第三届地下和空间粒子物理与宇宙物理前沿问题研讨会
Conference on frontiers of underground and space particle physics and cosmophysics
2024年5月7日-11日，四川西昌

中国锦屏地下实验室
China Jinping Underground Laboratory
雅砻江水电

EFT STUDIES OF NEUTRINOLESS DOUBLE BETA DECAY IN LR SYMMETRIC MODEL

DONG-LIANG FANG
IMP, CAS

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

Outline

* Matching to EFT
* Derivation of reaction matrix
* Many-body calculations
* Conclusion and perspective

LR symmetric model

PHYSICAL REVIEW D VOLUME 23, NUMBER 1 1 JANUARY 1981

Neutrino masses and mixings in gauge models with spontaneous parity violation

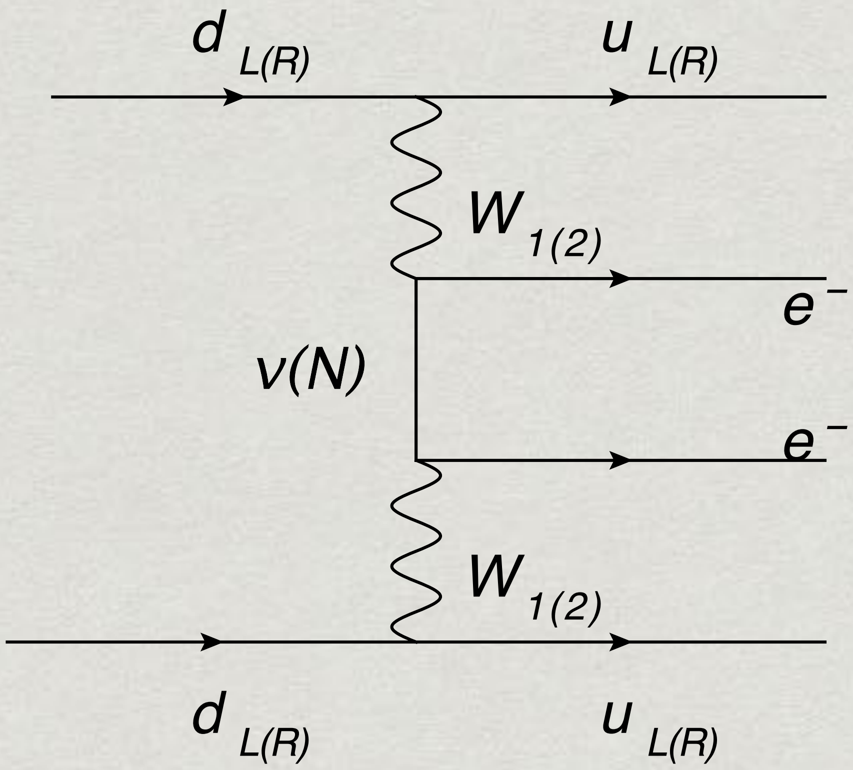
Rabindra N. Mohapatra*
Department of Physics, The City College of the City University of New York, New York, New York 10031

Goran Senjanović
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and Department of Physics, University of Maryland, College Park, Maryland 20742
(Received 8 August 1980)

- The LR symmetric SM is proposed by Mohapatra and Senjanovic
- One introduces the right-handed copies of neutrinos, gauge bosons as well as Higgs boson
- Besides a triplet Higgs boson has been introduced which gives rise of Majorana mass term of neutrino

Neutrinoless double beta decay

Doi et al. PTPS83,1(1985)



- Neutrinoless double beta decay related terms
  - Mass terms:

$$\nu_{eL} = \sum_{j=1}^{3} (U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C),$$

$$\nu_{eR} = \sum_{j=1}^{3} (T^*_{ej}(\nu_{jL})^C + V^*_{ej}N_{jR}).$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

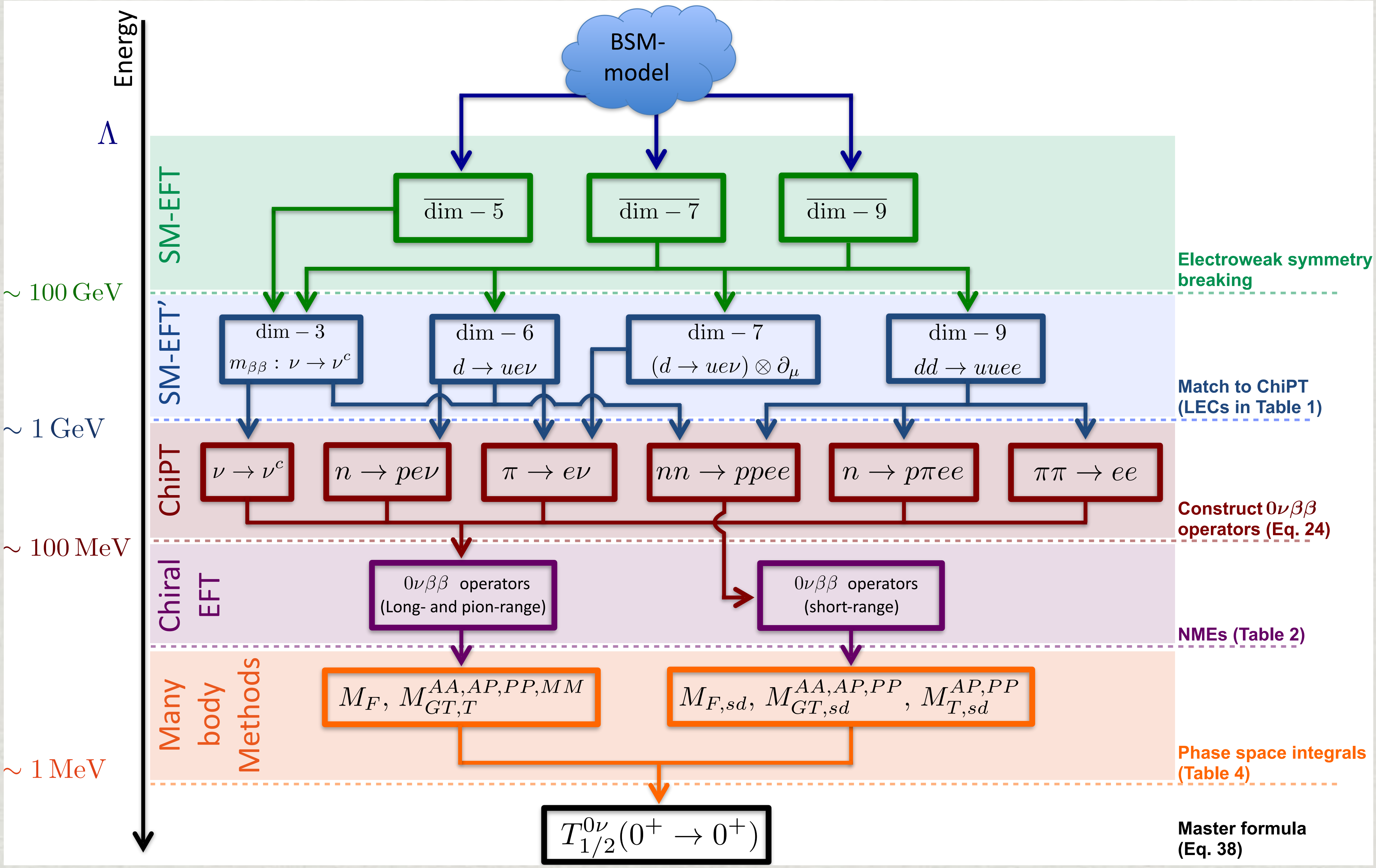
  - Weak current:

$$H^\beta = \frac{G_\beta}{\sqrt{2}}[j_L^\rho J_{L\rho}^\dagger + \chi j_L^\rho J_{R\rho}^\dagger + \eta j_R^\rho J_{L\rho}^\dagger + \lambda j_R^\rho J_{R\rho}^\dagger + \text{H.c.}]$$

