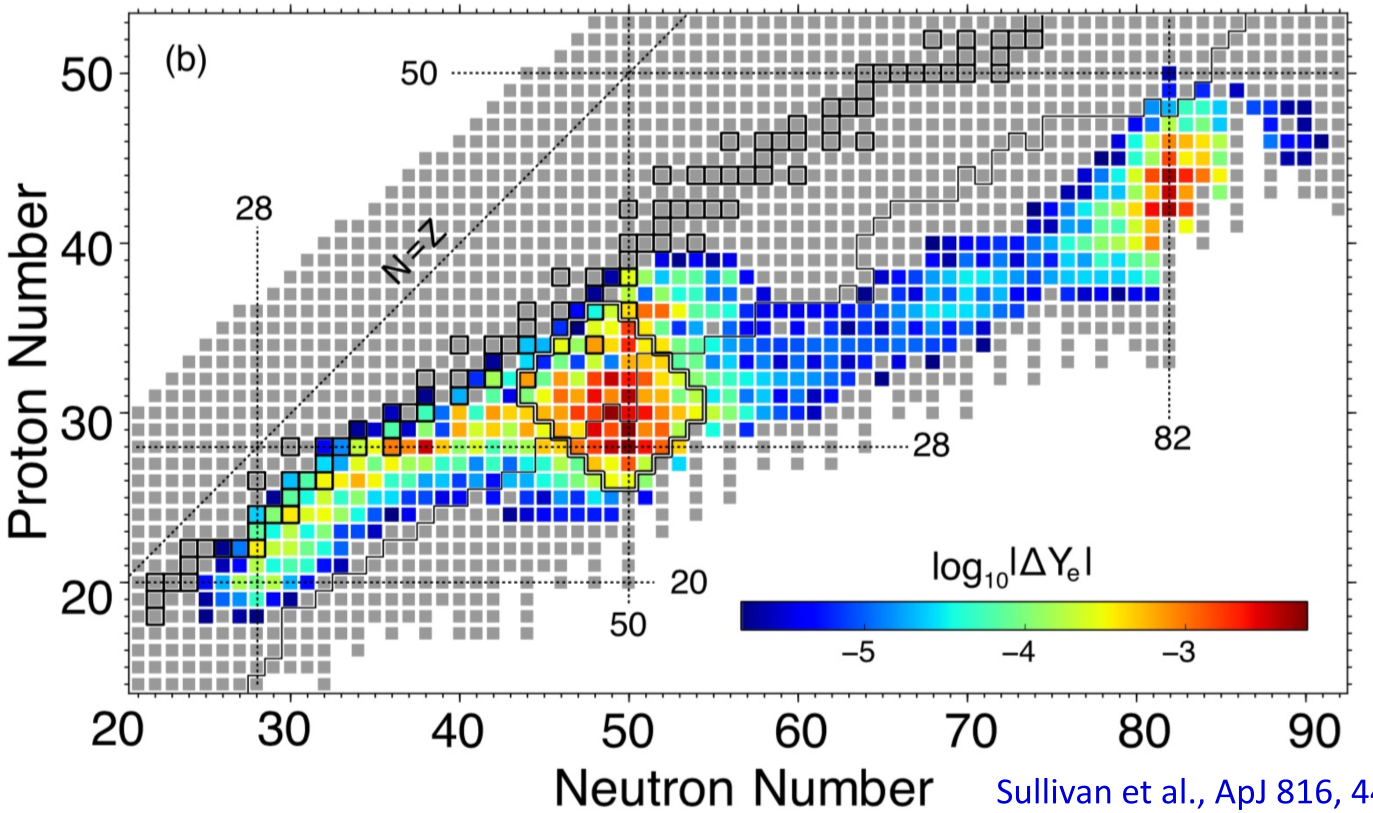
↓

affect the strength of bounce shock and supernova evolution

Langanke, Acta Physica Polonica B, 39, 2008

Important electron-capturing nuclei

Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction (Y_e) up to neutrino trapping



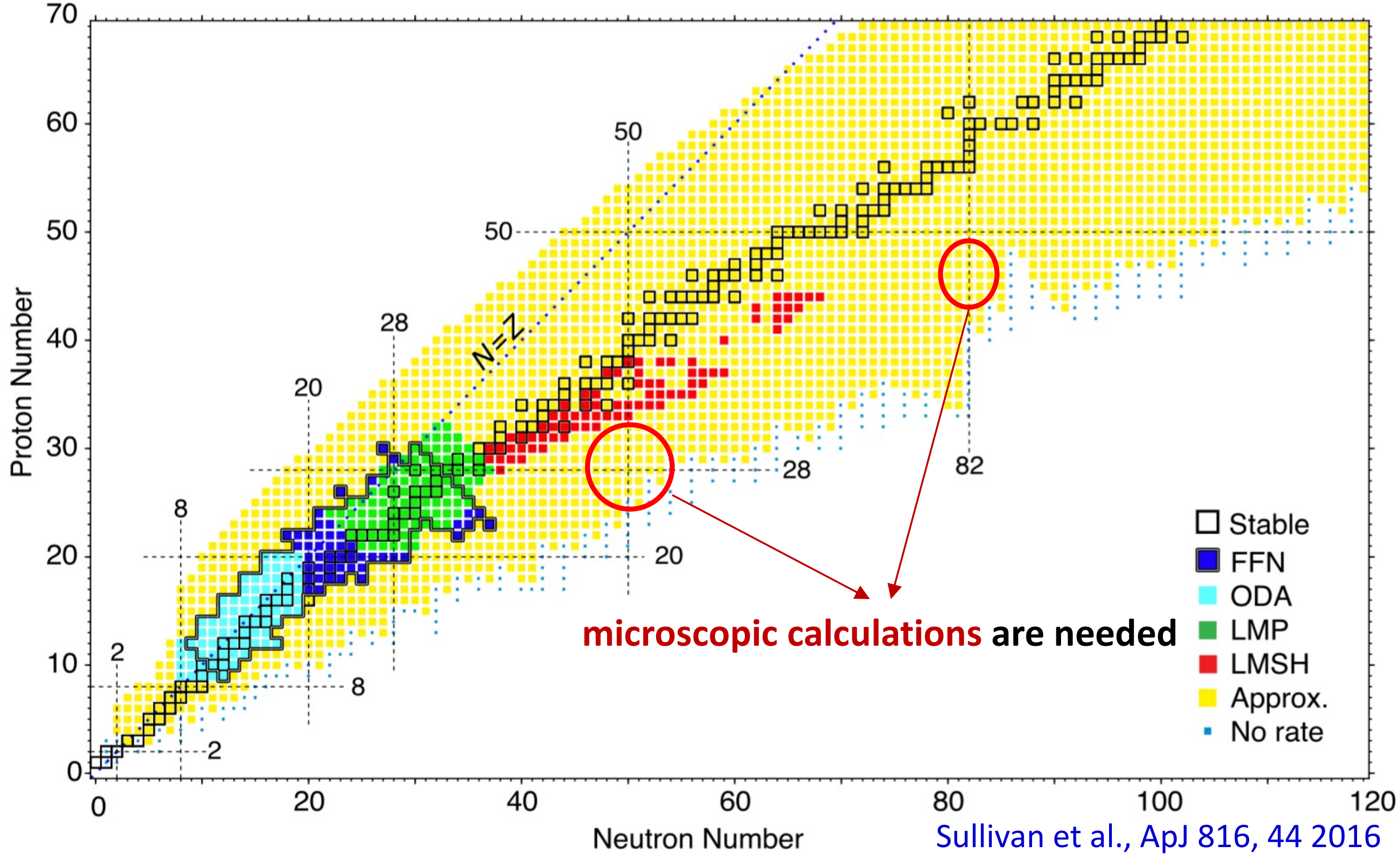
Sullivan et al., ApJ 816, 44 2016

- The integrated contribution to core deleptonization up to neutrino trapping

$$Y_e(t = t_{\text{trapping}}) \simeq Y_e(t = 0) - \sum_i \Delta Y_e^i$$

- Primary contributors: neutron rich nuclei near **N=50** and N=82 closed neutron shells

