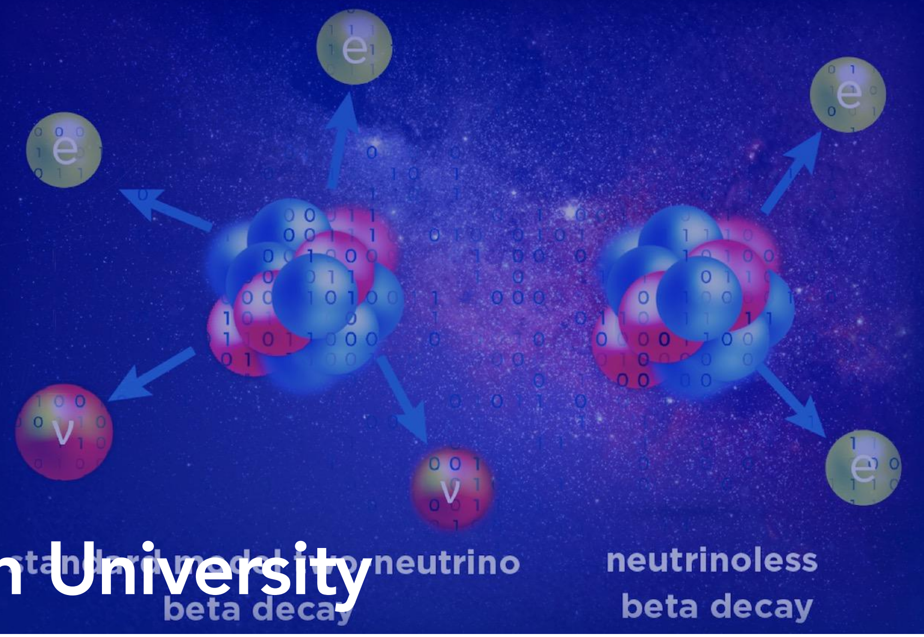
Neutrinoless double-β decay and the challenge it poses for nuclear physics

Changfeng Jiao

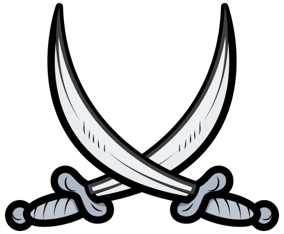
School of Physics and Astronomy, Sun Yat-sen University



May 7-11th, 2024, COUSP2024 @ Xichang, Sichuan, China

Is neutrino a majorana fermion?

Neutrino oscillations → Neutrinos are not massless



No dirac mass term naturally ← Left-handed neutrinos only

To solve the problem in a natural way,

We can construct left- and right-handed Majorana mass terms with charge conjugate transformation,

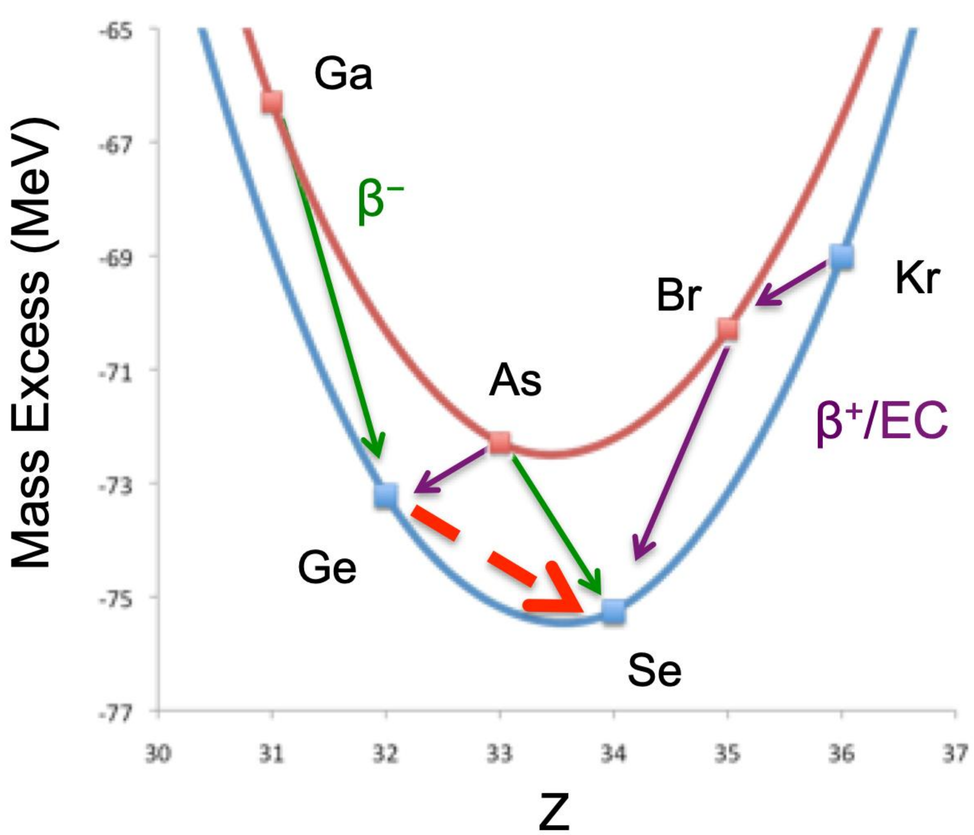
only if neutrinos are Majorana particles.

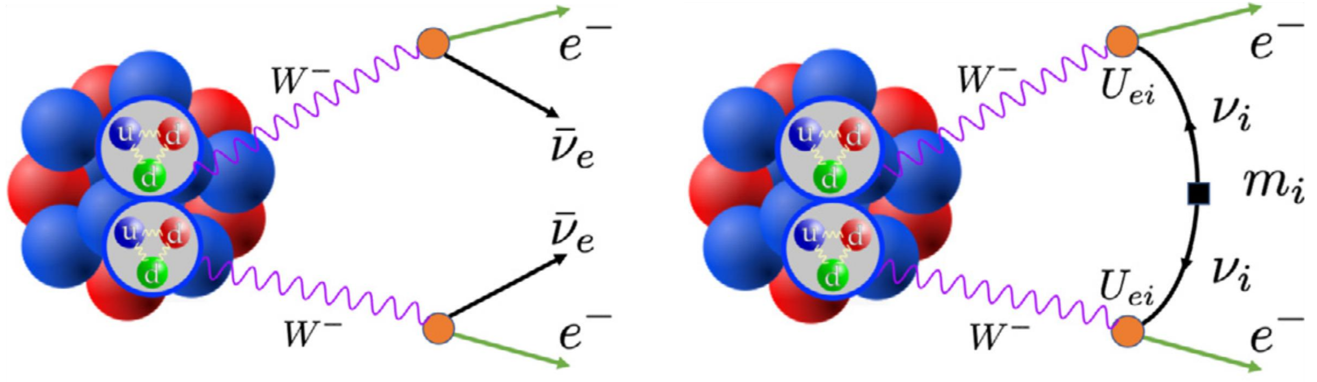
If it is true, then

- Why neutrons have masses
- Leptogenesis
- Beyond standard model

Probes: Neutrinoless double-β decay (0νββ decay)

In certain even-even nuclei, β decay is energetically forbidden, because *m*(*Z*, *A*) < *m*(*Z*+1, *A*), while double-β decay, from a nucleus of (*Z*, *A*) to (*Z*+2, *A*), is allowed.