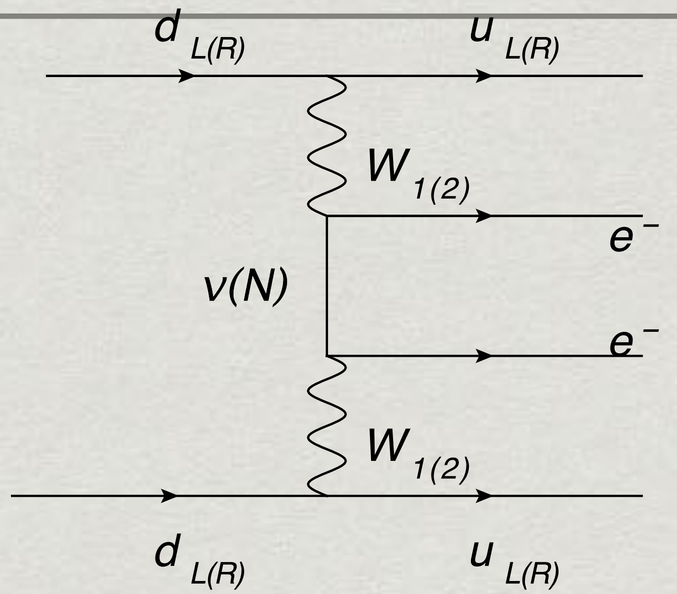
$$\eta \simeq -\tan\zeta \quad \lambda \simeq (M_{W_1}/M_{W_2})^2$$

Matching

Cirigliano et al. JHEP12,097(2018)



SMEFT



Cirigliano et al. JHEP12,097(2018)

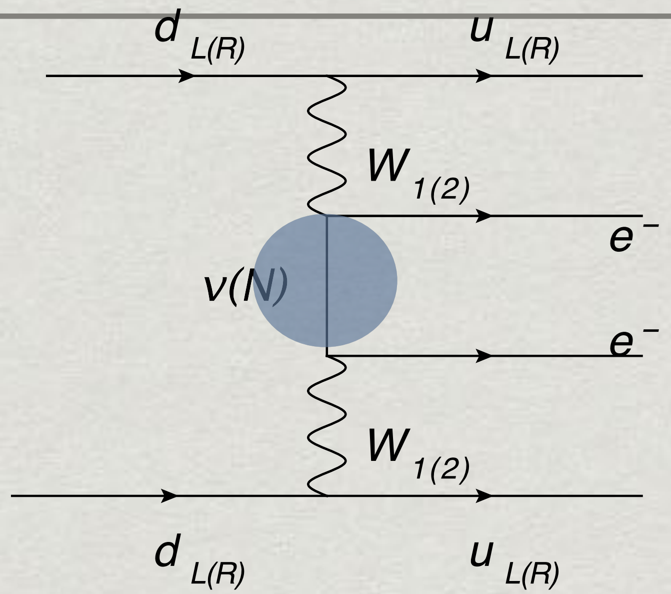
- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)}\epsilon_{kl}\epsilon_{mn}(L_k^T C L_m)H_l H_n$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe}\epsilon_{ij}\epsilon_{mn}(L_i^T C\gamma_\nu e)H_j H_m(D^\mu H_n)$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H}\epsilon_{ij}(L_i^T C\gamma_\mu e)(\bar{d}\gamma^\mu u)H_j$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud}\bar{e}Ce\bar{u}\gamma_\mu d\bar{u}\gamma^\mu d$

    $\mathscr{C}^{(9)}_{eeHud}\bar{e}Ce\bar{u}\gamma_\mu d((iD^\mu H)^\dagger \tilde{H})$

    $\mathscr{C}^{(9)}_{eeHD}\bar{e}Ce((iD^\mu H)^\dagger \tilde{H})^2$

SMEFT



Cirigliano et al. JHEP12,097(2018)

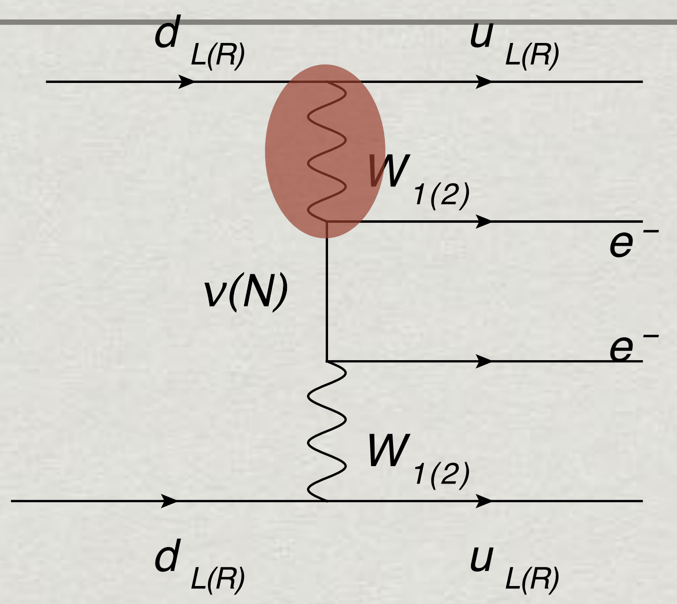
- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)}\epsilon_{kl}\epsilon_{mn}(L_k^T C L_m)H_l H_n$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe}\epsilon_{ij}\epsilon_{mn}(L_i^T C\gamma_\nu e)H_j H_m(D^\mu H_n)$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H}\epsilon_{ij}(L_i^T C\gamma_\mu e)(\bar{d}\gamma^\mu u)H_j$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud}\bar{e}Ce\bar{u}\gamma_\mu d\bar{u}\gamma^\mu d$

    $\mathscr{C}^{(9)}_{eeHud}\bar{e}Ce\bar{u}\gamma_\mu d((iD^\mu H)^\dagger\tilde{H})$