Theoretical study of electron-capture rates



Sullivan et al., ApJ 816, 44 2016

- Approx. - Approximate Rates estimated by $\lambda = \frac{(\ln 2)B}{K}\left(\frac{T}{m_e c^2}\right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$
- Fitted by shell model calculation for nuclei not far from stability line

  ⇒ For neutron rich nuclei, the formulas is not a good approximation

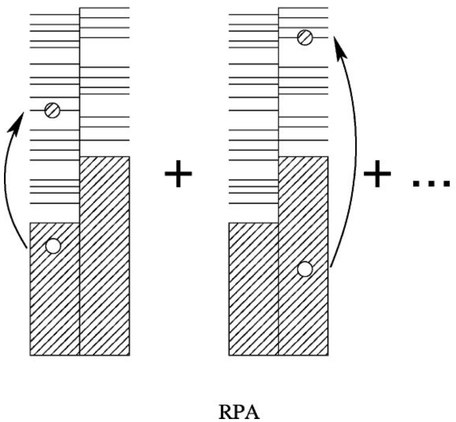
Langanke et al., PRL 90, 241102 (2003)

Random Phase Approximation (RPA)

- **RPA:** widely used for the description of spin-isospin excitations
  - **The RPA excited state is generated by**

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi} a_i^\dagger a_m$$

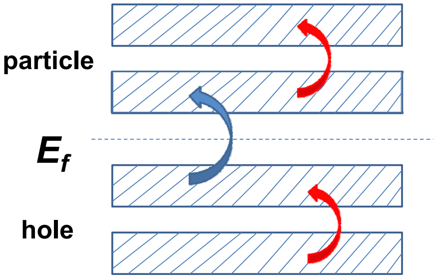
  - **Full 1p1h configuration space ⇒ almost whole nuclear chart**



To study the electron capture in core-collapse supernova, inclusion of temperature effect is necessary! (T ~ 0 – 2 MeV)

- **Finite Temperature RPA (FTRPA):** takes into account temperature self-consistently both in Hartree and RPA level
  - **Temperature is introduced by thermal occupation of each nucleon**

$$f_{p(n)} = \frac{1}{1 + \exp(\frac{\epsilon_{p(n)} - \mu_{p(n)}}{kT})}$$

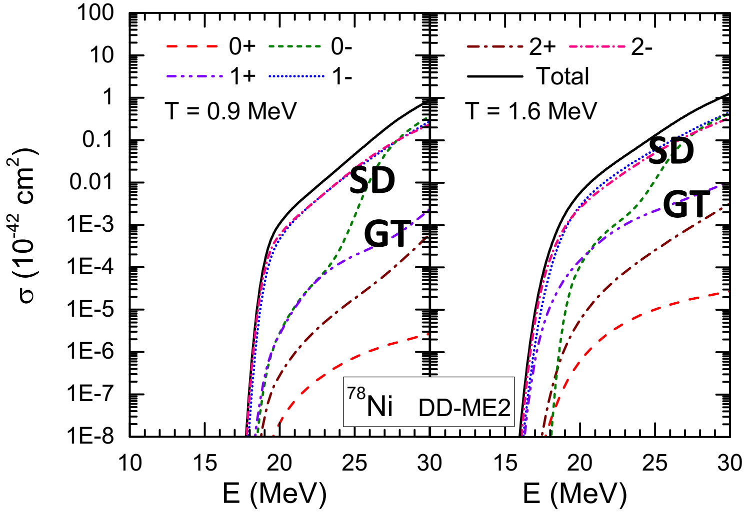


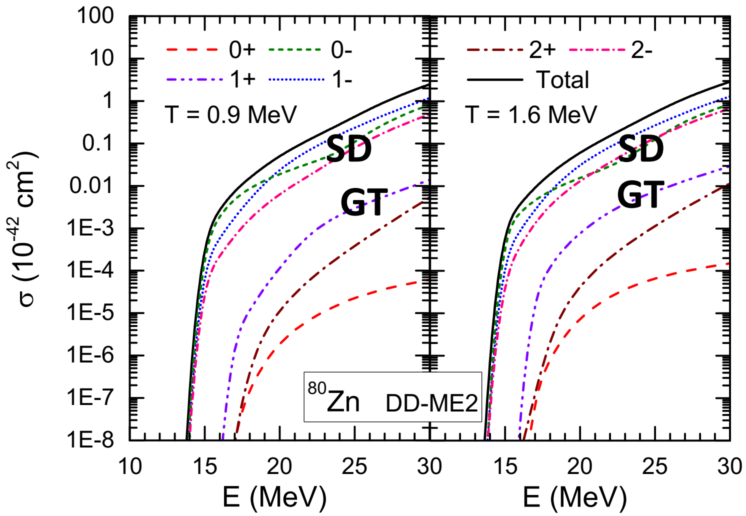
  - **Configuration space: p-h, p-p, and h-h pairs**