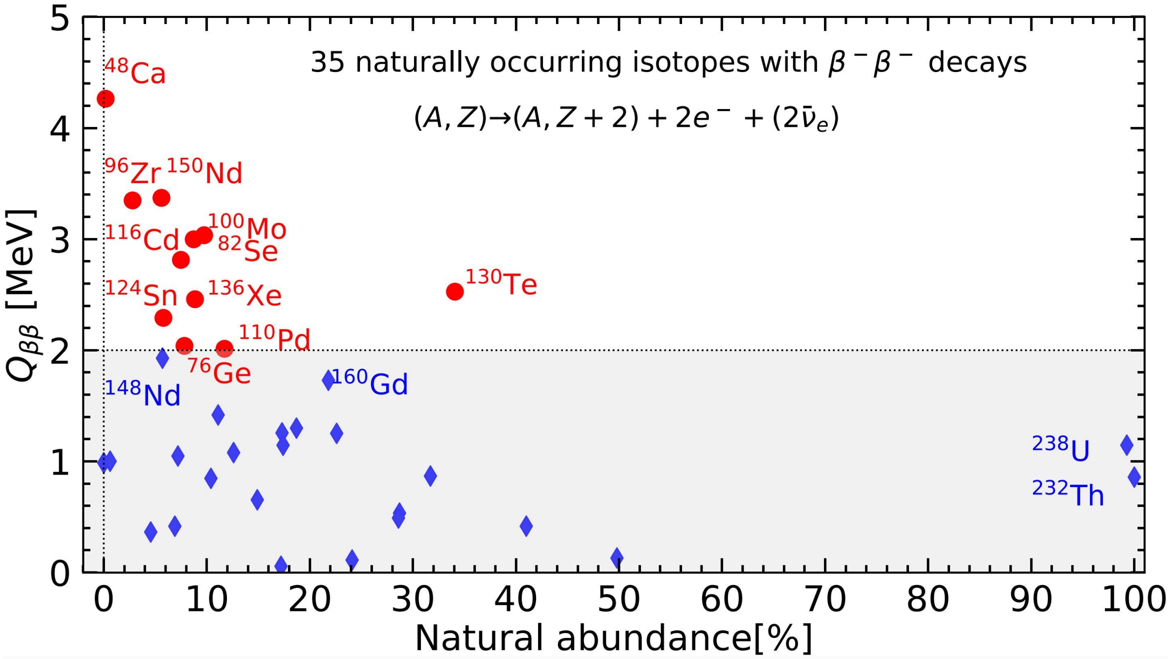
2νββ: observed **0νββ: not yet**

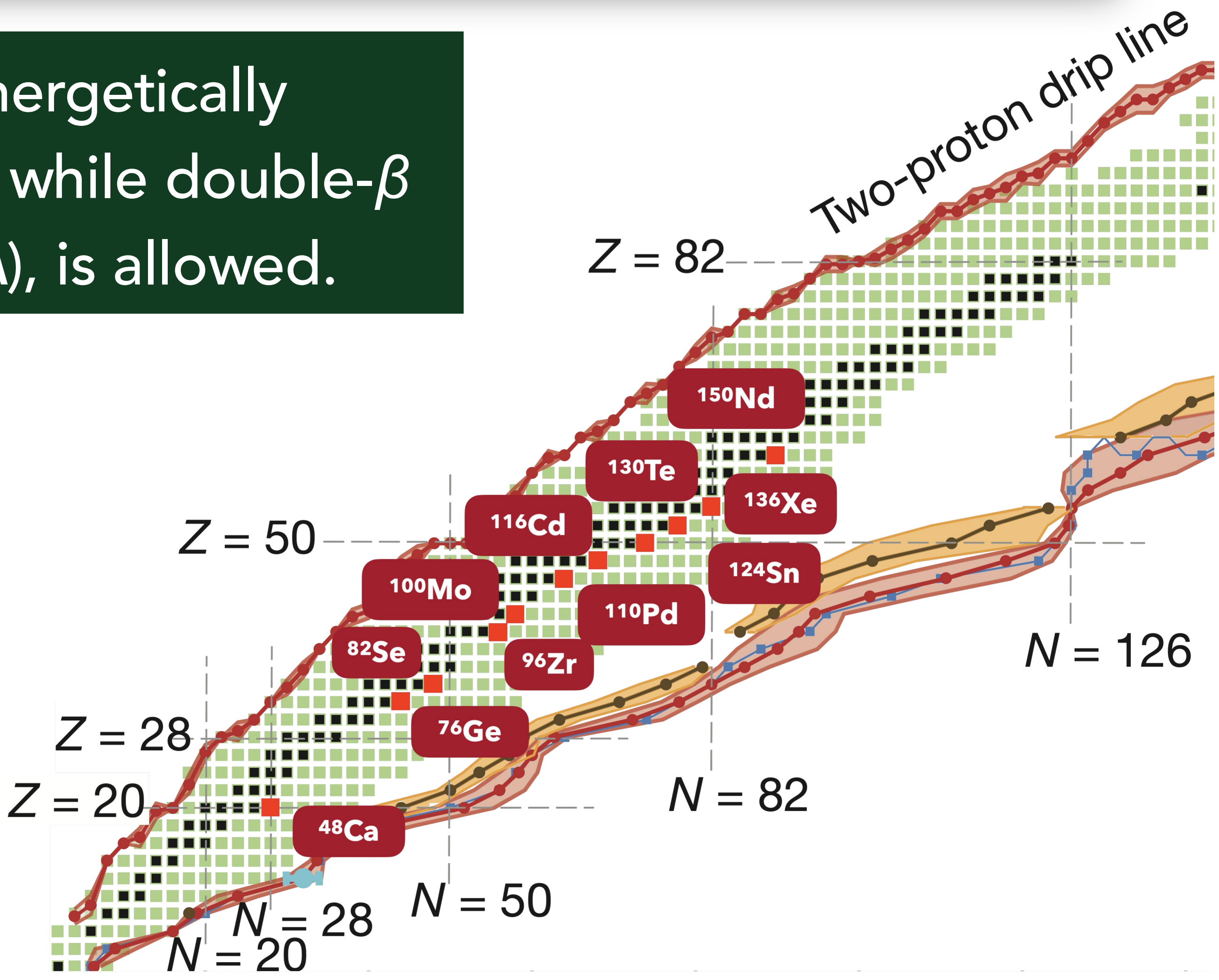
0νββ decay is interesting since:

- ☐ Lepton-number violation, baryongenesis.
- ☐ May be the only way to determine whether neutrino is a Majorana Fermion.

Probes: Neutrinoless double-β decay (0νββ decay)

In certain even-even nuclei, β decay is energetically forbidden, because *m*(*Z*, *A*) < *m*(*Z*+1, *A*), while double-β decay, from a nucleus of (*Z*, *A*) to (*Z*+2, *A*), is allowed.

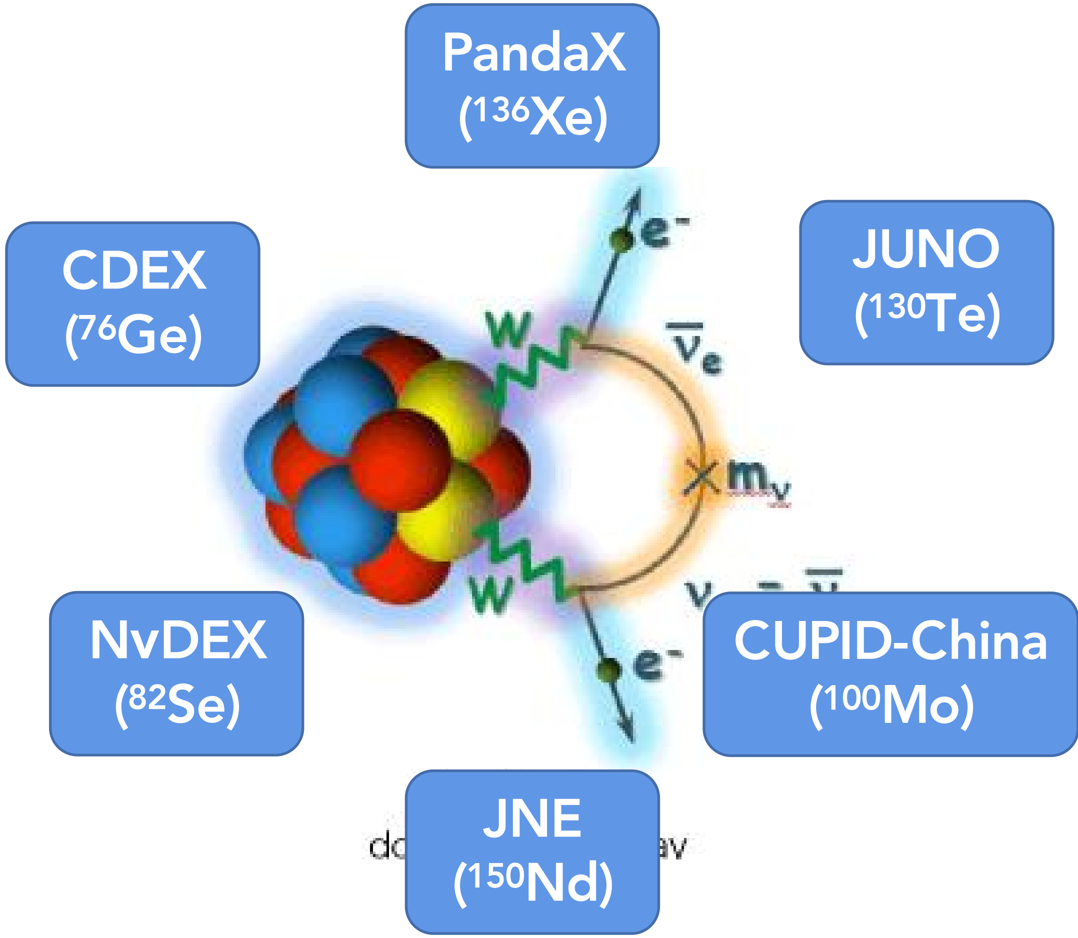




0νββ decay experiments

Overseas

	Isotope	Technique	mass (0νββ isotope)	Status
CANDLES	Ca-48	305 kg CaF$_2$ crystals - liq. scint	0.3 kg	Construction
CARVEL	Ca-48	^{48}CaWO$_4$ crystal scint.	~ ton	R&D
GERDA I	Ge-76	Ge diodes in LAr	15 kg	Complete
GERDA II	Ge-76	Point contact Ge in LAr	31	Operating
MAJORANA DEMONSTRATOR	Ge-76	Point contact Ge	25 kg	Operating
LEGEND	Ge-76	Point contact	**~ ton**	R&D
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Complete
SuperNEMO Demonstrator	Se-82	Foils with tracking	7 kg	Construction
SuperNEMO	Se-82	Foils with tracking	100 kg	R&D
LUCIFER (CUPID)	Se-82	ZnSe scint. bolometer	18 kg	R&D
AMoRE	Mo-100	CaMoO$_4$ scint. bolometer	1.5 - 200 kg	R&D
LUMINEU (CUPID)	Mo-100	ZnMoO$_4$ / Li$_2$MoO$_4$ scint. bolometer	1.5 - 5 kg	R&D
COBRA	Cd-114,116	CdZnTe detectors	10 kg	R&D
CUORICINO, CUORE-0	Te-130	TeO$_2$ Bolometer	10 kg, 11 kg	Complete
CUORE	Te-130	TeO$_2$ Bolometer	**206 kg**	Operating
CUPID	Te-130	TeO$_2$ Bolometer & scint.	**~ ton**	R&D
SNO+	Te-130	0.3% ^{nat}Te suspended in Scint	**160 kg**	Construction
EXO200	Xe-136	Xe liquid TPC	79 kg	Operating
nEXO	Xe-136	Xe liquid TPC	**~ ton**	R&D
KamLAND-Zen (I, II)	Xe-136	2.7% in liquid scint.	**380 kg**	Complete
KamLAND2-Zen	Xe-136	2.7% in liquid scint.	**750 kg**	Upgrade
NEXT-NEW	Xe-136	High pressure Xe TPC	5 kg	Operating
NEXT	Xe-136	High pressure Xe TPC	100 kg - **ton**	R&D
PandaX - 1k	Xe-136	High pressure Xe TPC	~ **ton**	R&D
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D



Domestic

The importance of NME in 0νββ decay

From neutrino oscillations we know

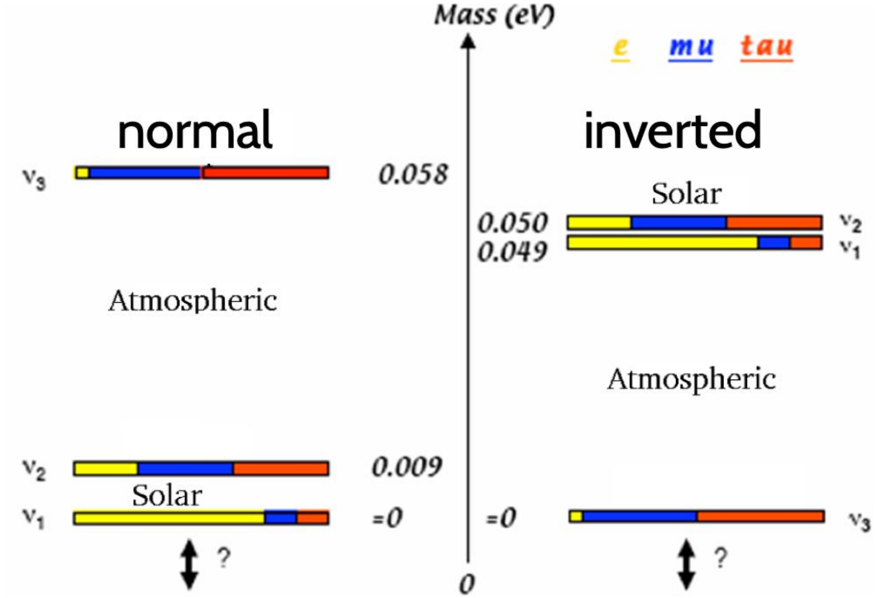
$$\Delta m^2_{\rm sun} \simeq 75\ {\rm meV}^2 \qquad \Delta m^2_{\rm atm} \simeq 2400\ {\rm meV}^2$$