    $\mathscr{C}^{(9)}_{eeHD}\bar{e}Ce((iD^\mu H)^\dagger\tilde{H})^2$

SMEFT



Cirigliano et al. JHEP12,097(2018)

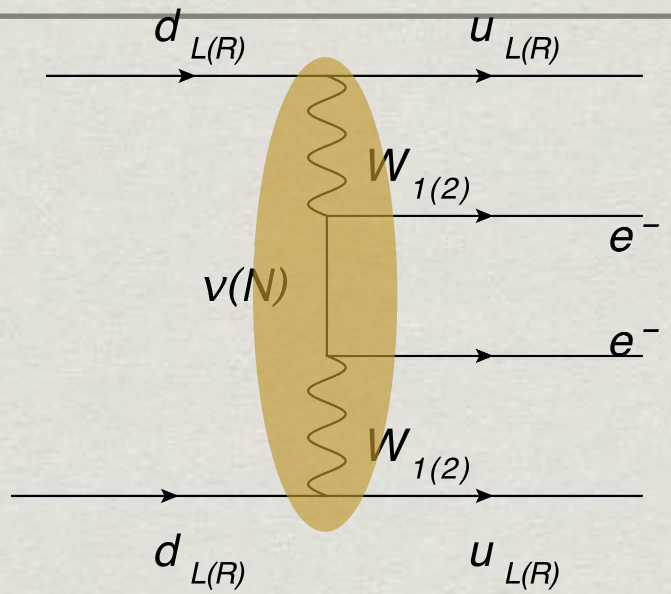
- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)}\epsilon_{kl}\epsilon_{mn}(L_k^T C L_m)H_l H_n$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe}\epsilon_{ij}\epsilon_{mn}(L_i^T C\gamma_\nu e)H_j H_m(D^\mu H_n)$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H}\epsilon_{ij}(L_i^T C\gamma_\mu e)(\bar{d}\gamma^\mu u)H_j$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud}\bar{e}Ce\bar{u}\gamma_\mu d\bar{u}\gamma^\mu d$

    $\mathscr{C}^{(9)}_{eeHud}\bar{e}Ce\bar{u}\gamma_\mu d((iD^\mu H)^\dagger\tilde{H})$

    $\mathscr{C}^{(9)}_{eeHD}\bar{e}Ce((iD^\mu H)^\dagger\tilde{H})^2$

SMEFT



Cirigliano et al. JHEP12,097(2018)

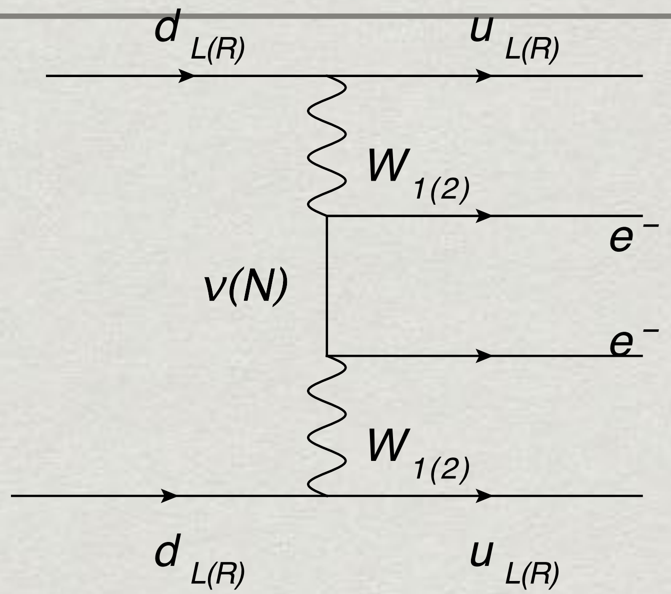
- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)}\epsilon_{kl}\epsilon_{mn}(L_k^T C L_m)H_l H_n$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe}\epsilon_{ij}\epsilon_{mn}(L_i^T C\gamma_\nu e)H_j H_m(D^\mu H_n)$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H}\epsilon_{ij}(L_i^T C\gamma_\mu e)(\bar{d}\gamma^\mu u)H_j$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud}\bar{e}Ce\bar{u}\gamma_\mu d\bar{u}\gamma^\mu d$

    $\mathscr{C}^{(9)}_{eeHud}\bar{e}Ce\bar{u}\gamma_\mu d((iD^\mu H)^\dagger \tilde{H})$

    $\mathscr{C}^{(9)}_{eeHD}\bar{e}Ce((iD^\mu H)^\dagger \tilde{H})^2$

SMEFT



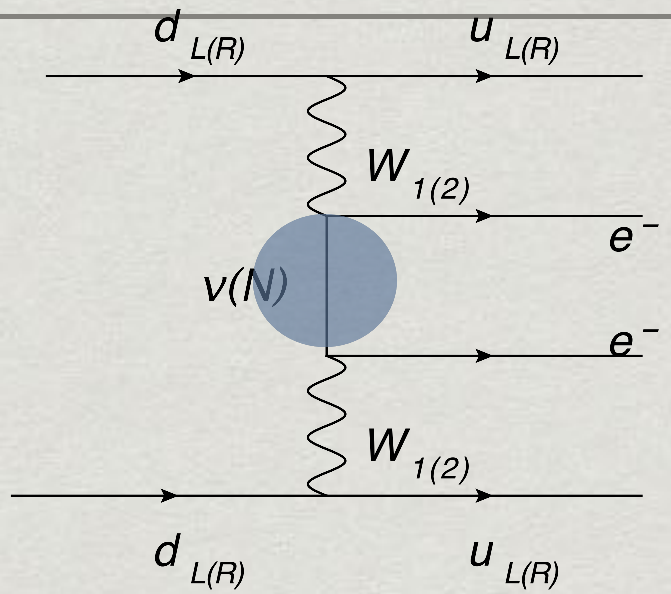
Cirigliano et al. JHEP12,097(2018)

- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)} = \frac{1}{2} M_D^T M_{\nu_R}^{-1} M_D - \frac{\sqrt{2} v_L e^{i\theta_L}}{v^2} M_L$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe} = \frac{2i\xi e^{i\alpha}}{(1+\xi^2) v_R^2} (M_D^T M_{\nu_R}^{-1})_{ee}$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H} = \frac{1}{v_R^2} (V_R^{ud})^* (M_D^T M_{\nu_R}^{-1})_{ee}$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud} = -\frac{1}{2 v_R^4} (V_R^{ud})^2 [(M_{\nu_R}^\dagger)^{-1} + \frac{2}{m^2_{\Delta_R}} M_{\nu R}]$

$$\mathscr{C}^{(9)}_{eeHud} = -4 \frac{\xi e^{-i\alpha}}{1+\xi^2} \frac{\mathscr{C}^{(9)}_{eeud}}{V_R^{ud}} \qquad \mathscr{C}^{(9)}_{eeHud} = 4 \frac{\xi^2 e^{-2i\alpha}}{(1+\xi^2)^2} \frac{\mathscr{C}^{(9)}_{eeud}}{(V_R^{ud})^2}$$