N. Paar et al., PRC 80, 055801 (2009)
Y. F. Niu et al., PLB 681, 315 (2009)

Electron capture cross sections

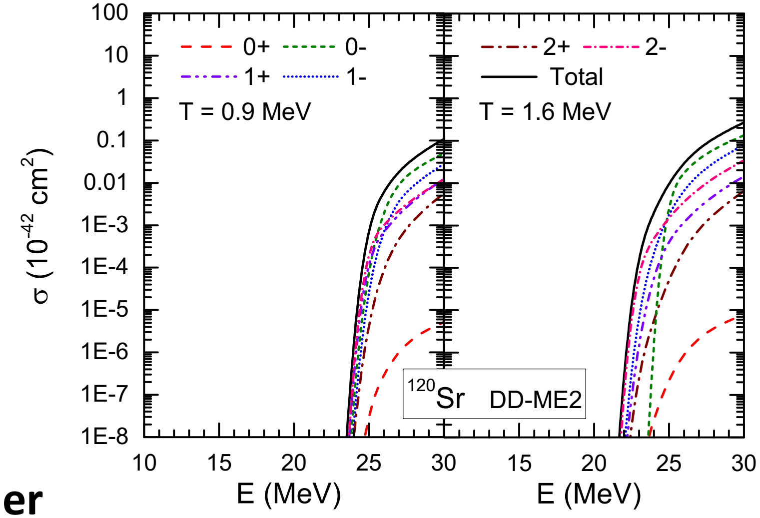
N = 50

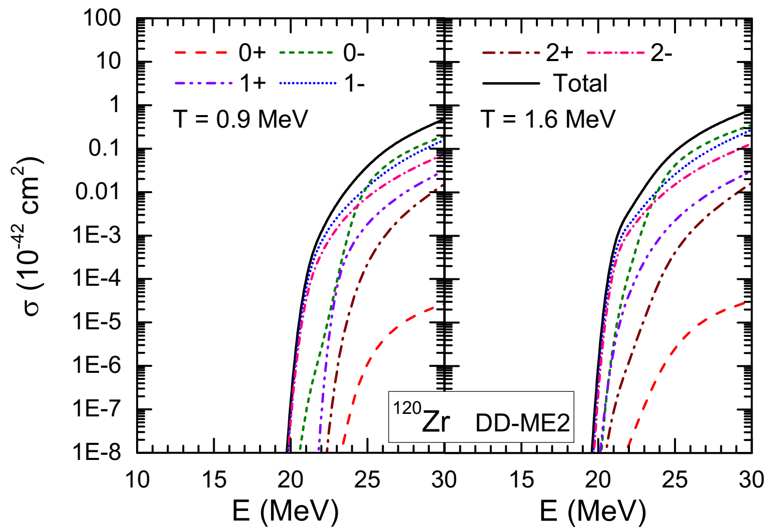




smaller →

N ≃ 82

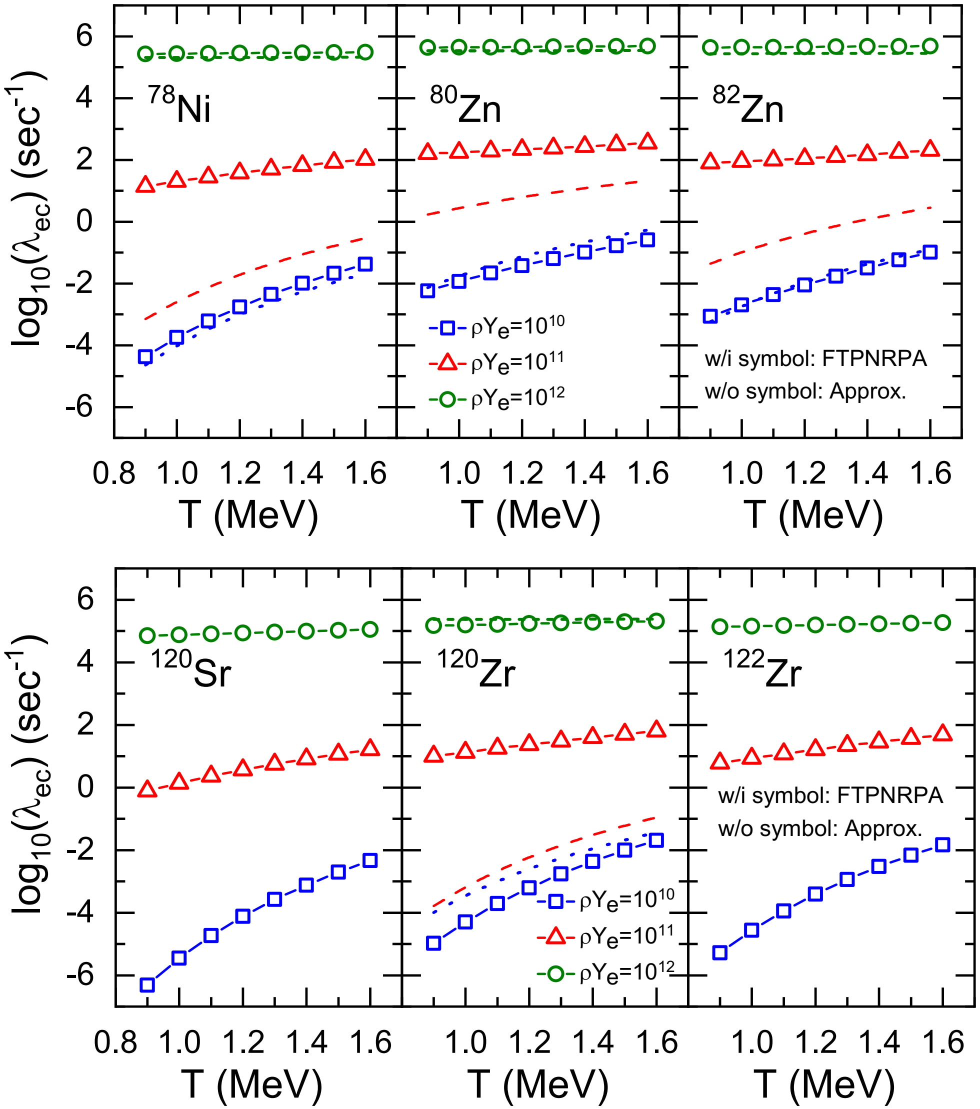




- For these neutron rich nuclei, spin dipole transitions dominate the cross section
- Even at high temperatures, GT transitions are not considerably unblocked

Electron capture rates

Electron capture rates at different stellar environment



- With the increase of electron density, the EC rates are increased by several orders of magnitude.
- At lower electron densities, the EC rates have big increase with temperature, but at high densities, the rate is not sensitive to temperature.
- Approx.

$$\lambda = \frac{(\ln 2)B}{K}\left(\frac{T}{m_e c^2}\right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$$

Rates from approximation formula at 10^{11} g/cm^3 is much underestemated compared to our results

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- Nuclear weak-interaction processes
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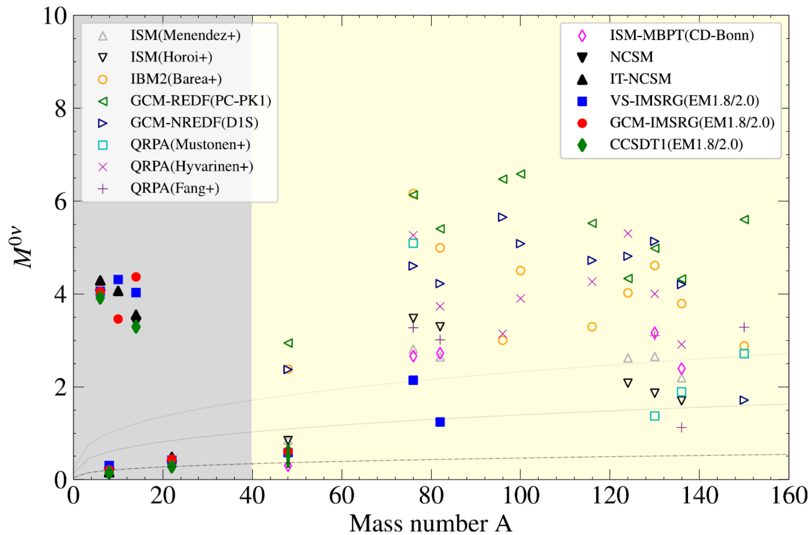
- Electron capture in stellar environment
- **Nuclear double β decay**
- Summary and Perspective

Neutrinoless double beta decay

- **Challenge to nuclear physicists**



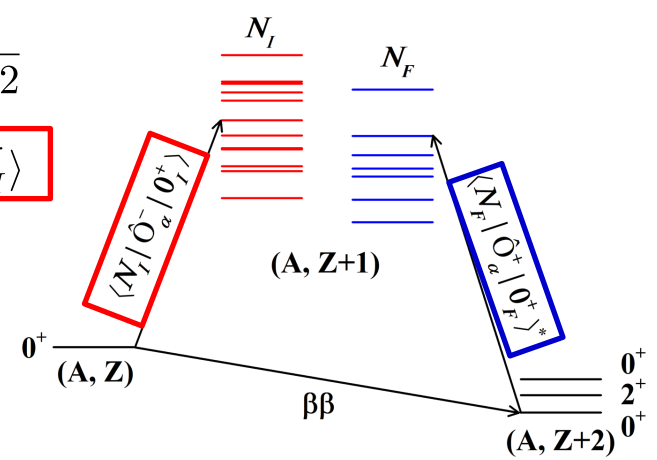
$$\left(T_{1/2}^{0\nu}\right)^{-1} = G_{0\nu}|M^{0\nu}|^2 m_{\beta\beta}^2$$

Discrepancies of nuclear matrix elements (NMEs) obtained by different nuclear models are large!

J. M. Yao, J. Meng, Y. F. Niu, and P. Ring, Prog. Phys. Nucl. Phys. 126, 103965 (2022)

- **NME** $M^{0\nu} \equiv -M_{\rm F}^{0\nu} + M_{\rm GT}^{0\nu} + M_{\rm T}^{0\nu}$