We can get the mixing angles

$$m_{\beta\beta} \equiv \left| \sum_k m_k U^2_{ek} \right|$$

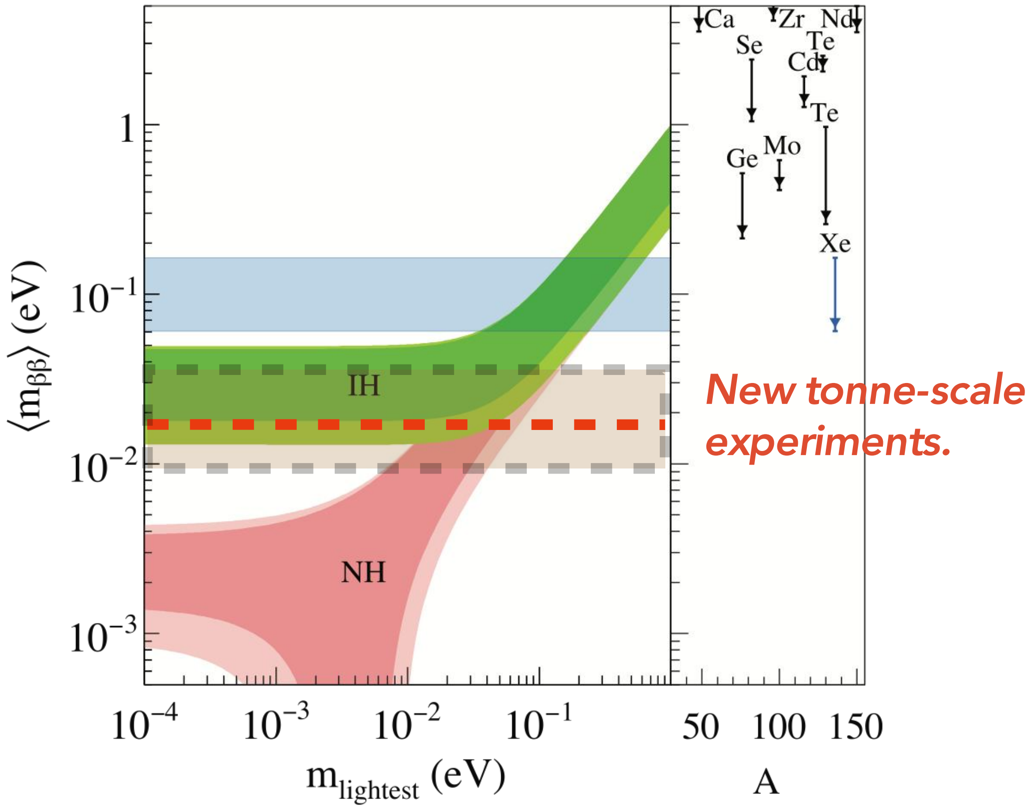
U_{ek} from PMNS matrix

But we *don't* know the absolute mass scale and mass hierarrchy.



0νββ decay can help since

$$[T^{0\nu}_{1/2}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$



The large uncertainty comes from NMEs.

Probes: Neutrinoless double-β decay (0νββ decay)

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

$$M^{0\nu} = M_{\rm GT}^{0\nu} - \frac{g_V^2}{g_A^2}M_{\rm F}^{0\nu} + M_{\rm T}^{0\nu}$$

with

$$M_{\rm GT}^{0\nu} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\rm GT}(|q|)\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

$$M_{\rm F}^{0\nu} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\rm F}(|q|)}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

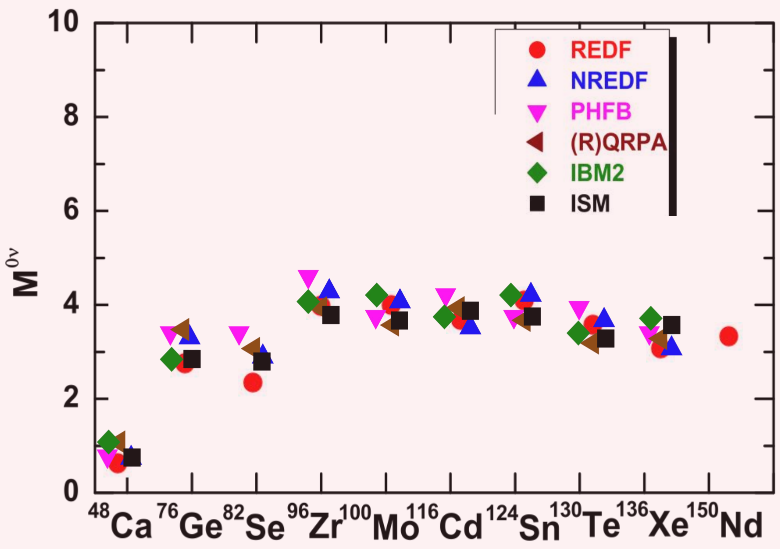
$$M_{\rm T}^{0\nu} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_2(|q|r_{ab})h_{\rm T}(|q|)[3\vec{\sigma}_j\cdot\hat{r}_{ab}\vec{\sigma}_k\cdot\hat{r}_{ab}-\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

Lines of attack:

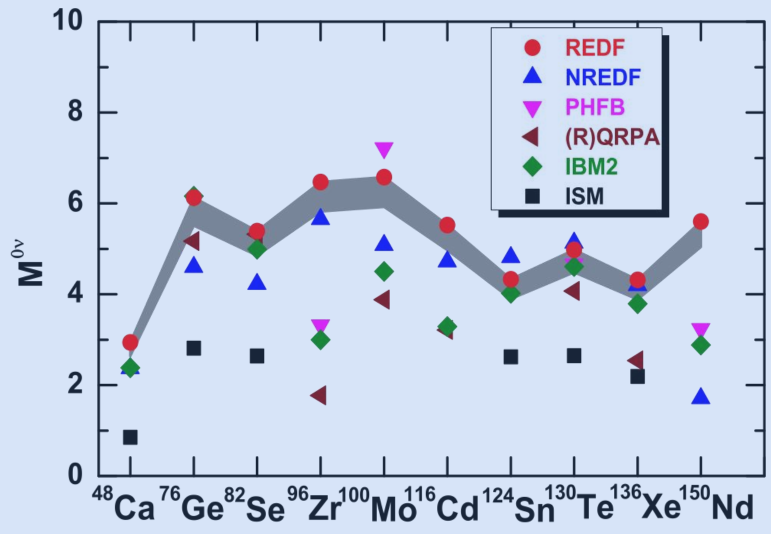
- Construct effective operator.
- *Good initial and final ground-state wave functions: nuclear structure.*

Current status of calculated NMEs in 0νββ decay

What we hope:



What we had got:



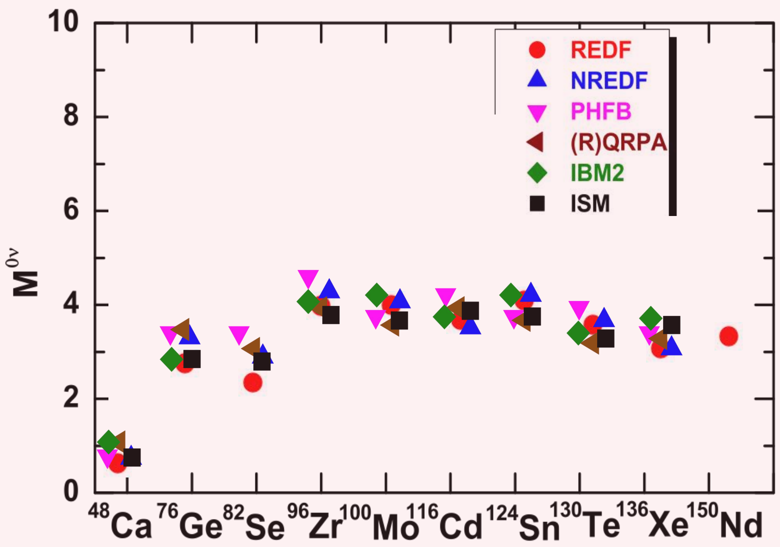
It poses challenges for nuclear structure studies:

- ☐ Some **omits the correlations** underlying nuclear structure aspects.
- ☐ Some limits the correlations in **a small model space.**

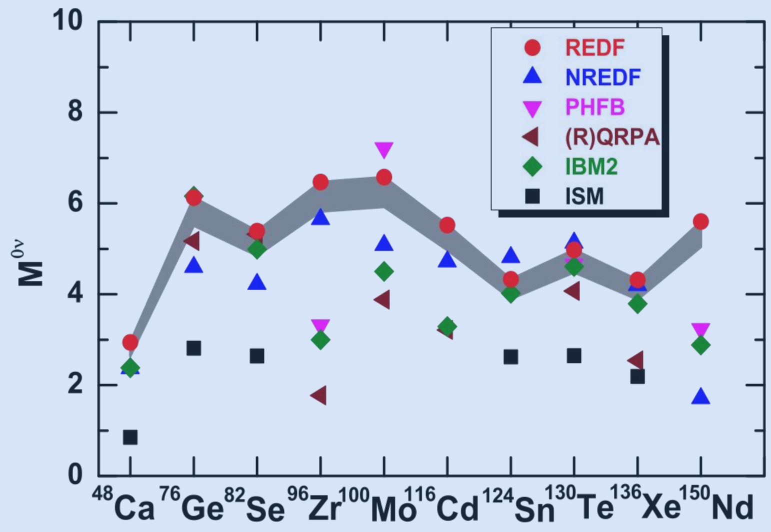
Does the discrepancy come from methods, or the interactions they use?

Current status of calculated NMEs in 0νββ decay

What we hope:



What we had got:



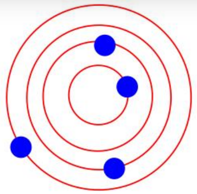
In short, we need:

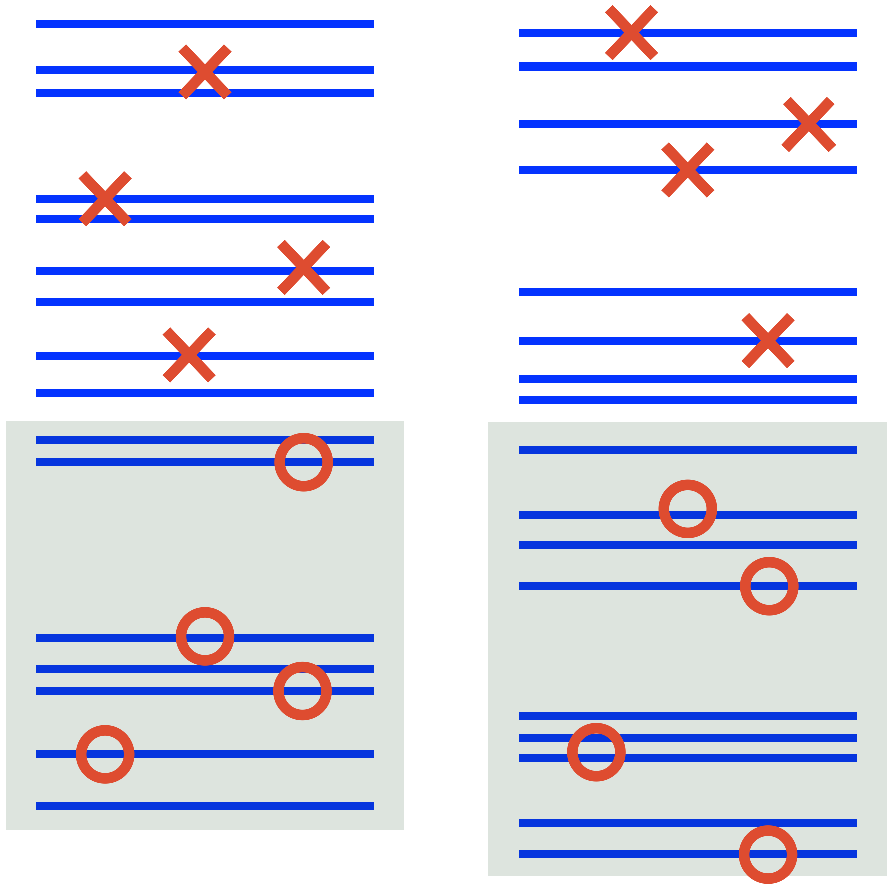
- ☐ Understanding the effect from collective correlations on NMEs.
- ☐ Understanding the effect from enlarging the model space.