SMEFT



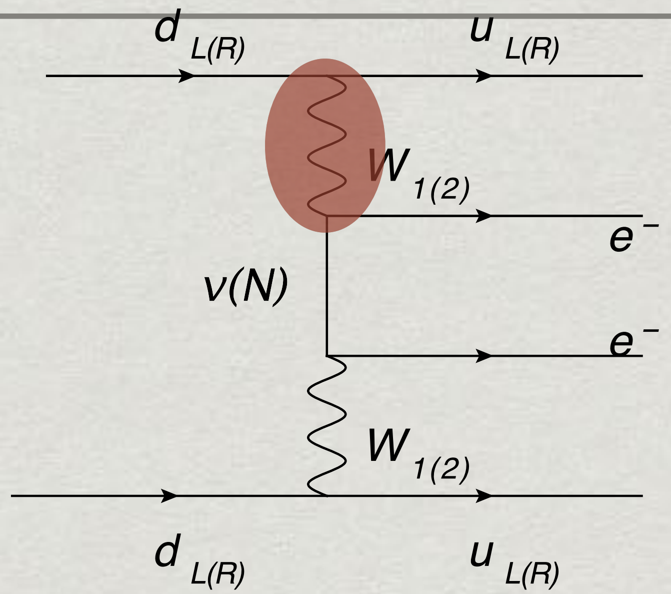
Cirigliano et al. JHEP12,097(2018)

- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)} = \frac{1}{2} M_D^T M_{\nu_R}^{-1} M_D - \frac{\sqrt{2} v_L e^{i\theta_L}}{v^2} M_L$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe} = \frac{2i\xi e^{i\alpha}}{(1+\xi^2) v_R^2} (M_D^T M_{\nu_R}^{-1})_e e$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H} = \frac{1}{v_R^2} (V_R^{ud})^* (M_D^T M_{\nu_R}^{-1})_{ee}$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud} = -\frac{1}{2v_R^4} (V_R^{ud})^2 [(M_{\nu_R}^\dagger)^{-1} + \frac{2}{m_{\Delta_R}^2} M_{\nu R}]$

$\mathscr{C}^{(9)}_{eeHud} = -4 \frac{\xi e^{-i\alpha}}{1+\xi^2} \frac{\mathscr{C}^{(9)}_{eeud}}{V_R^{ud}}$ $\quad$ $\mathscr{C}^{(9)}_{eeHud} = 4 \frac{\xi^2 e^{-2i\alpha}}{(1+\xi^2)^2} \frac{\mathscr{C}^{(9)}_{eeud}}{(V_R^{ud})^2}$

SMEFT

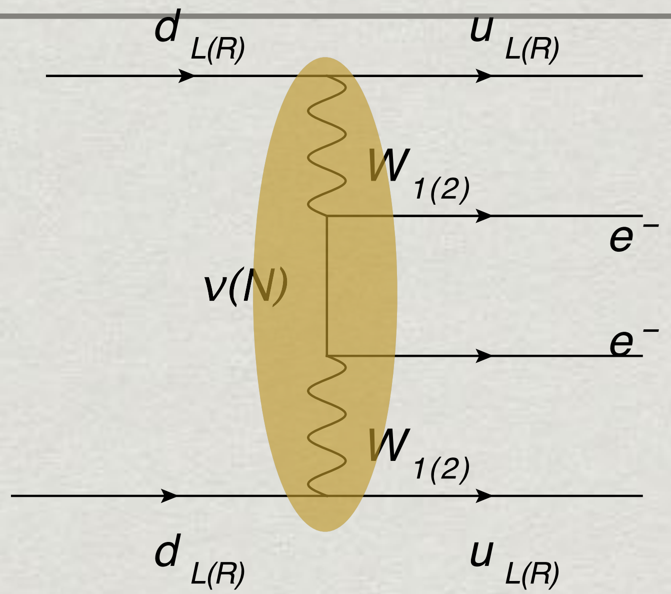


Cirigliano et al. JHEP12,097(2018)

- Matching to SMEFT
  - Dim-5: $$\mathscr{C}^{(5)} = \frac{1}{2} M_D^T M_{\nu_R}^{-1} M_D - \frac{\sqrt{2} v_L e^{i\theta_L}}{v^2} M_L$$
  - Dim-7: $$\mathscr{C}^{(7)}_{LHDe} = \frac{2i\xi e^{i\alpha}}{(1+\xi^2) v_R^2} (M_D^T M_{\nu_R}^{-1})_e e$$
  $$\mathscr{C}^{(7)}_{Leu\bar{d}H} = \frac{1}{v_R^2} (V_R^{ud})^* (M_D^T M_{\nu_R}^{-1})_{ee}$$
  - Dim-9: $$\mathscr{C}^{(9)}_{eeud} = -\frac{1}{2 v_R^4} (V_R^{ud})^2 [(M_{\nu_R}^\dagger)^{-1} + \frac{2}{m_{\Delta_R}^2} M_{\nu R}]$$

$$\mathscr{C}^{(9)}_{eeHud} = -4 \frac{\xi e^{-i\alpha}}{1+\xi^2} \frac{\mathscr{C}^{(9)}_{eeud}}{V_R^{ud}} \qquad \mathscr{C}^{(9)}_{eeHud} = 4 \frac{\xi^2 e^{-2i\alpha}}{(1+\xi^2)^2} \frac{\mathscr{C}^{(9)}_{eeud}}{(V_R^{ud})^2}$$

SMEFT



Cirigliano et al. JHEP12,097(2018)

- Matching to SMEFT
  - Dim-5: $\mathscr{C}^{(5)} = \frac{1}{2} M_D^T M_{\nu_R}^{-1} M_D - \frac{\sqrt{2} v_L e^{i\theta_L}}{v^2} M_L$
  - Dim-7: $\mathscr{C}^{(7)}_{LHDe} = \frac{2i\xi e^{i\alpha}}{(1+\xi^2) v_R^2} (M_D^T M_{\nu_R}^{-1})_{ee}$

    $\mathscr{C}^{(7)}_{Leu\bar{d}H} = \frac{1}{v_R^2} (V_R^{ud})^* (M_D^T M_{\nu_R}^{-1})_{ee}$
  - Dim-9: $\mathscr{C}^{(9)}_{eeud} = -\frac{1}{2v_R^4} (V_R^{ud})^2 [(M_{\nu_R}^\dagger)^{-1} + \frac{2}{m_{\Delta_R}^2} M_{\nu R}]$

$$\mathscr{C}^{(9)}_{eeHud} = -4 \frac{\xi e^{-i\alpha}}{1+\xi^2} \frac{\mathscr{C}^{(9)}_{eeud}}{V_R^{ud}} \qquad \mathscr{C}^{(9)}_{eeHud} = 4 \frac{\xi^2 e^{-2i\alpha}}{(1+\xi^2)^2} \frac{\mathscr{C}^{(9)}_{eeud}}{(V_R^{ud})^2}$$