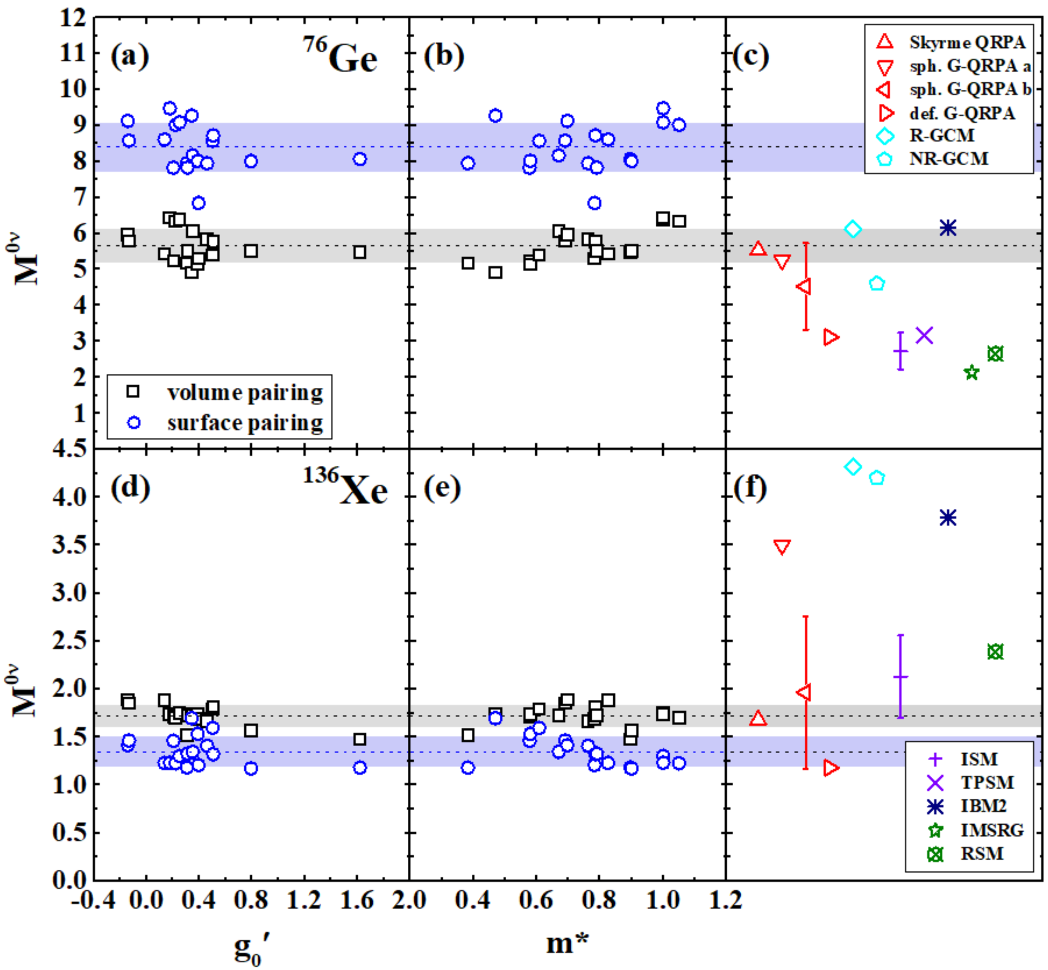
$$M^{o\nu} = \frac{8R_0}{g_A^2(0)} \int dqq \sum_{N_F N_I} \sum_{\alpha={\rm F,GT}} \sum_{LM} \sum_{m,n}^{A} \frac{h_\alpha(q^2)}{q + E_N - (E_I + E_F)/2} \langle N_F|\mathcal{O}^+_{\alpha,LM,m}|0_F^+\rangle \langle N_F|N_I\rangle \langle N_I|\mathcal{O}^-_{\alpha,LM,n}|0_I^\dagger\rangle$$



$$\mathcal{O}_{\rm F}^{\pm} = j_L(qr)Y_{LM}(\hat{\boldsymbol{r}})\tau^{\pm},$$
$$\mathcal{O}_{\rm GT}^{\pm} = j_L(qr)Y_{LM}(\hat{\boldsymbol{r}})\boldsymbol{\sigma}\tau^{\pm}.$$

$$h_F(\boldsymbol{q}^2) = -g_V^2$$
$$h_{GT}(\boldsymbol{q}^2) = g_A^2 - g_A g_P \frac{\boldsymbol{q}^2}{3m_p} + g_P^2 \frac{\boldsymbol{q}^4}{12m_p^2} + g_M^2 \frac{\boldsymbol{q}^2}{6m_p^2}$$

Uncertainties of $M^{0\nu}$ studied by Skyrme QRPA model



Nucleus	Volume pairing ^{76}Ge	Volume pairing ^{136}Xe	Surface pairing ^{76}Ge	Surface pairing ^{136}Xe
$\overline{M}^{0\nu}$	5.65	1.72	8.40	1.35
σ of $M^{0\nu}$	0.45	0.11	0.66	0.15

- Although the effective mass m^* and Landau parameter g_0' span a wide range, for each kind of pp interaction, σ is only around 10% of $\overline{M}^{0\nu}$.
- Uncertainties come from pairing interaction.

- 18 Skyrme interactions
- 2 pairing interactions

W. L. Lv, Y. F. Niu, D. L. Fang, J. M. Yao, C. L. Bai, and J. Meng, Phys. Rev. C 108, L051304 (2023)

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Summary and Perspectives

Summary

- Based on QRPA model and beyond, we have studied
  - ✓ nuclear beta decay half-lives
  - ✓ electron capture rates in stellar environment
  - ✓ NME of neutrinoless double beta decay

Perspective

- Based on QRPA model and beyond, we could also study
  - ✓ neutrino-nucleus scattering cross section
- The improvement by QPVC model
  - ✓ electron capture rates
  - ✓ NME of $0\nu\beta\beta$

Acknowledgement

Collaborators:

PKU: J. Meng

Anhui Uni.: Z. M. Niu

LZU: W. H. Long, W. L. Lv, L. Guo

IMP: D. L. Fang, X. D. Tang, B. S. Gao, K. A. Li

Sichuan Uni. : C. L. Bai

Milan Univ. : G. Colo, E. Vigezzi

Aizu Univ. and RIKEN: H. Sagawa

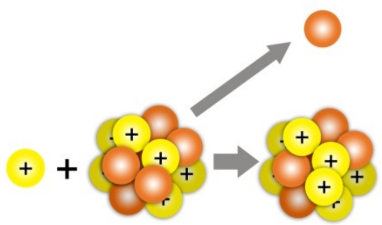
Zagreb Univ.: N. Paar, T. Niksic, D. Vretenar

Thank you!

Gamow-Teller Transitions (T⁻ direction)

- **Gamow-Teller Resonance (T⁻ direction)**

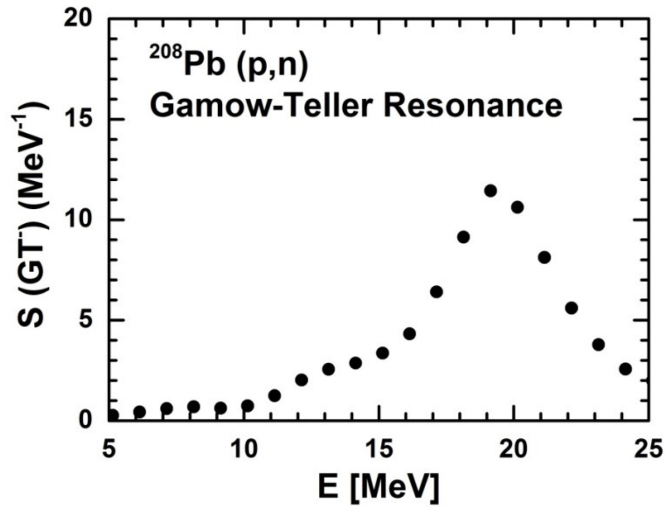
(p,n) reaction



$$\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$$

Strong interaction

Gamow-Teller transition



Wakasa, et al., PRC 85, 064606 (2012)

operator

$$\hat{O}_{\mathrm{GT}^-} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_-(i)$$

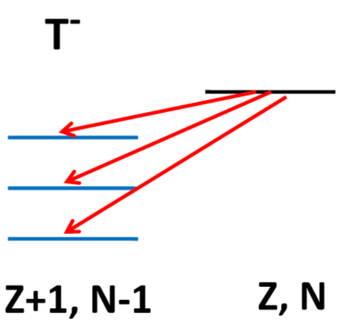
Transition probability

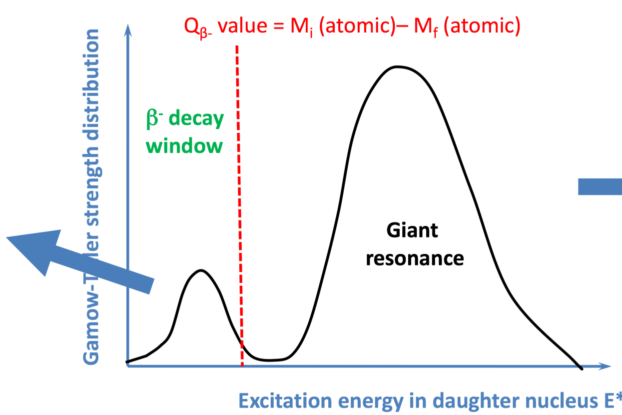
$$B(\mathrm{GT}^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$