And a better effective interaction.

Nuclear models applied on calculations of NME

Some models are built on single independent-particle state.





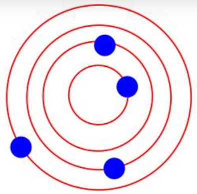
Starting from one Slater determinant, e.g., the HF state $|\psi_0\rangle$, the ground state

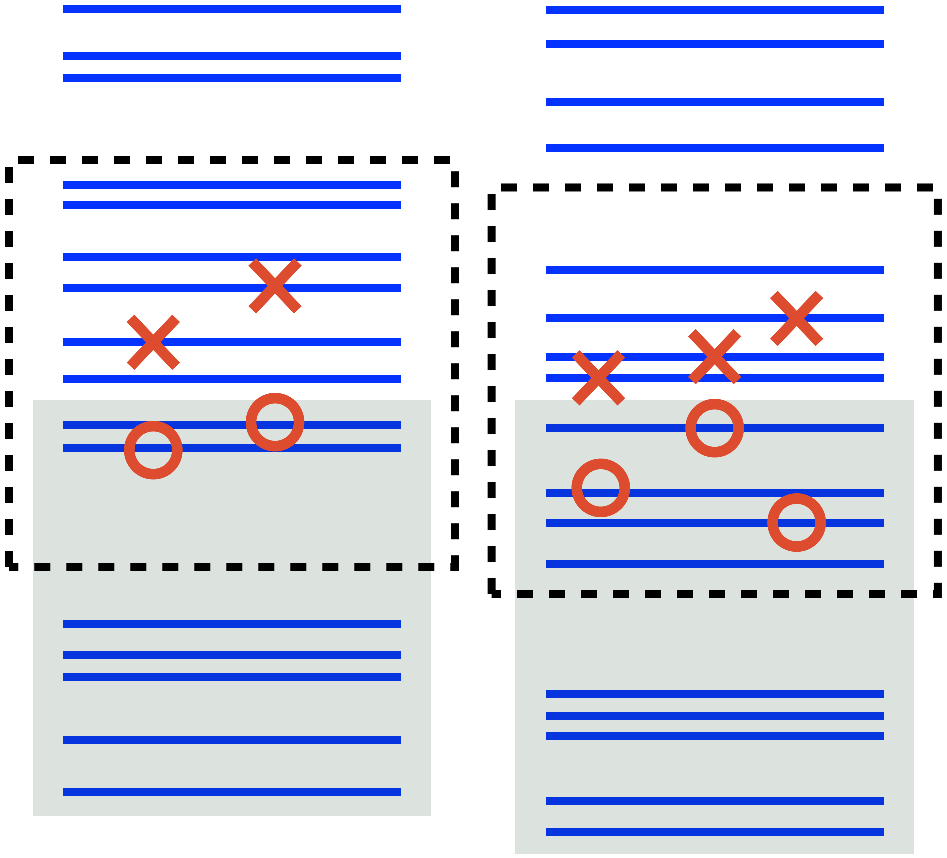
$$|0\rangle = |\psi_0\rangle + \sum_{mi} C^0_{mi} a^\dagger_m a_i |\psi_0\rangle + \frac{1}{4} \sum_{mnij} C^0_{mn,ij} a^\dagger_m a^\dagger_n a_i a_j |\psi_0\rangle + \cdots$$

But exact diagonalization in the complete Hilbert space is not solvable.

Nuclear models applied on calculations of NME

Some models are built on single independent-particle state.





Interacting shell model (ISM)

- Same starting point $|0\rangle$.
- Instead of solving Schrödinger equation in complete Hilbert space, one restricts the dynamics in a configuration space.

$$H|\Phi_i\rangle = E_i|\Phi_i\rangle \to H_{\text{eff}}|\bar{\Phi}_i\rangle = E_i|\bar{\Phi}_i\rangle$$

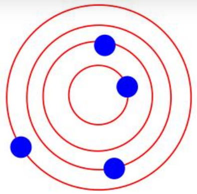
Configuration interaction of orthonormal Slater determinants:

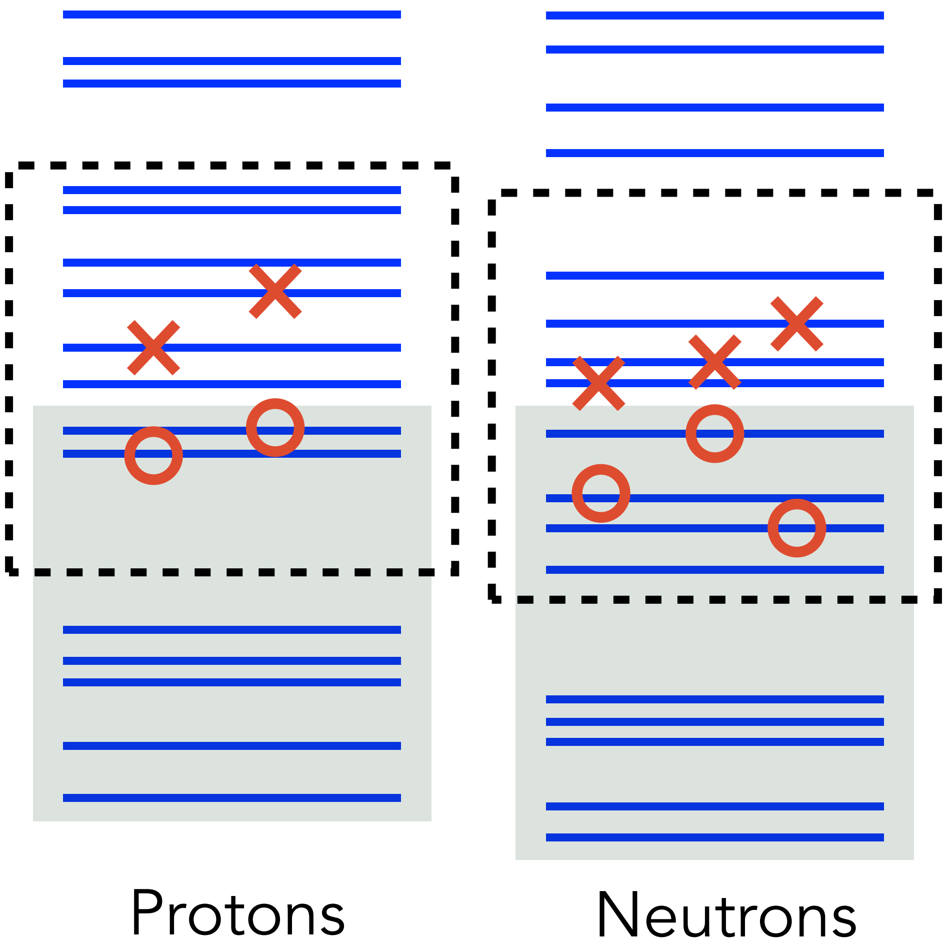
$$|\bar{\Phi}_i\rangle = \sum_j c_{ij}|\psi_j\rangle, \qquad \langle\psi_j|\psi_k\rangle = \delta_{jk}$$

Diagonalizing the H_{eff} in the orthonormal basis.

Nuclear models applied on calculations of NME

Some models are built on single independent-particle state.





Interacting shell model (ISM)

Pros:

- Arbitrarily complex correlations within the model space.

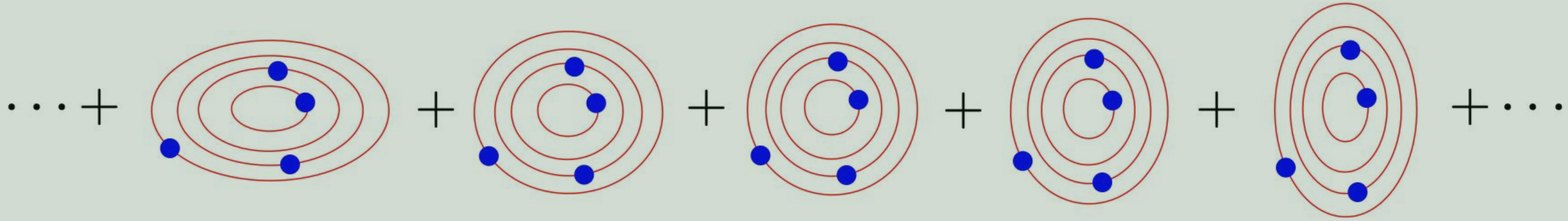
Cons:

- Relatively small configuration spaces.
  - *At present most of the 0νββ decay NME calculations carried out by ISM are limited in one single shell.*

The Other Way Around...

Generator-coordinate method (GCM)

Instead of configuration interaction with orthogonal states, one can diagonalize the Hamiltonian in a set of *non-orthogonal* basis.



$$|\Phi\rangle = \sum_j c_j |\psi_j\rangle, H_{jk} = \langle j|H|k\rangle$$

$$\sum_k H_{jk} c_k = E \sum_k N_{jk} c_k, N_{jk} = \langle j|k\rangle$$

The non-orthogonal states can be generated to give different quantities of many-body correlations as collective coordinates (***fluctuations of deformation, pairing...***).

Hamiltonian-based projected generator-coordinate method

- Using a realistic effective Hamiltonian.
- ***Trying to include all possible correlations.*** (For now, we pick the most important ones)

  $\mathcal{O}_1 = Q_{20}, \quad \mathcal{O}_2 = Q_{22},$ **quadrupole correlations**

  $\mathcal{O}_3 = \frac{1}{2}(P_0 + P_0^\dagger), \quad \mathcal{O}_4 = \frac{1}{2}(S_0 + S_0^\dagger),$ **proton-neutron pairing correlations**
- HFB states with multipole constraints

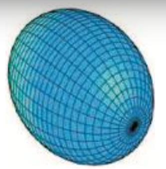
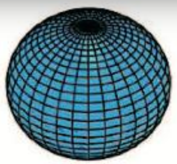
  $$\langle H' \rangle = \langle H_{\text{eff}} \rangle - \lambda_Z(\langle N_Z \rangle - Z) - \lambda_N(\langle N_N \rangle - N) - \sum_i \lambda_i(\langle \mathcal{O}_i \rangle - q_i),$$
- Angular momentum and particle number projection $|JMK;NZ;q\rangle = \hat{P}^J_{MK}\hat{P^N}\hat{P^Z}|\Phi(q)\rangle$
- Configuration mixing within generator-coordinate method (GCM)

GCM wavefunction: $|\Psi^J_{NZ\sigma}\rangle = \sum_{K,q} f^{JK}_\sigma(q)|JMK;NZ;q\rangle$