LEFT

Cirigliano et al. JHEP12,097(2018)

- Matching operators after EWSB, we focus on long-range mechanism with light neutrinos:

- Dim-3:

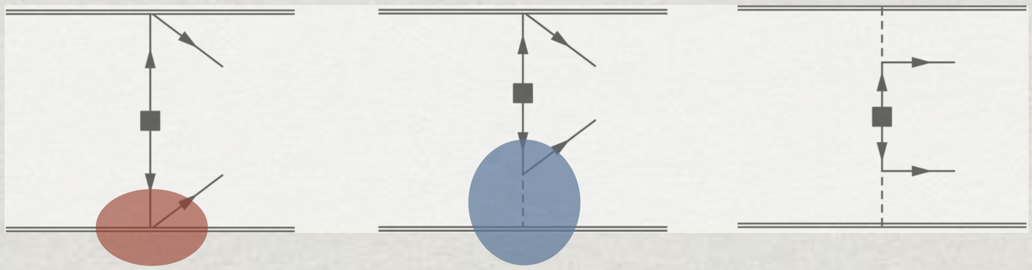
$$m_{\beta\beta}\nu_{eL}^{T}C\nu_{eL} \qquad m_{\beta\beta} = -v^2(\mathscr{C}^{(5)})_{ee}$$

- Dim-6:

$$C_{VL}^{(6)}\bar{u}_L\gamma^\mu d_L\bar{e}_R\gamma_\mu C\bar{\nu}_L^T \qquad C_{VL}^{(6)} = -iV_L^{ud}\frac{v^3}{\sqrt{2}}(\mathscr{C}_{LHDe})^*$$

$$C_{VR}^{(6)}\bar{u}_R\gamma^\mu d_R\bar{e}_R\gamma_\mu C\bar{\nu}_L^T \qquad C_{VR}^{(6)} = \frac{v^3}{\sqrt{2}}(\mathscr{C}_{LeudH}^{(7)})^*$$

Chiral EFT



- The mesonic chiral Lagrangian at LO

$$\mathscr{L}_\pi = \frac{F_0^2}{4} Tr[(D_\mu U)^\dagger D^\mu U] + \frac{F_0^2}{4} Tr[U^\dagger \chi + U \chi^\dagger]$$

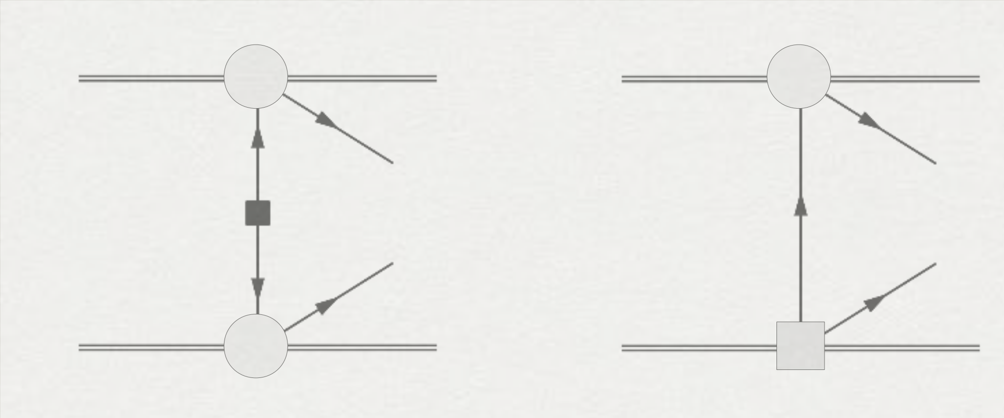
- The baryonic chiral Lagrangian at LO

$$\mathscr{L}_{\pi N}^{(1)} = i\bar{N} v \cdot DN + g_A \bar{N} S \cdot uN + c_5 \bar{N} \hat{\chi}_+ N + \dots$$

- NLO

$$\mathscr{L}_{\pi N}^{(2)} = \frac{1}{2m_N}(v^\mu v^\nu - g^{\mu\nu})(\bar{N} D_\mu D_\nu N) - \frac{g_M}{4m_N} \epsilon^{\mu\nu\alpha\beta} v_\alpha \bar{N} S_\beta f_{\mu\nu}^+ N \dots$$

Chiral EFT



- χEFT Lagrangian for these weak decay vertices is

$$\mathcal{A}^{n\to pe^-\nu} = \bar{N}\tau^+\left[\frac{l_\mu + r_\mu}{2}J_V^\mu + \frac{l_\mu - r_\mu}{2}J_A^\mu\right]N$$

- the lepton currents are introduced as external fields

$$l_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[-2vV_{ud}\bar{e}_L\gamma_\mu\nu_L + v\,C_{\rm VL}^{(6)}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

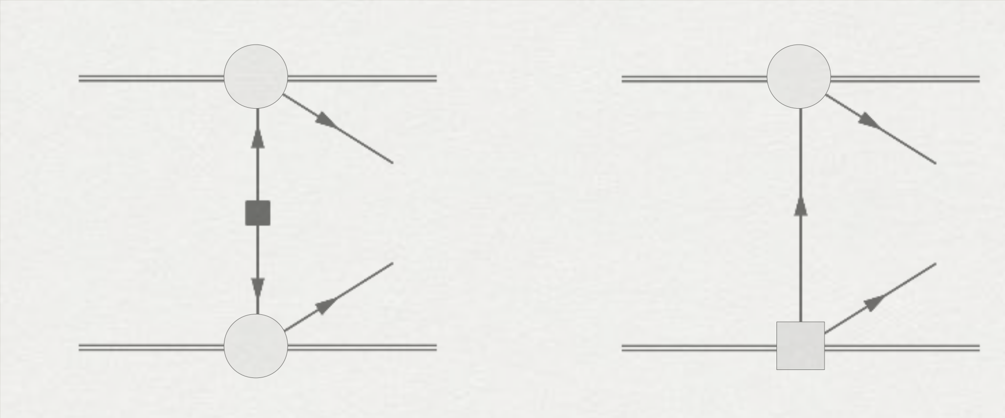
$$r_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[v\,C_{\rm VR}^{(6)}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

- And corresponding nuclear current

$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + \frac{ig_M(\mathbf{q}^2)}{m_N}\varepsilon^{\mu\nu\alpha\beta}v_\alpha S_\beta q_\nu\,,$$

$$J_A^\mu = -g_A(\mathbf{q}^2)\left(2S^\mu - \frac{v^\mu}{2m_N}2S\cdot(p+p')\right) + \frac{g_P(\mathbf{q}^2)}{2m_N}2q^\mu\,S\cdot q\,,$$

Chiral EFT



- χEFT Lagrangian for these weak decay vertices is

$$\mathcal{A}^{n\to pe^-\nu} = \bar{N}\tau^+\left[\frac{l_\mu + r_\mu}{2}J_V^\mu + \frac{l_\mu - r_\mu}{2}J_A^\mu\right]N$$