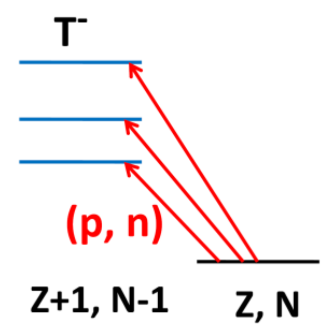
Transition strength S

= smoothed B

- **β-decay dominated by low-energy GT transition**



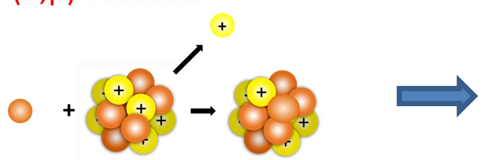




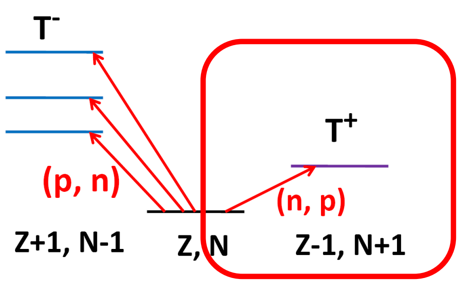
Gamow-Teller Transitions **(T^{+} direction)**

- **Gamow-Teller Resonance (T^{+} direction)**

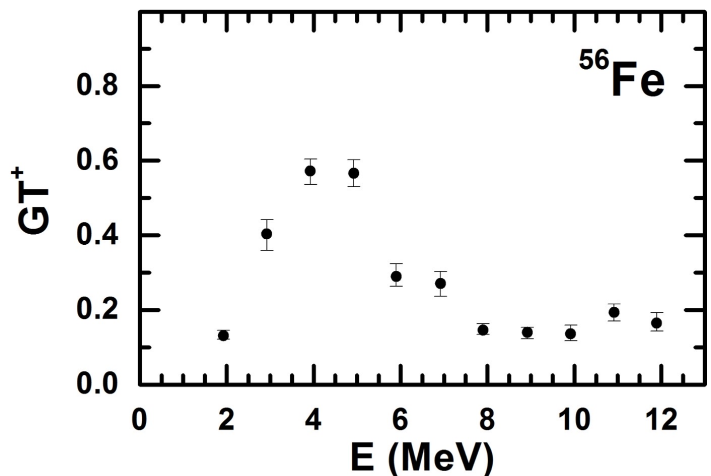
(n,p) reaction



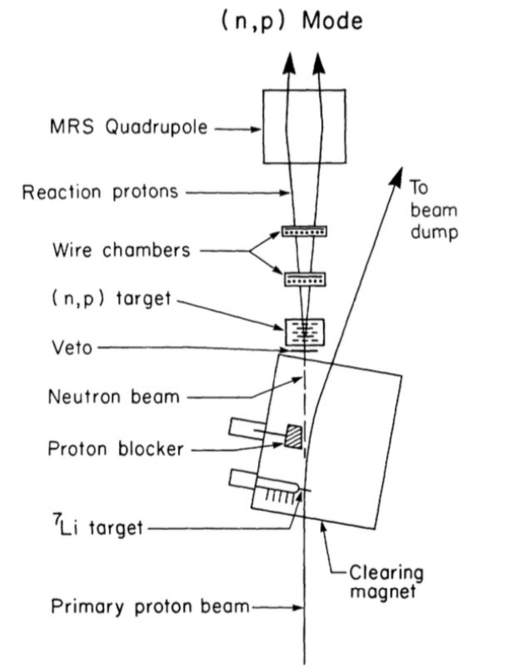
$$\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$$



El-Kateb, et al., PRC 49, 3128 (1994)

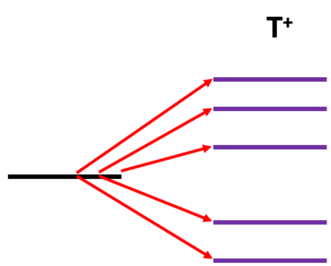


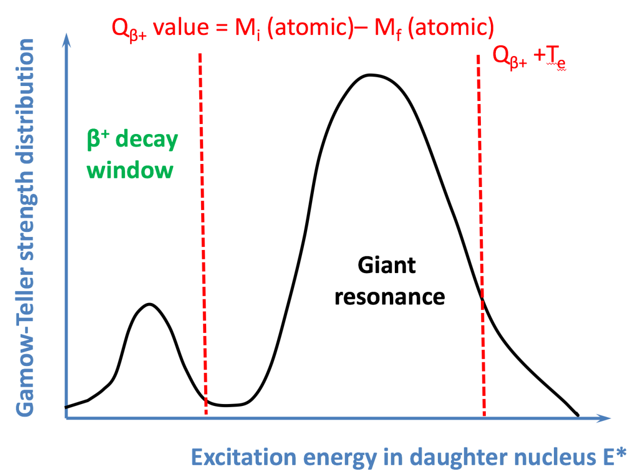
operator $$\hat{O}_{\mathrm{GT}+} = \sum_{i=1}^{A} \sigma(i)\tau_{+}(i)$$



Schematic layout of the TRIUMF (n,p) facility.

- **Electron Capture** **dominated by GT transition**





Stellar Environment

$\rho Ye = 3.75*10^{11}$ g/cm^3,

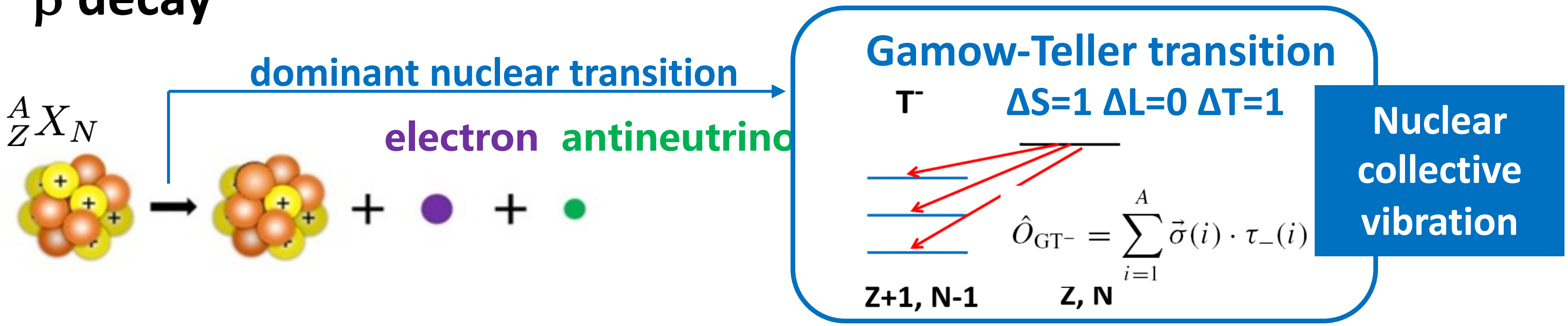
kT = 1.44 MeV

μ_e = 25.66 MeV

$$\rho Y_e = \frac{1}{\pi^2 N_A}\left(\frac{m_e c}{\hbar}\right)^3 \int_0^\infty (S_e - S_p) p^2\, dp$$