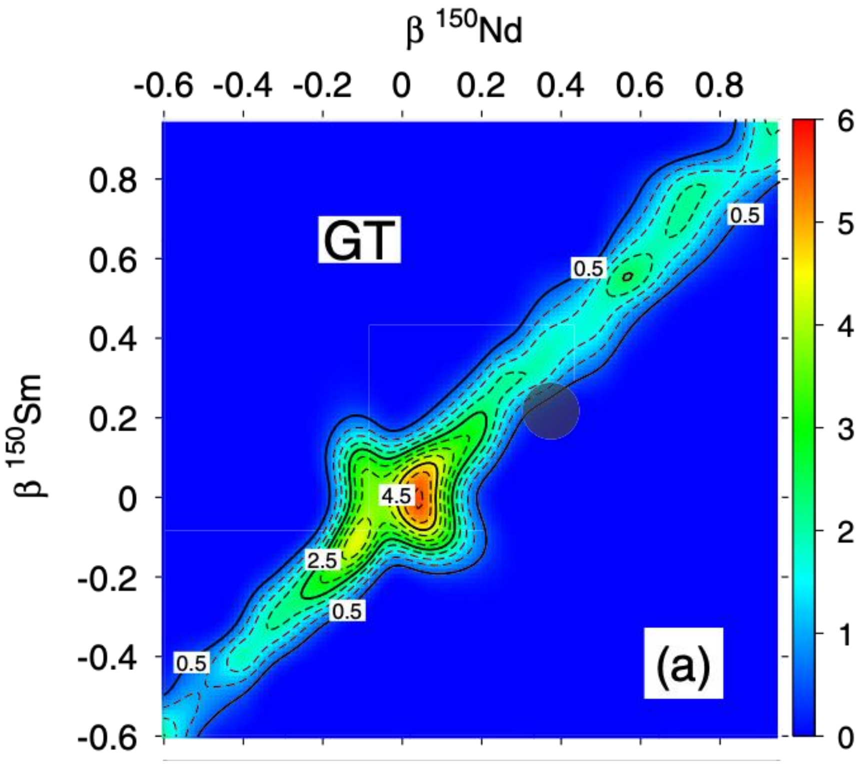
Hill-Wheeler equation: $\sum_{K',q'} \{\mathcal{H}^J_{KK'}(q;q') - E^J_\sigma \mathcal{N}^J_{KK'}(q;q')\} f^{JK'}_\sigma(q') = 0$

$0\nu\beta\beta$ NME: $M^{0\nu\beta\beta}_\xi = \langle \Psi^{J=0}_{N_f Z_f}|\hat{O}^{0\nu\beta\beta}_\xi|\Psi^{J=0}_{N_i Z_i}\rangle$

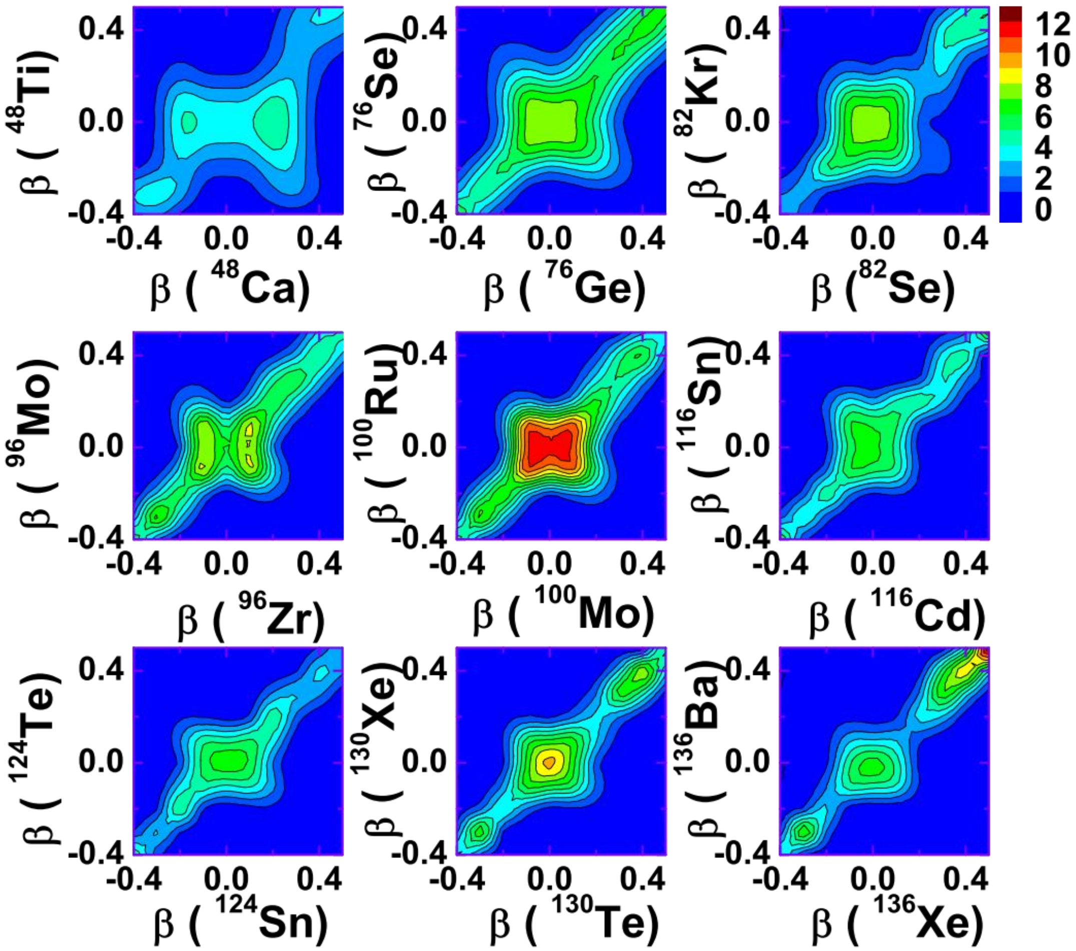
Which correlations are the most relevant to NMEs?

Axial deformation prolate   oblate

If parent and daughter nuclei have different axial deformation, **NMEs are suppressed.**



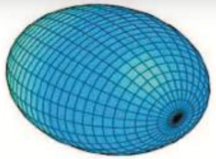
T. Rodriguez *et. al.*, PRL **105**, 252503 (2010)



J. Yao, *et. al.*, PRC **91**, 024316 (2015).

Which correlations are the most relevant to NMEs?

Triaxial deformation



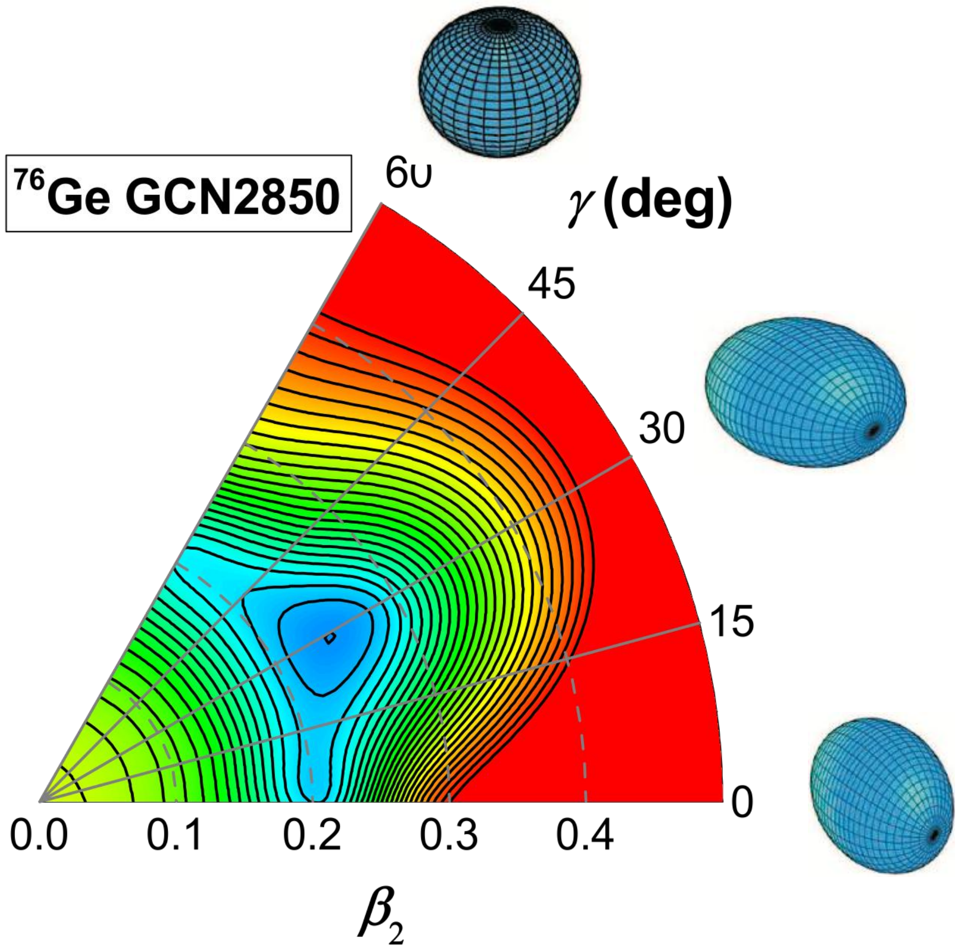
triaxial shape

Both theory and experiment indicate that ^{76}Ge and ^{76}Se are triaxially deformed, but the effect on $0\nu\beta\beta$ NMEs has never been investigated.

TABLE I. Matrix elements $M^{0\nu}$ produced in the GCM by GCN2850 and JUN45 for the decay of ^{76}Ge, with and without triaxial deformation as a generator coordinate, and by those same interactions with exact diagonalization.

	GCN2850	JUN45
Axial GCM	2.93	3.51
Triaxial GCM	2.56	3.16

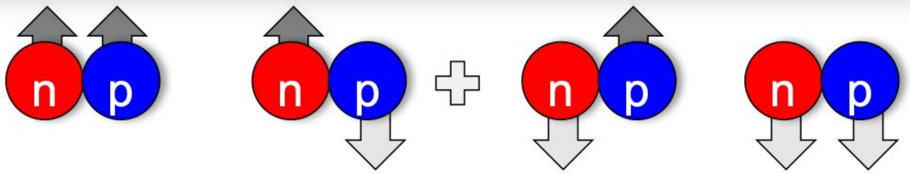
If triaxial deformation is included,
NMEs are slightly suppressed by 10~15%



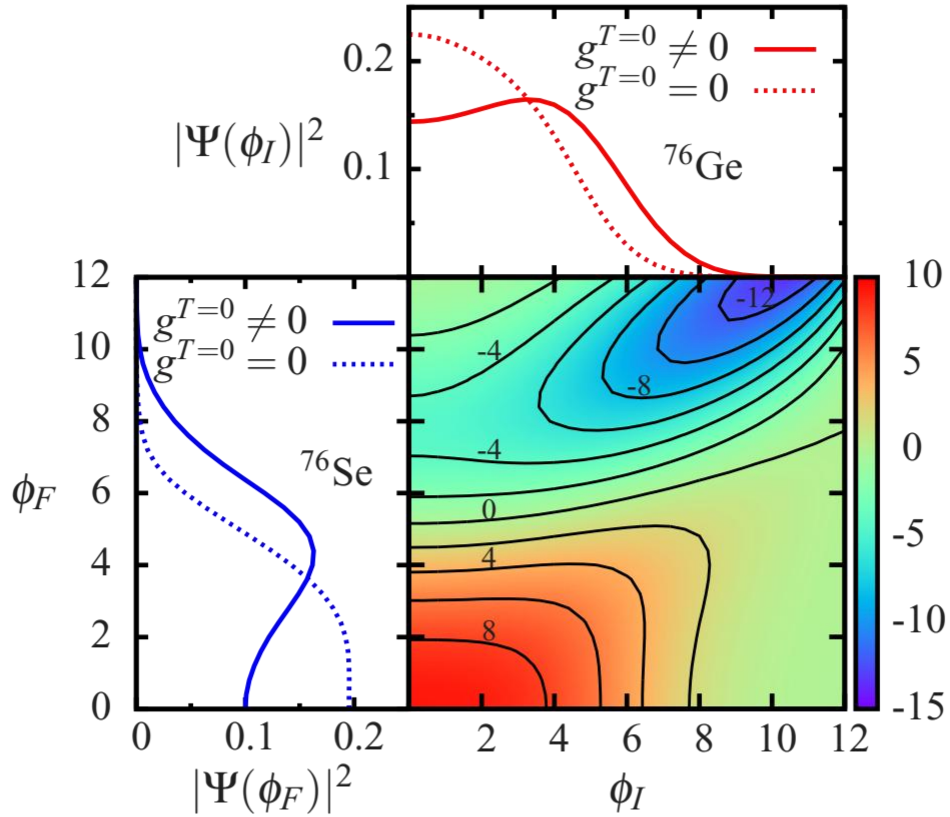
CFJ, J. Engel, J. D. Holt, PRC **96**, 054310 (2017).