- the lepton currents are introduced as external fields

$$l_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[-2vV_{ud}\bar{e}_L\gamma_\mu\nu_L + v\,C_{\rm VL}^{(6)}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

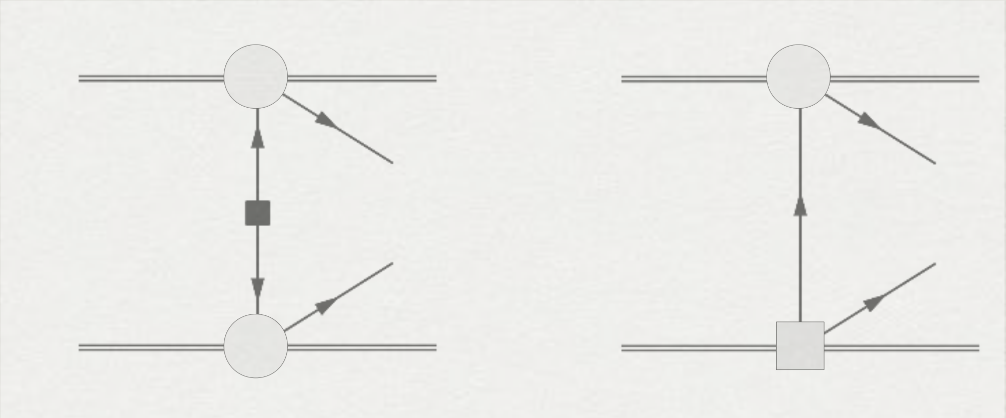
$$r_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[v\,C_{\rm VR}^{(6)}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

- And corresponding nuclear current

$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + \frac{ig_M(\mathbf{q}^2)}{m_N}\varepsilon^{\mu\nu\alpha\beta}v_\alpha S_\beta q_\nu\,,$$

$$J_A^\mu = -g_A(\mathbf{q}^2)\left(2S^\mu - \frac{v^\mu}{2m_N}2S\cdot(p+p')\right) + \frac{g_P(\mathbf{q}^2)}{2m_N}2q^\mu\, S\cdot q\,,$$

Chiral EFT



- χEFT Lagrangian for these weak decay vertices is

$$\mathcal{A}^{n\to pe^-\nu} = \bar{N}\tau^+\left[\frac{l_\mu + r_\mu}{2}J_V^\mu + \frac{l_\mu - r_\mu}{2}J_A^\mu\right]N$$

- the lepton currents are introduced as external fields

$$l_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[-2vV_{ud}\bar{e}_L\gamma_\mu\nu_L + v\,C^{(6)}_{\rm VL}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

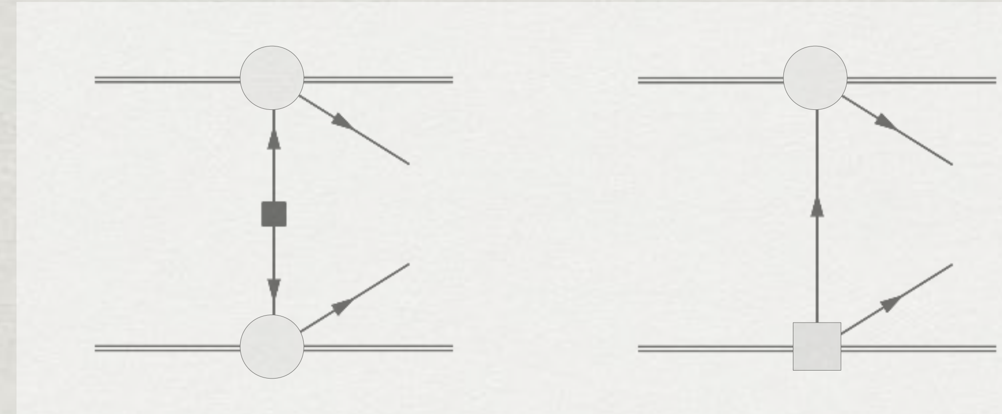
$$r_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[v\,C^{(6)}_{\rm VR}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

- And corresponding nuclear current

$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + \frac{ig_M(\mathbf{q}^2)}{m_N}\varepsilon^{\mu\nu\alpha\beta}v_\alpha S_\beta q_\nu\,,$$

$$J_A^\mu = -g_A(\mathbf{q}^2)\left(2S^\mu - \frac{v^\mu}{2m_N}2S\cdot(p+p')\right) + \frac{g_P(\mathbf{q}^2)}{2m_N}2q^\mu\,S\cdot q\,,$$

Chiral EFT



- χEFT Lagrangian for these weak decay vertices is

$$\mathcal{A}^{n\to pe^-\nu} = \bar{N}\tau^+\left[\frac{l_\mu + r_\mu}{2}J_V^\mu + \frac{l_\mu - r_\mu}{2}J_A^\mu\right]N$$

- the lepton currents are introduced as external fields

$$l_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[-2vV_{ud}\bar{e}_L\gamma_\mu\nu_L + v\,C^{(6)}_{\rm VL}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

$$r_\mu = \frac{2G_F}{\sqrt{2}v}(\tau^+)\left[v\,C^{(6)}_{\rm VR}\,\bar{e}_R\gamma_\mu C\bar{\nu}_L^T\right] + \text{h.c.}$$

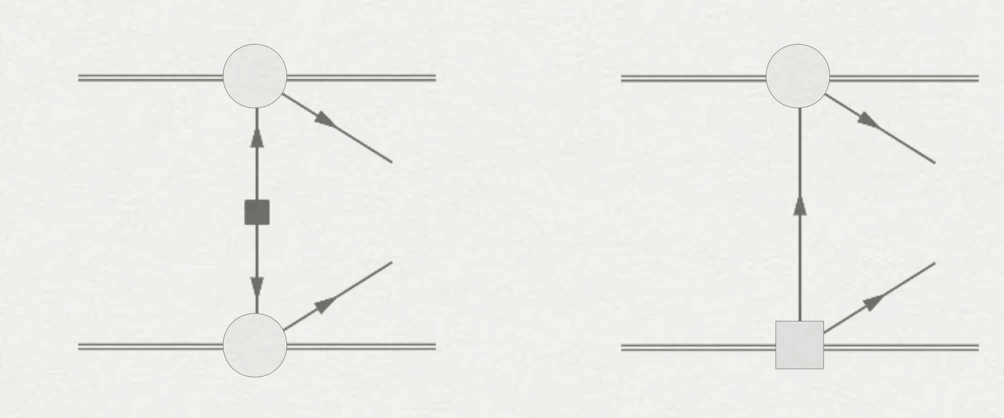
- And corresponding nuclear current

$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + \frac{ig_M(\mathbf{q}^2)}{m_N}\varepsilon^{\mu\nu\alpha\beta}v_\alpha S_\beta q_\nu\,,$$

$$J_A^\mu = -g_A(\mathbf{q}^2)\left(2S^\mu - \frac{v^\mu}{2m_N}2S\cdot(p+p')\right) + \frac{g_P(\mathbf{q}^2)}{2m_N}2q^\mu\,S\cdot q\,,$$