β Decay and Gamow-Teller transitions

- **β decay**



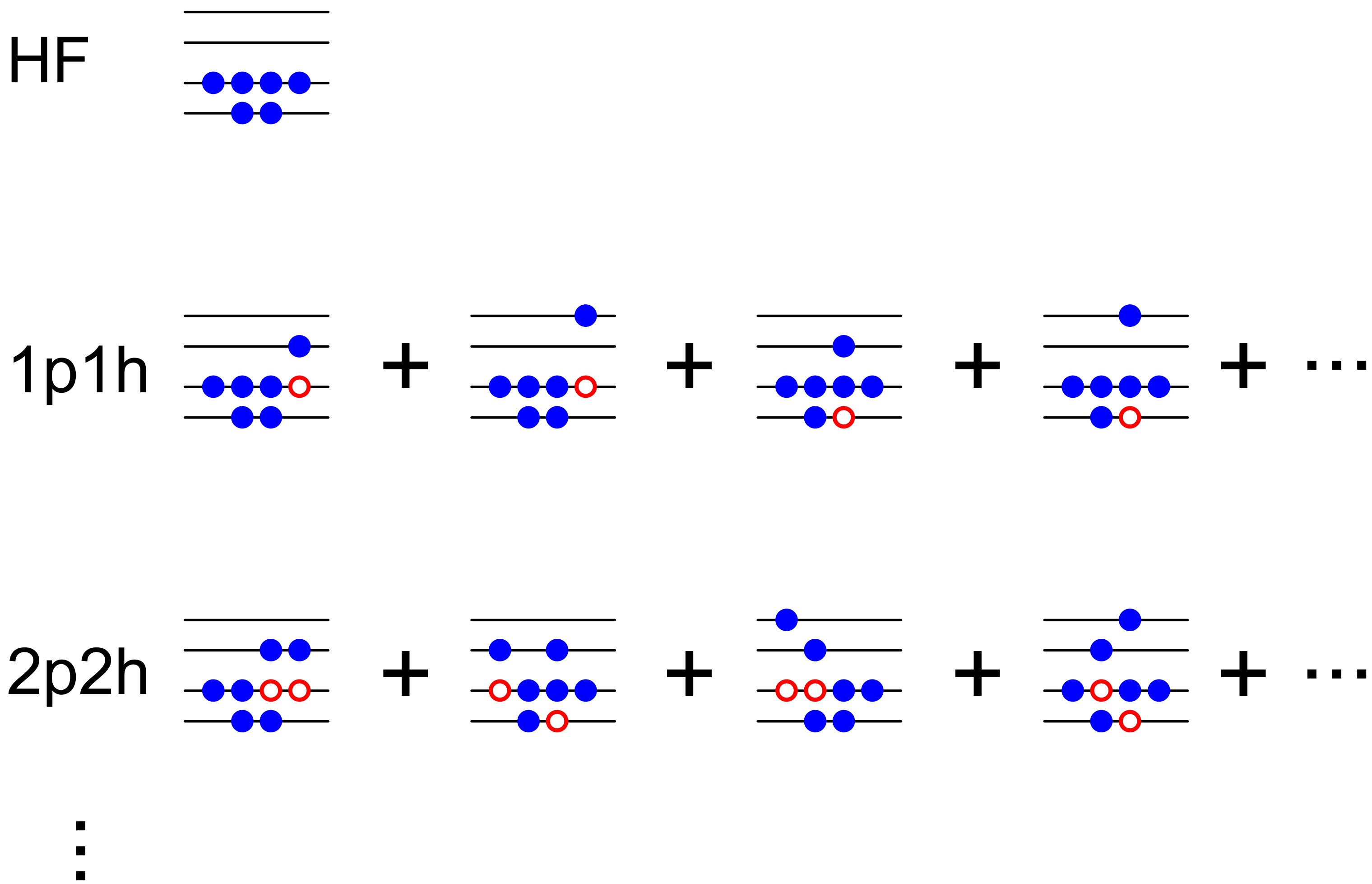
- **β-decay half life**

Weak decay constant (Particle Physics) × **Transition matrix element** × **Phase space (∝ decay energy E^5)** (Nuclear Physics)

The key is to describe nuclear collective vibrations correctly

Accurate nuclear model is needed!