Which correlations are the most relevant to NMEs?

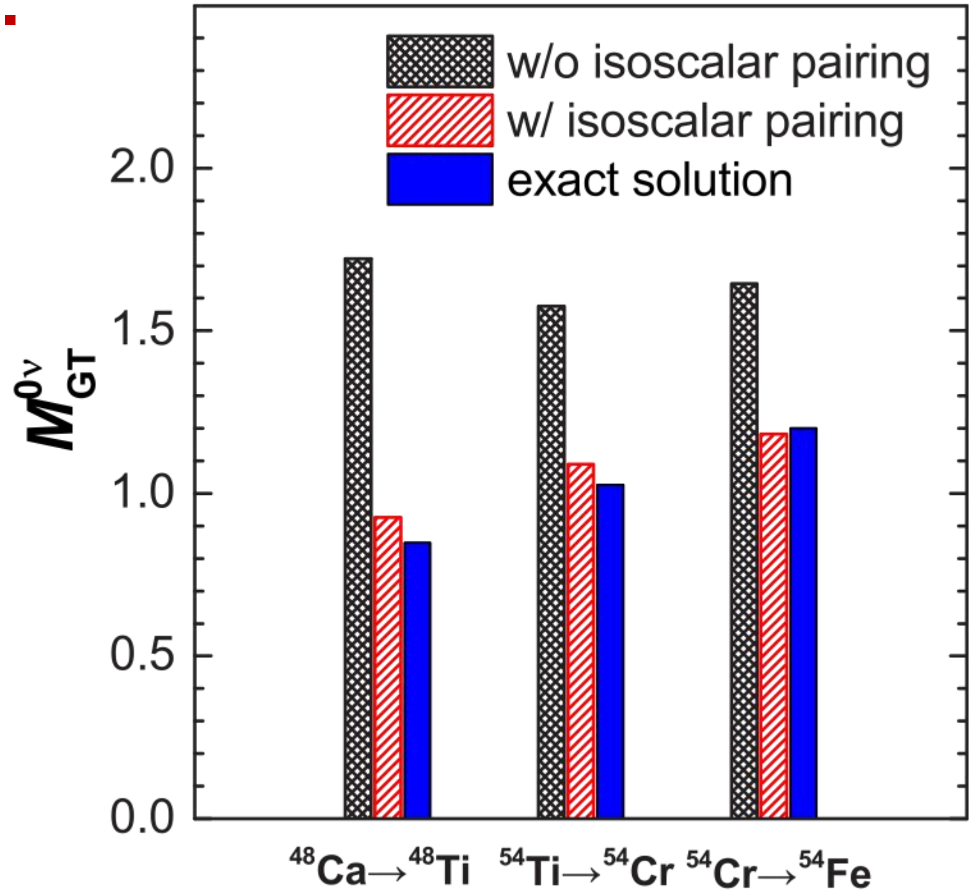
Proton-neutron pairing



If *pn* pairing is included, **NMEs are suppressed.**

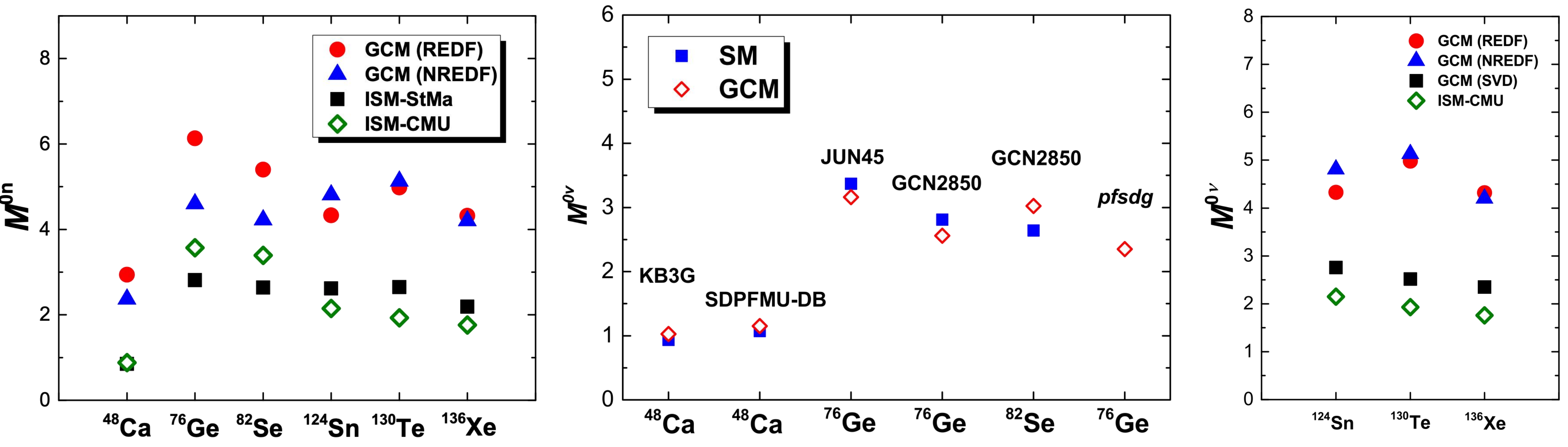


N. Hinohara et. al., PRC 90, 031301(R) (2014)



CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

If we treat these collective correlations correctly...



Axial deformation only

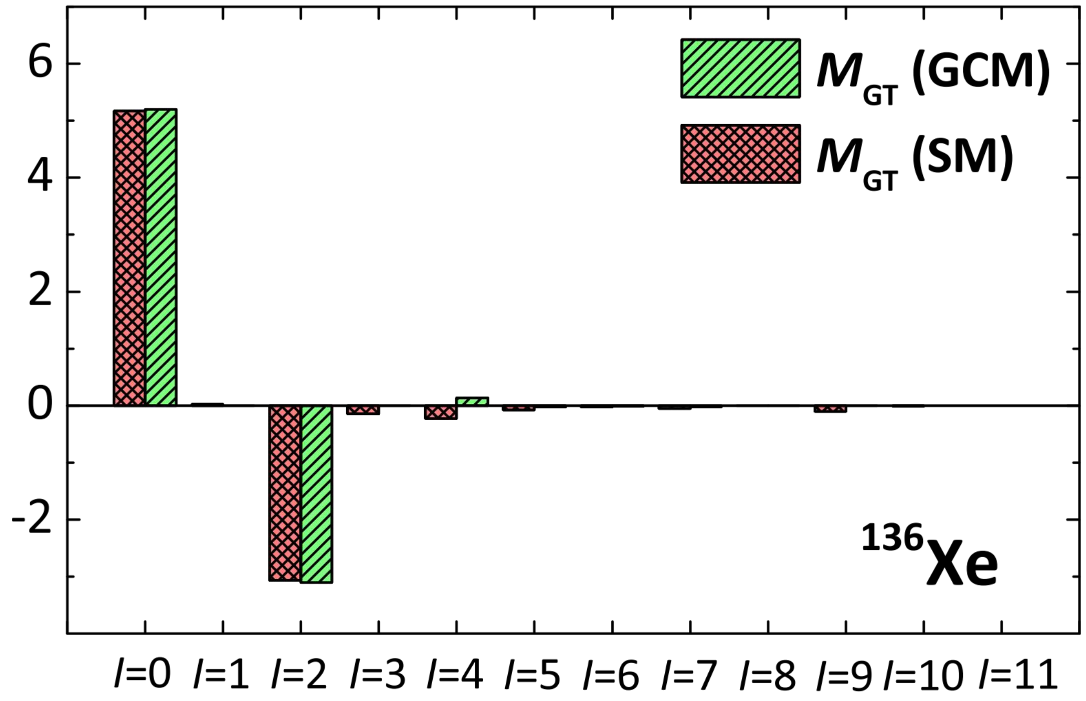
Axial deformation + triaxial deformation + *pn* pairing

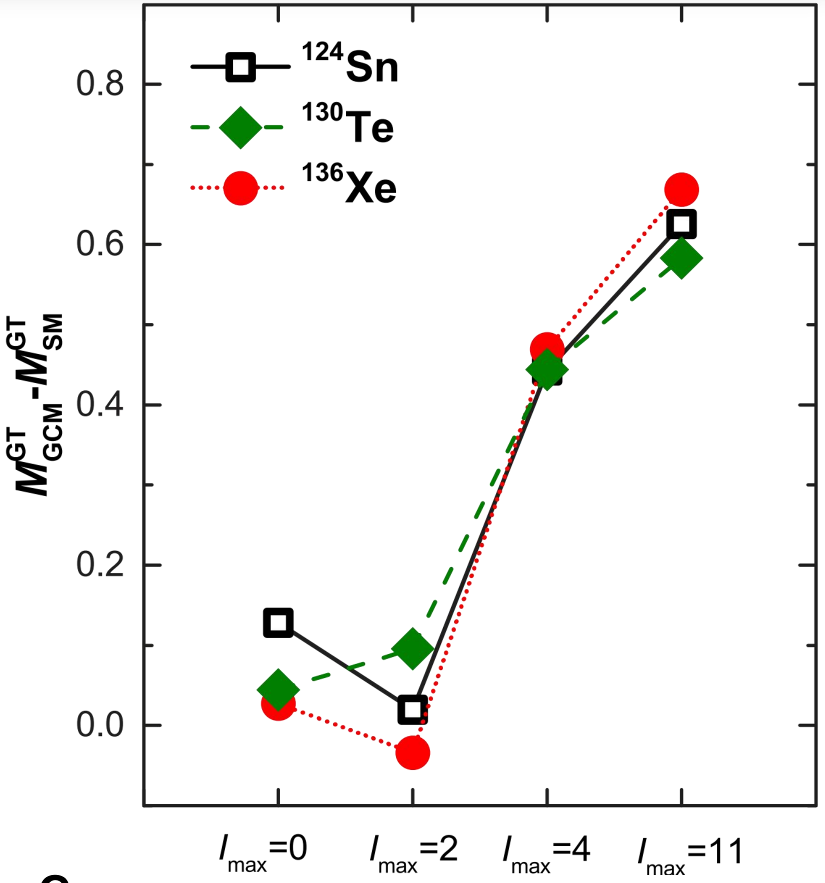
Most of the deviation between the GCM and the SM vanishes.

CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).
CFJ, M. Horoi, A. Neacsu, PRC 98, 064324 (2018).

If we treat these collective correlations correctly...

I-pair decomposition:





We barely capture the contributions with pair spin $I > 3$.

There must be something else.

Perhaps vibrational correlations or qp excitations

CFJ, M. Horoi, A. Neacsu, PRC 98, 064324 (2018)

If we further include 2-qp excitations via QTDA-driven GCM...

I proposed a novel idea to incorporate important correlations in GCM.

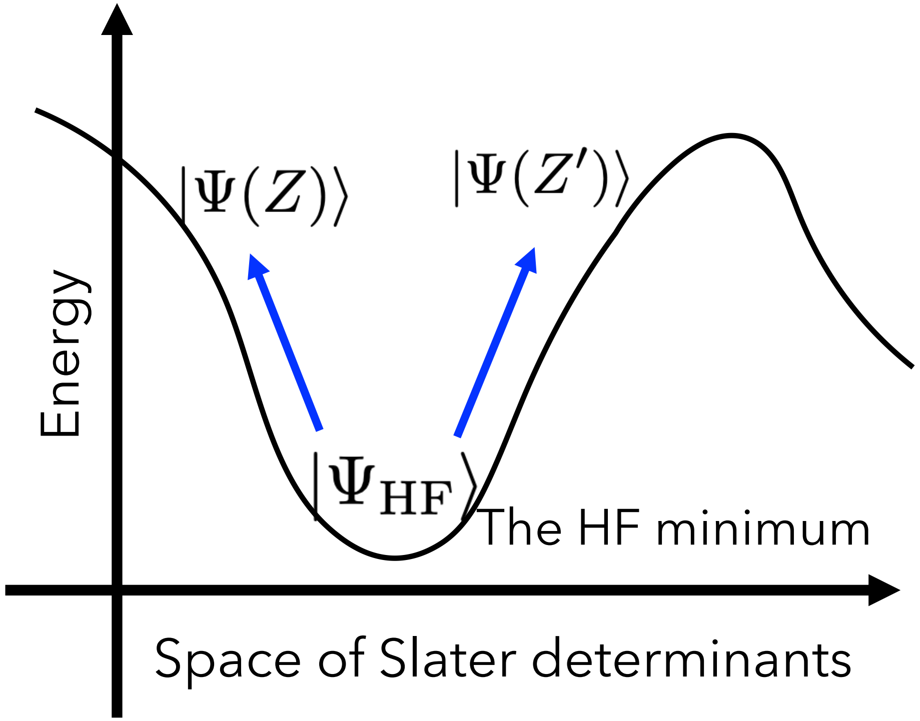
Starts from the HF minimum.

Apply Thouless evolution to explore the energy landscape

Thouless theorem:

$$\exp(\hat{Z})|\Psi\rangle = |\Psi'\rangle \equiv |\Psi(Z)\rangle$$

$\Rightarrow$ $|\Psi(Z)\rangle = \exp(\hat{Z})|\Psi_{\rm HF}\rangle$



Difine an energy landscape $E(Z) = \langle\Psi(Z)|\hat{H}|\Psi(Z)\rangle$ which can be expanded in *Z*. Note that the curvature around HF minimum approximates the landscape as a quadratic in *Z* and thus a multi-dimensional harmonic oscillator, leading to TDA/RPA and their quasiparticle extension.

If we further include 2-qp excitations via QTDA-driven GCM...

Here we generate non-orthogonal states by applying Thouless evolution with QTDA operators.

Low-lying excited states are approximated as linear combinations of two-quasiparticle excitations, represented by QTDA operator:

$$\hat{Z}_r = \frac{1}{2}\sum_{\alpha\alpha'} Z^r_{\alpha\alpha'}\hat{c}^\dagger_\alpha(0)\hat{c}^\dagger_{\alpha'}(0)$$

where $\hat{c}_\alpha(0) = \sum_\beta \hat{a}_\beta U^*_{\beta\alpha}(0) + \hat{a}^\dagger_\beta V^*_{\beta\alpha}(0)$

One computes the matrix elements of the Hamiltonian in a basis of two-quasiparticle excited states

$$A_{\alpha\alpha',\beta\beta'} = \langle\Phi_0|[\hat{c}_{\alpha'}(0)\hat{c}_\alpha(0), [\hat{H}, \hat{c}^\dagger_\beta(0)\hat{c}^\dagger_{\beta'}(0)]]|\Phi_0\rangle$$

We then solve $\sum_{\beta\beta'} A_{\alpha\alpha',\beta\beta'} Z^r_{\beta\beta'} = E^{\mathrm{QTDA}}_r Z^r_{\alpha\alpha'}.$

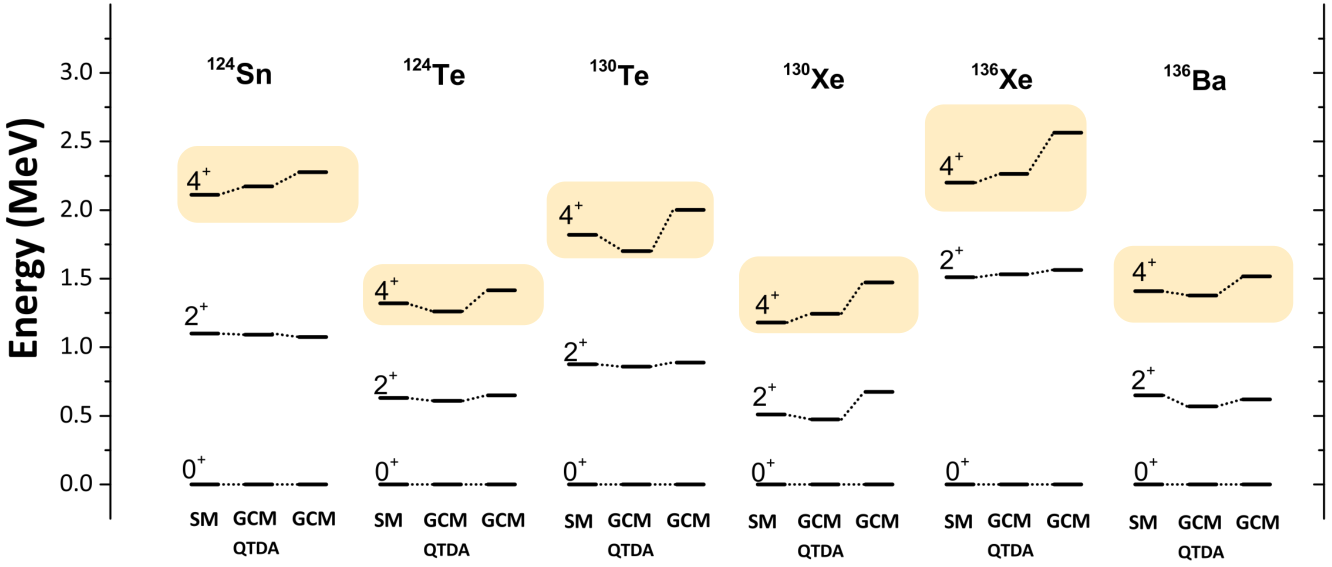
to find the coefficients $Z^r_{\alpha\alpha'}$ of QTDA operator, and apply Thouless theorem to get a new state

$$|\Phi_r\rangle = \exp\left(\lambda\hat{Z}_r\right)|\Phi_0\rangle$$

Then we project these states onto good quantum numbers and take them as basis states for GCM.

If we further include 2-qp excitations via QTDA-driven GCM...

We generate 2-qp configurations for GCM by applying Thouless evolution with QTDA operators obtained from solving the QTDA equation.



		$M^{0\nu}_{\rm GT}$	$M^{0\nu}_{\rm F}$	$M^{0\nu}_{\rm T}$	$M^{0\nu}$
^{124}Sn	CHFB-GCM	2.48	−0.51	−0.03	2.76
	QTDA-GCM	2.08	−0.73	−0.01	2.53
	SM	1.85	−0.47	−0.01	2.15
^{130}Te	CHFB-GCM	2.25	−0.47	−0.02	2.52
	QTDA-GCM	1.97	−0.69	−0.01	2.39
	SM	1.66	−0.44	−0.01	1.94
^{136}Xe	CHFB-GCM	2.17	−0.32	−0.02	2.35
	QTDA-GCM	1.65	−0.50	−0.01	1.96
	SM	1.50	−0.40	−0.01	1.76

Next move: QRPA operators instead of QTDA operators.

CFJ and C.W. Johnson, PRC 100, 031303(R) (2019)

What is the effect from the enlargement of the model space?

Now we extend the Hamiltonian-based GCM calculation to full *pf-sdg* two-shell space.
It is unreachable by the interacting shell model currently.

- There is no *a priori* effective Hamiltonian in this model space.

  We use extended Krenciglowa-Kuo method of many-body perturbation theory to derive a realistic Hamiltonian from the Chiral EFT.

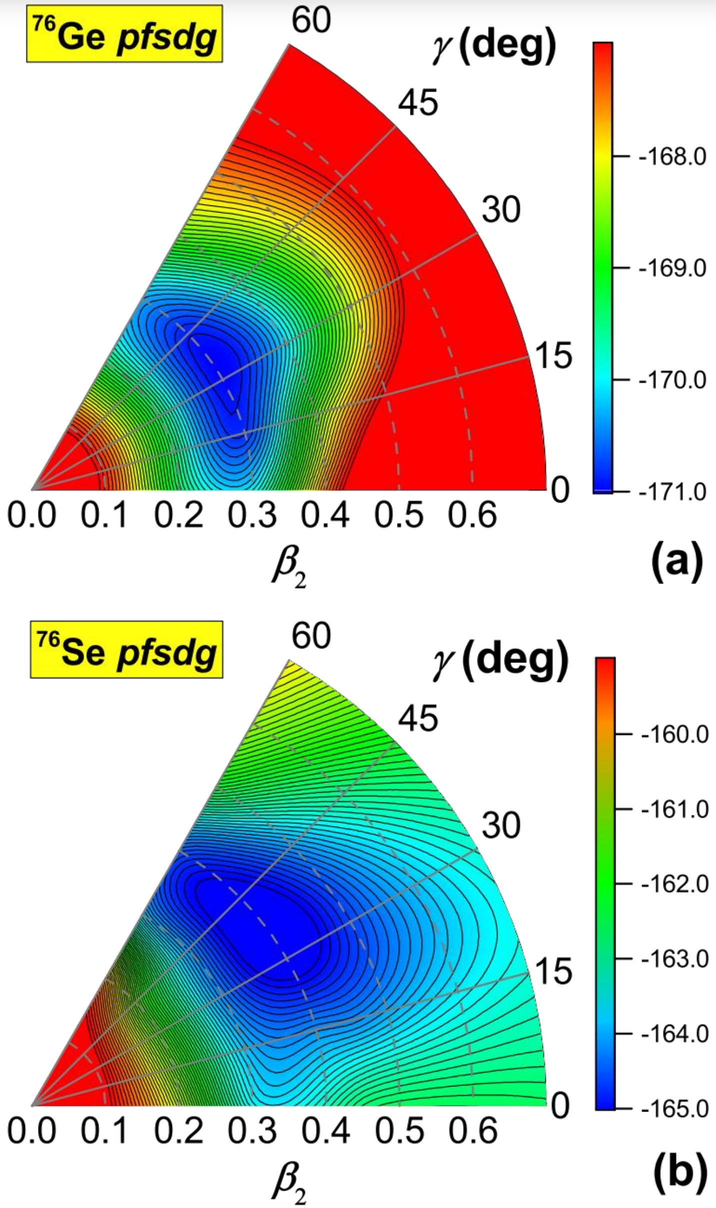
  N. Tsunoda *et. al.*, PRC 89, 024313 (2014).