Decay width

Doi et al. PTPS83,1(1985)



- The decay width can be obtained from S-matrix theory

$$d\Gamma_{0\nu} = 2\pi \sum_{\text{spin}} |R_{0\nu}|^2 \delta(\varepsilon_1 + \varepsilon_2 + E_f - M_i)\, d\Omega_{e_1} d\Omega_{e_2}$$

- The reaction matrix element can be expressed as

$$R_{0\nu} = \frac{1}{\sqrt{2}} \int dx \int dy \langle p_1 p_2; f \,|\, T\{ e^{iH_0(x_0 - y_0)} H_{int}(\overrightarrow{x}) H_{int}(\overrightarrow{y}) \} \,|\, i \rangle$$

- This is a typical second order process

Decay width

- After tedious derivation, we come to

$$\Gamma^{0\nu} = \frac{|m_{\beta\beta}|^2}{m_e^2}\mathscr{C}_{mm} + |\frac{C_{VL}^{(6)}}{2V_{ud}}|^2\mathscr{C}_{\eta\eta} + |\frac{C_{VR}^{(6)}}{2V_{ud}}|^2\mathscr{C}_{\lambda\lambda}$$

$$+Re(\frac{m_{\beta\beta}C_{VR}^{(6)}}{2m_eV^{ud}})\mathscr{C}_{m\lambda} - Re(\frac{m_{\beta\beta}C_{VL}^{(6)}}{2m_eV^{ud}})\mathscr{C}_{m\eta} - Re(\frac{C_{VL}^{(6)}C_{VR}^{(6)}}{4|V^{ud}|^2})\mathscr{C}_{\lambda\eta}$$

- This agrees with earlier calculations based on LR symmetric model

Decay width

$$
\begin{aligned}
\mathcal{C}_{mm} &= \mathcal{G}_{01}|M_m^{0\nu}|^2 \\
\mathcal{C}_{m\lambda} &= -\mathcal{G}_{03}M_m^{0\nu}M_{\omega-}^{0\nu} + \mathcal{G}_{04}M_m^{0\nu}M_{q+}^{0\nu} \\
\mathcal{C}_{m\eta} &= \mathcal{G}_{03}M_m^{0\nu}M_{\omega+}^{0\nu} - \mathcal{G}_{04}M_m^{0\nu}M_{q-}^{0\nu} - \mathcal{G}_{05}M_m^{0\nu}M_P^{0\nu} \\
&+ \mathcal{G}_{06}M_m^{0\nu}M_R^{0\nu} \\
\mathcal{C}_{\lambda\lambda} &= \mathcal{G}_{02}|M_{\omega-}^{0\nu}|^2 + \mathcal{G}_{011}|M_{q+}^{0\nu}|^2 \\
\mathcal{C}_{\eta\eta} &= \mathcal{G}_{02}|M_{\omega+}^{0\nu}|^2 + \mathcal{G}_{011}|M_{q-}^{0\nu}|^2 + \mathcal{G}_{08}|M_P^{0\nu}|^2 \\
&+ \mathcal{G}_{09}|M_R^{0\nu}|^2 - \mathcal{G}_{07}M_P^{0\nu}M_R^{0\nu} \\
\mathcal{C}_{\lambda\eta} &= -2\mathcal{G}_{02}M_{\omega-}^{0\nu}M_{\omega+}^{0\nu} - \mathcal{G}_{010}(M_{q+}^{0\nu}M_{\omega+}^{0\nu} + M_{q-}^{0\nu}M_{\omega-}^{0\nu}) \\
&\quad - 2\mathcal{G}_{011}M_{q+}^{0\nu}M_{q-}^{0\nu}
\end{aligned}
$$

- Here G’s are phase space factors and M’s the matrix elements

NME

$$M_m^{0\nu} = -M_F + M_{GT} + M_T$$
$$M_{\omega\pm}^{0\nu} = M_{\omega GT\pm} + M_{\omega T\pm} \pm M_{\omega F}$$
$$M_{q\pm}^{0\nu} = \frac{1}{3m_e R}(M_{qGT\pm} - 6M_{qT\pm} \pm 3M_{qF})$$
$$M_R^{0\nu} = \frac{1}{m_e R}(M_{RGT} + M_{RT})$$
$$M_P^{0\nu} = \frac{1}{m_e R} M_P$$

$$M_{iGT} = M_{iGT}^{AA} + M_{iGT}^{AP} + M_{iGT}^{PP} + M_{iGT}^{MM}$$
$$M_{iT} = M_{iT}^{AA} + M_{iT}^{AP} + M_{iT}^{PP} + M_{iT}^{MM}$$
$$i = m, \omega, q$$

- Detailed expressions for NMEs

Master formula

- Besides this S-matrix derivation, there are also the so-called master formula

$$\left(T^{0\nu}_{1/2}\right)^{-1} = g_A^4\Big\{G_{01}\ \left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2(G_{01} - G_{04})\mathrm{Re}\mathcal{A}_\nu^*\mathcal{A}_R + 4G_{02}\,|\mathcal{A}_E|^2$$
$$+2G_{04}\left[|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left(\mathcal{A}_{m_e}^*(\mathcal{A}_\nu + \mathcal{A}_R)\right)\right]$$
$$-2G_{03}\,\mathrm{Re}\left[(\mathcal{A}_\nu + \mathcal{A}_R)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$
$$+G_{09}\,|\mathcal{A}_M|^2 + G_{06}\,\mathrm{Re}\left[(\mathcal{A}_\nu - \mathcal{A}_R)\mathcal{A}_M^*\right]\Big\}.$$

- Where the amplitudes A are all sums of known nuclear matrix elements from mass mechanism

Nuclear many-body methods

- For double beta decay calculations, various many-body approaches have been adopted:
  - Nuclear Shell Model
  - Quasi-particle Random phase approximation (QRPA)
  - Generator coordinator method (GCM)
  - Interacting Boson model (IBM-2)
  - ……