Schematic picture for collective excitations



del

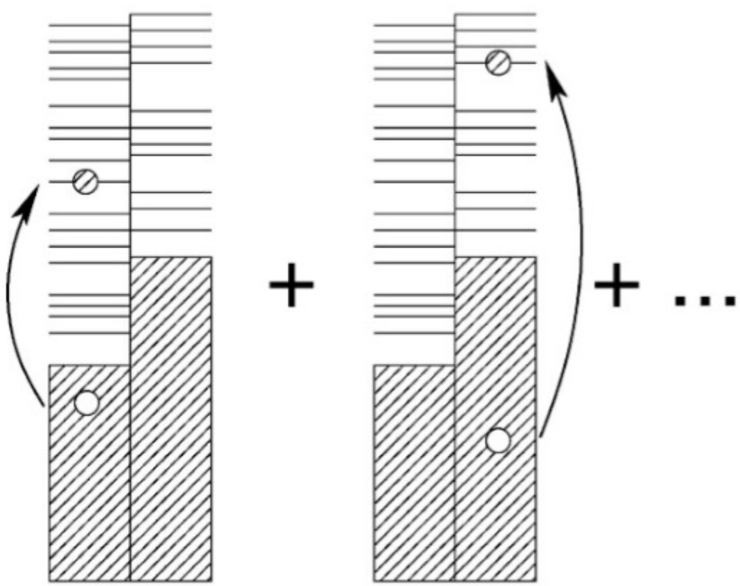
clei

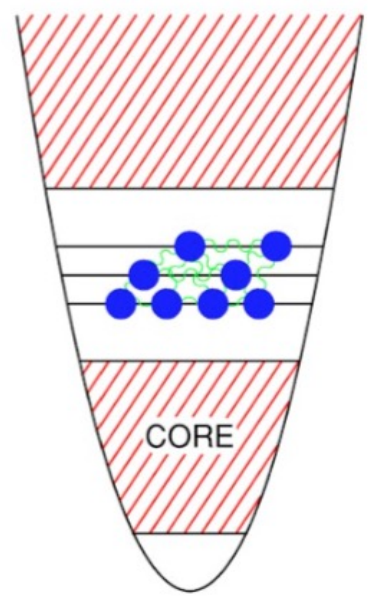
997; Caurier: RMP 2005

Phase Approximation (RPA) based on density funct

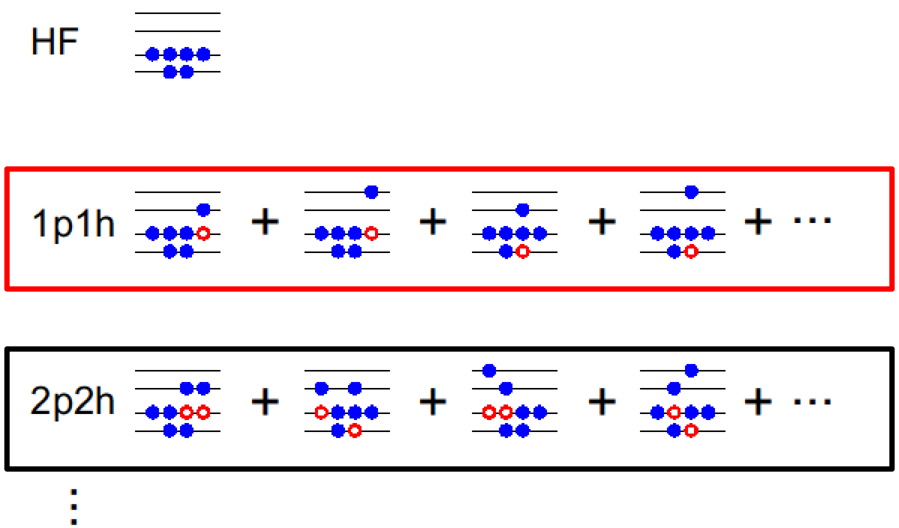
elativistic density functional

vistic density functional



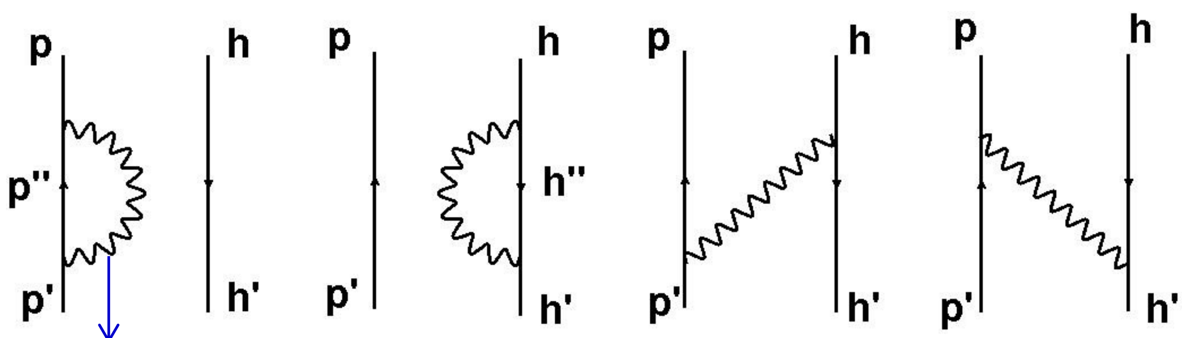


Something in between? --- RPA+PVC model



RPA

- **Second RPA** drozdz et al., PR 197, 1 (1990)
  Gambacurta et al., PRC 81, 054312 (2010)
- **RPA + PVC (particle vibration coupling)**

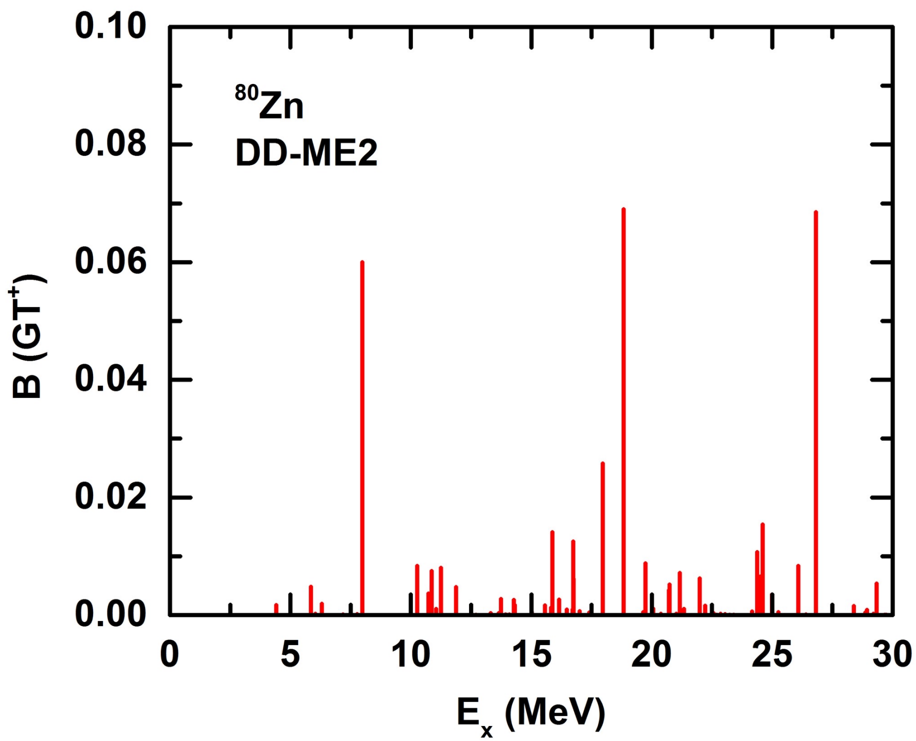


Low-lying vibration phonons |N>

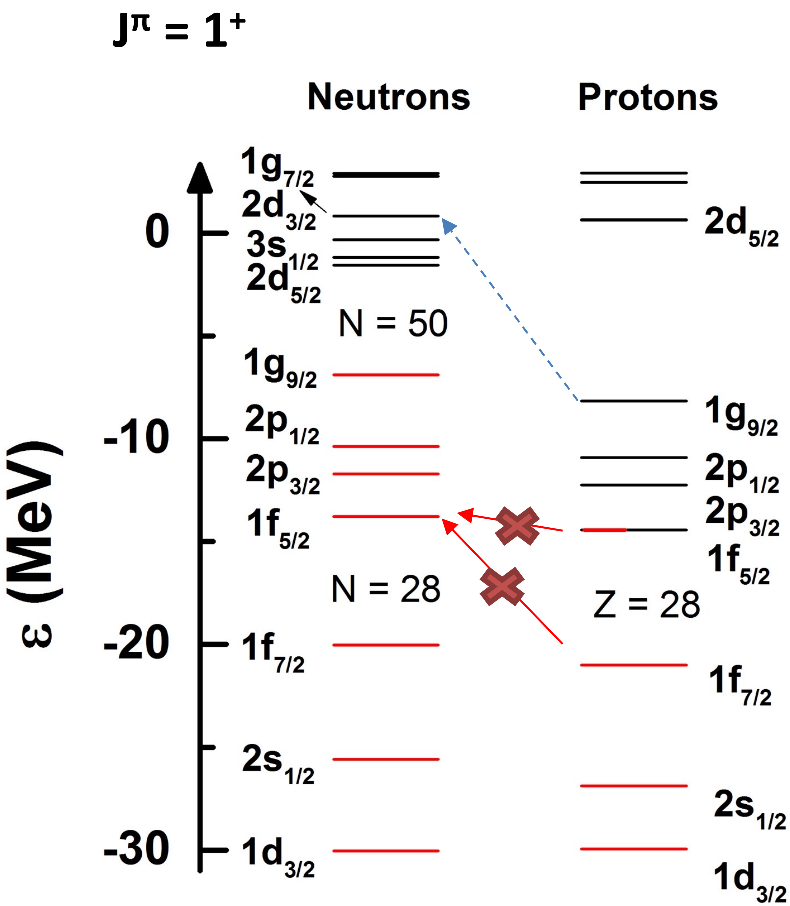
$$W^{\downarrow}_{ph,p'h'}(\omega) = \sum_N \frac{\langle ph|V|N\rangle\langle N|V|p'h'\rangle}{\omega - \omega_N}$$

Gamow-Teller strength distribution (T$^+$)

- **GT operator** $$\hat{F}^{\pm}_{\mathrm{GT}} = \sum_{i=1}^{A} \sigma(i)\tau_{\pm}(i)$$



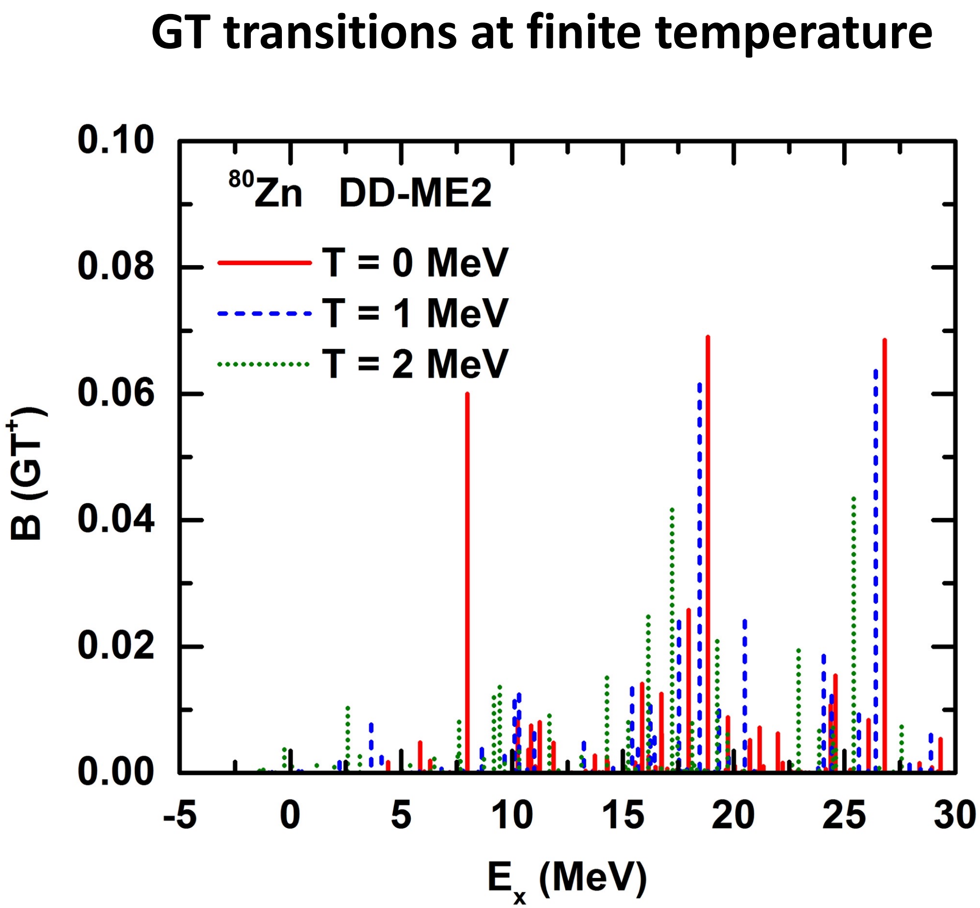
with respect to daughter nucleus ^{80}Cu