Results

Fang et al. In preparation

NME			^{76}Ge→^{76}Se		^{82}Se→^{82}Kr		^{130}Te→^{130}Xe		^{136}Xe→^{136}Ba	
			jun45	jj44b	jun45	jj44b	jj55a	GCN50:82	jj55a	GCN50:82
M_m	F		-0.665	-0.601	-0.624	-0.523	-0.668	-0.701	-0.574	-0.567
	GT	AA	3.584	3.278	3.360	2.860	3.147	3.180	2.648	2.549
		AP	-1.090	-0.960	-1.021	-0.834	-0.979	-1.034	-0.820	-0.829
		PP	0.344	0.300	0.321	0.261	0.313	0.335	0.260	0.268
		MM	0.247	0.215	0.229	0.188	0.227	0.244	0.188	0.194
		total	3.085	2.833	2.889	2.474	2.708	2.724	2.277	2.183
	T	AP	-0.013	-0.004	-0.014	-0.012	0.008	0.015	0.002	0.014
		PP	0.002	-0.001	0.003	0.003	-0.006	-0.007	-0.003	-0.006
		MM	-0.001	-0.000	-0.001	-0.002	0.003	0.003	0.001	0.002
		total	-0.012	-0.004	-0.013	-0.010	0.004	0.010	-0.000	0.010
$M_{\omega\pm}$	F		-0.637	-0.575	-0.597	-0.500	-0.637	-0.669	-0.545	-0.540
	GT	AA	3.276	2.980	3.073	2.596	2.883	2.931	2.427	2.351
		AP	-1.044	-0.919	-0.978	-0.798	-0.939	-0.993	-0.786	-0.795
		PP	0.333	0.290	0.310	0.252	0.303	0.324	0.252	0.259
		MM	0.239	0.208	0.221	0.181	0.220	0.236	0.182	0.188
		GT_+total	2.803	2.558	2.626	2.231	2.466	2.498	2.075	2.002
		GT_-total	2.325	2.172	2.184	1.789	2.026	2.026	2.711	2.626
	T	AP	-0.012	-0.003	-0.013	-0.011	0.009	0.015	0.003	0.014
		PP	0.002	-0.001	0.003	0.003	-0.006	-0.007	-0.003	-0.006
		MM	-0.001	-0.000	-0.001	-0.002	0.003	0.003	0.001	0.002
		T_+total	-0.011	-0.004	-0.012	-0.010	0.005	0.010	0.000	0.010
		T_-total	-0.0013	-0.004	-0.014	-0.014	-0.001	0.004	-0.001	0.006

- Mass term and ω term are basically the same

Results

NME			^{76}Ge→^{76}Se		^{82}Se→^{82}Kr		^{130}Te→^{130}Xe		^{136}Xe→^{136}Ba	
			jun45	jj44b	jun45	jj44b	jj55a	GCN50:82	jj55a	GCN50:82
	F		-0.379	-0.351	-0.359	-0.304	-0.408	-0.417	-0.358	-0.342
$M_{q\pm}$	GT	AA	3.210	2.981	3.016	2.605	2.781	2.751	2.348	2.209
		AP	4.842	4.317	4.571	3.741	4.267	4.425	3.607	3.563
		PP	-1.943	-1.706	-1.829	-1.479	-1.731	-1.827	-1.454	-1.468
		MM	-1.874	-1.636	-1.745	-1.426	-1.708	-1.825	-1.419	-1.456
		GT_+total	7.983	7.228	7.502	6.293	7.026	7.173	5.920	5.760
		GT_-total	4.235	3.956	4.012	3.441	3.610	3.523	3.082	2.848
	T	AA	-0.056	-0.033	-0.055	-0.042	-0.031	-0.009	-0.031	0.002
		AP	0.004	-0.001	0.006	0.008	-0.018	-0.018	-0.007	-0.015
		PP	0.000	0.001	-0.001	-0.003	0.007	0.005	0.002	0.003
		MM	0.000	-0.000	-0.000	-0.001	0.001	0.001	0.000	0.001
		T_+total	-0.051	-0.034	-0.050	-0.035	-0.043	-0.023	-0.036	-0.012
		T_-total	-0.051	-0.034	-0.050	-0.037	-0.041	-0.021	-0.036	-0.009
M_R	GT		4.256	3.713	4.037	3.314	4.686	5.048	3.948	4.080
	T		0.014	0.004	0.018	0.028	-0.056	-0.056	-0.014	-0.042
M_P			-0.431	-0.279	-0.428	-0.152	-0.498	-0.425	-0.289	-0.255

- MM becomes LO for q term
- Larger R term than expected

Results

		rough estimation			^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
		lepton	nuclear	$\mathcal{R}$	$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$	
$\mu_{\beta\beta}$		$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	0.24	5.62 5.16		1.02	5.26 4.50		1.43	5.04 5.11		1.46	4.25 4.10	
					r_e	r_N	r_R	r_e	r_N	r_R	r_e	r_N	r_R	r_e	r_N	r_R
$C^{(6)}_{VL}$	M_ω	$\mathcal{O}(\epsilon_{12}/m_e)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	1.25	0.78 0.78	0.98 0.98	1.85	0.78 0.78	1.45 1.44	1.61	0.78 0.77	1.25 1.24	1.57	0.78 0.77	1.22 1.21
	M_q	$\mathcal{O}(\omega R)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(1)$	0.010	55.1 53.6	0.53 0.51	0.012	54.0 52.6	0.65 0.63	0.013	44.2 43.7	0.59 0.58	0.013	43.4 42.5	0.58 0.57
$C^{(6)}_{VR}$	M_ω	$\mathcal{O}(\epsilon_{12}/m_e)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	1.25	0.69 0.69	0.86 0.86	1.85	0.69 0.69	1.27 1.27	1.61	0.66 0.66	1.07 1.06	1.57	0.66 0.66	1.04 1.04
	M_q	$\mathcal{O}(\omega R)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(1)$	0.010	38.1 38.1	0.36 0.36	0.012	37.7 37.4	0.45 0.45	0.013	31.3 29.6	0.41 0.39	0.013	31.4 29.0	0.42 0.39
	M_R	$\mathcal{O}(1)$	$\mathcal{O}(q^2/(M_N m_e))$	$\mathcal{O}(\varepsilon^{-1})$	3.02	64.6 63.7	195.3 192.4	2.96	63.5 66.4	187.8 196.4	2.97	65.5 71.8	194.7 213.4	2.97	67.8 73.4	201.6 218.2
	M_P	$\mathcal{O}(\alpha Z)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(\varepsilon^{-1})$	0.34	7.40 5.21	2.49 1.75	0.33	7.65 3.18	2.50 1.04	0.27	7.97 6.71	2.19 1.84	0.25	5.41 4.94	1.37 1.25

- Dominance of R term in C_{VR} and coexistence of w and q terms in C_{VL}

Results

Chen et al. in preparation

	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$m\eta$	0.919	0.923	0.921	0.920
	0.919	0.921	0.920	0.919
$m\lambda$	−4.558	−6.255	−2.730	−2.340
	−2.013	−2.334	−1.507	−1.324
$\eta\eta$	0.851	0.856	0.853	0.858
	0.839	0.842	0.841	0.838
$\lambda\lambda$	3.830	3.682	4.984	4.584
	5.616	6.291	7.237	6.342
$\lambda\eta$	5.394	6.941	6.914	6.613
	3.792	4.295	3.285	4.166

- A comparison with the so-called master formula

Conclusions

- EFT studies of neutrinoless double beta decay agrees well with previous model studies
- We give related NMEs with shell model calculations and compare the relative magnitude of each term
- The mater formula offers very good approximations
- Two frames are equally efficient for double beta decay studies
- Constraints on Wilson coefficients by neutrinoless double beta decay is on going

谢谢