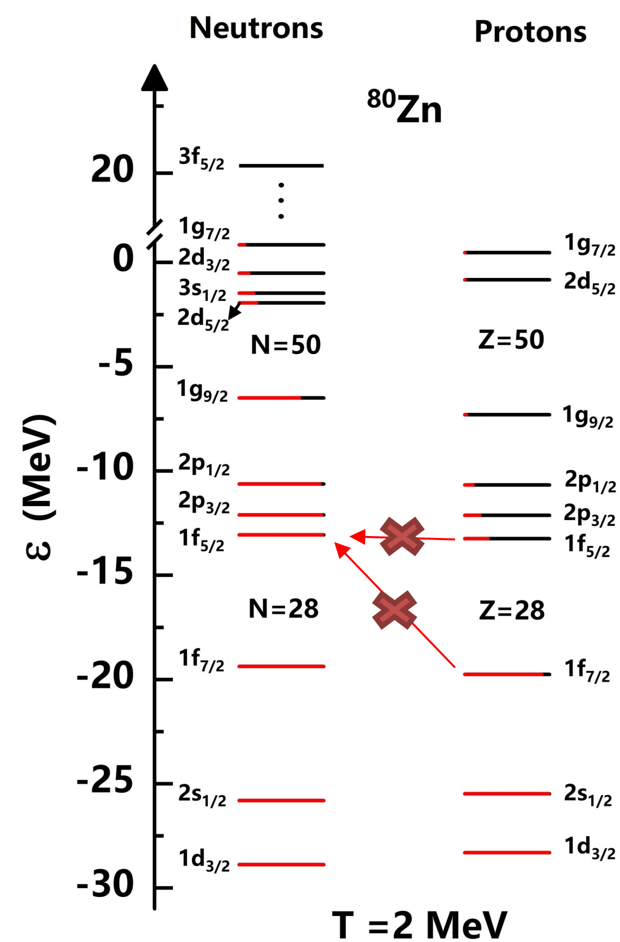


✓ GT$^+$ transitions are almost blocked (Ikeda sum rule = 60)

✓ Pairing correlations or transitions across major shells make little transition strength possible

Temperature effects



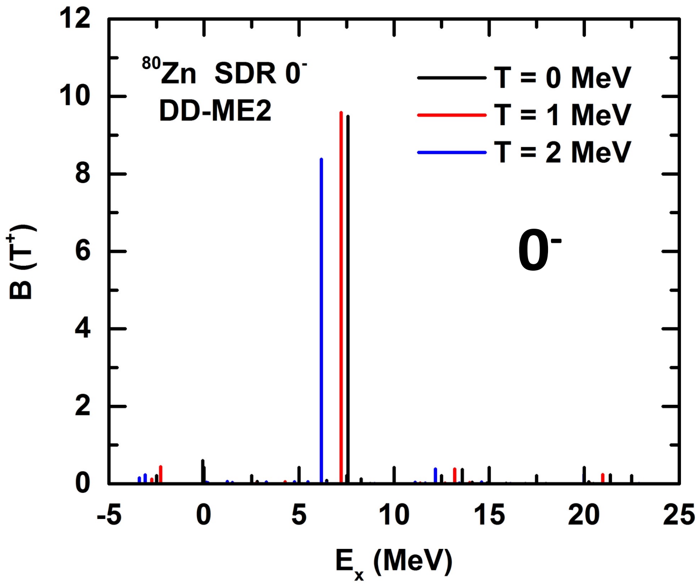


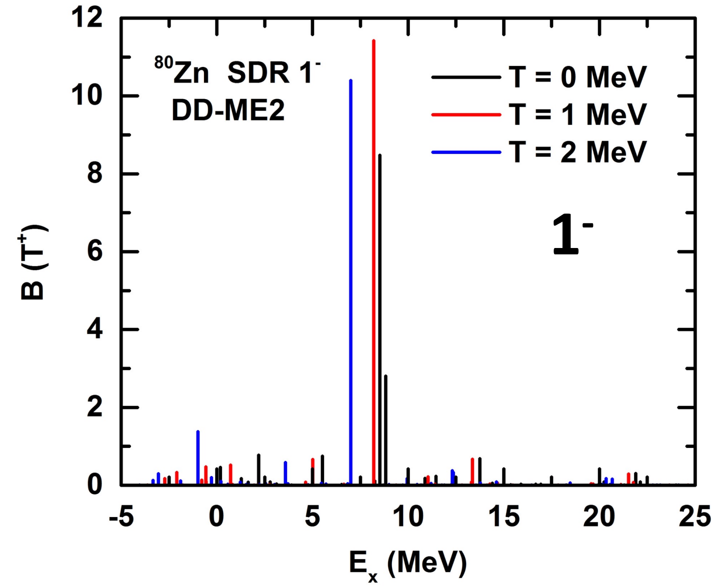
✓ Even temperature cannot unblock the GT$^+$ transition due to large neutron excess

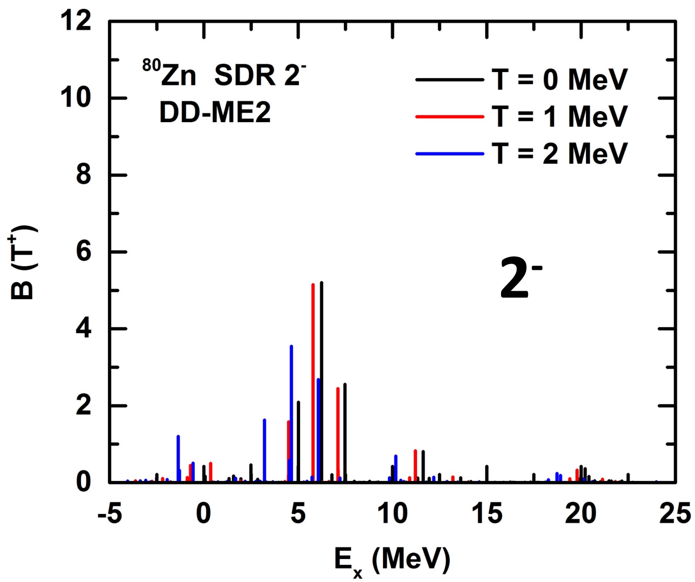
➢ In stellar environment, GT$^+$ still cannot contribute much to EC rates

Temperature effects

Spin Dipole Transitions at finite temperature







- ✓ Spin-Dipole transitions have significant strength, and hence will dominate EC cross section of ^{80}Zn
- ✓ Temperature decreases energies, but changes are small.

L. Guo, W. L. Lv, Y. F. Niu, D. L. Fang, B.S. Gao, K. A. Li, and X. D. Tang, PRC 107, 014318 (2023)