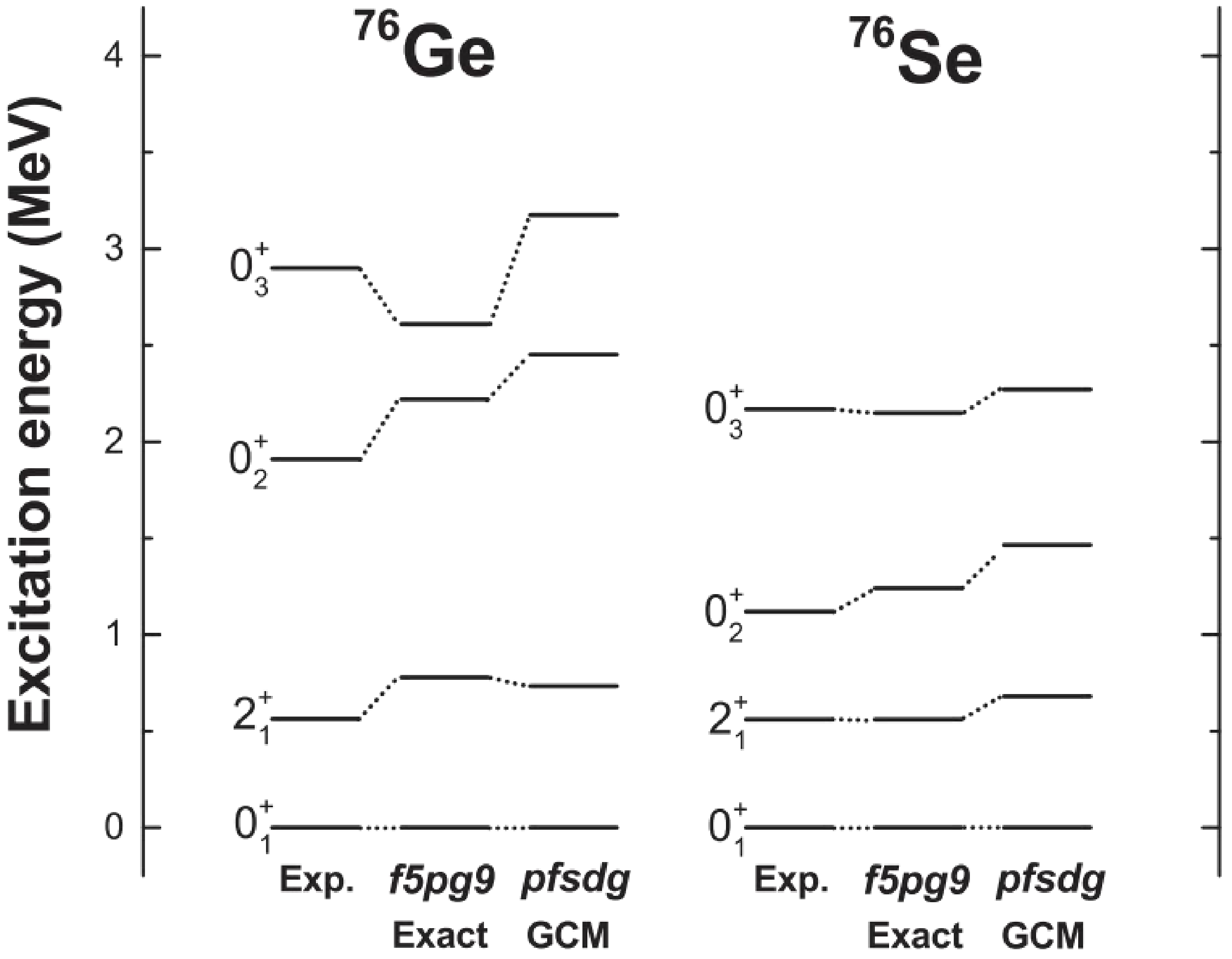
  Begin with the 1.8/2.0 NN+3N chiral interaction.

  J. Simonis *et. al.*, PRC 93, 011302 (2016).

  The initial 3N force reduces to effective 0-, 1-, 2-body parts via normal ordering with respect to the reference state.

What is the effect from the enlargement of the model space?





CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

What is the effect from the enlargement of the model space?

TABLE II. GCM results for the Gamow-Teller ($M^{0\nu}_{GT}$), Fermi ($M^{0\nu}_{F}$), and tensor ($M^{0\nu}_{T}$) $0\nu\beta\beta$ matrix elements for the decay of ^{76}Ge in two shells, without and with triaxial deformation.

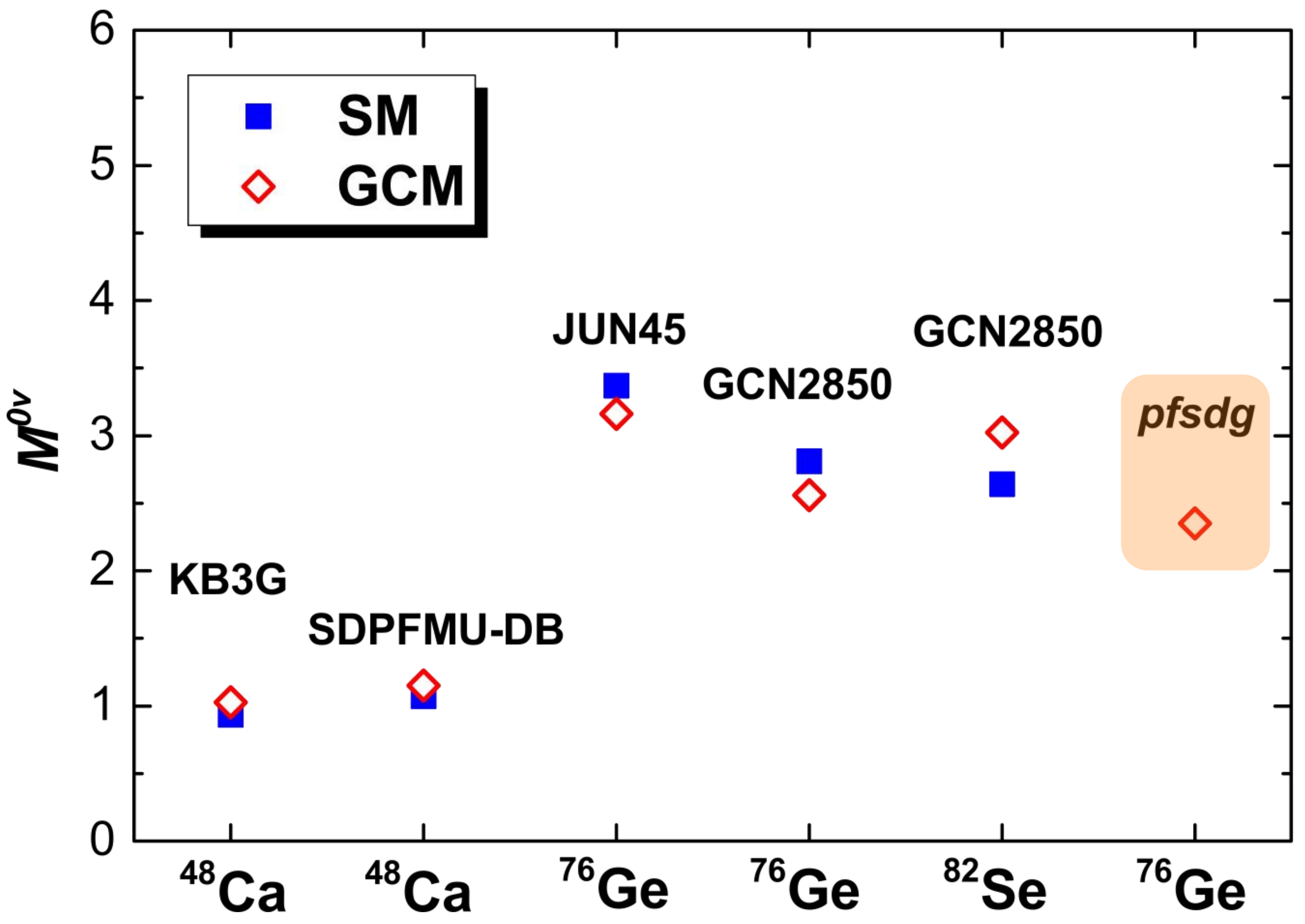
	Axial	Triaxial
$M^{0\nu}_{GT}$	3.18	1.99
$-\frac{g_V^2}{g_A^2} M^{0\nu}_{F}$	0.55	0.38
$M^{0\nu}_{T}$	–0.01	–0.02
Total $M^{0\nu}$	3.72	2.35

Triaxially-deformed result is suppressed: Larger space captures more effect from triaxial deformation.

Axially-deformed result is enhanced: Larger space captures more like-particle pairing.

CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

What is the effect from the enlargement of the model space?



The *pfsdg* "two-shell" result is slightly smaller than single-shell result.

Enlarging the space further may not dramatically change NMEs.

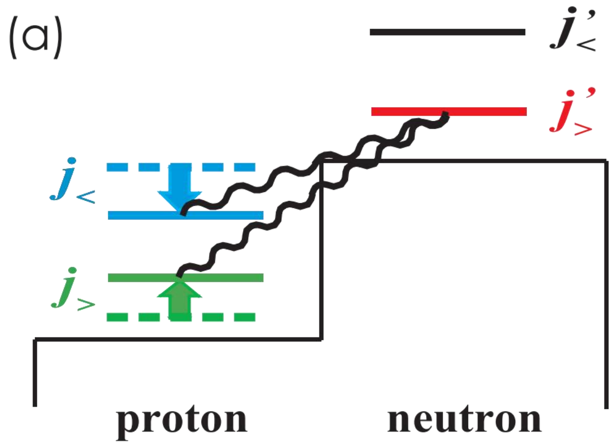
Better effective interactions derived from the Chiral EFT via non-pertubative *ab initio* methods are in progress…

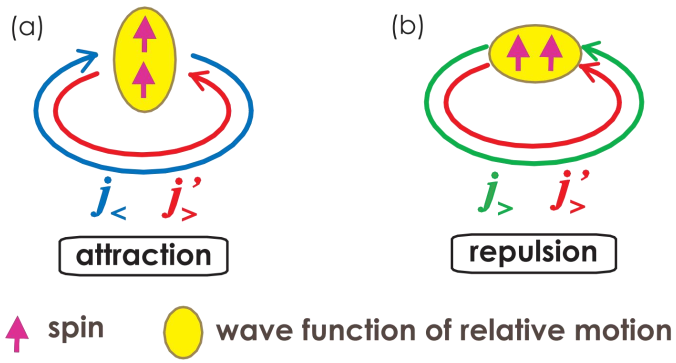
CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

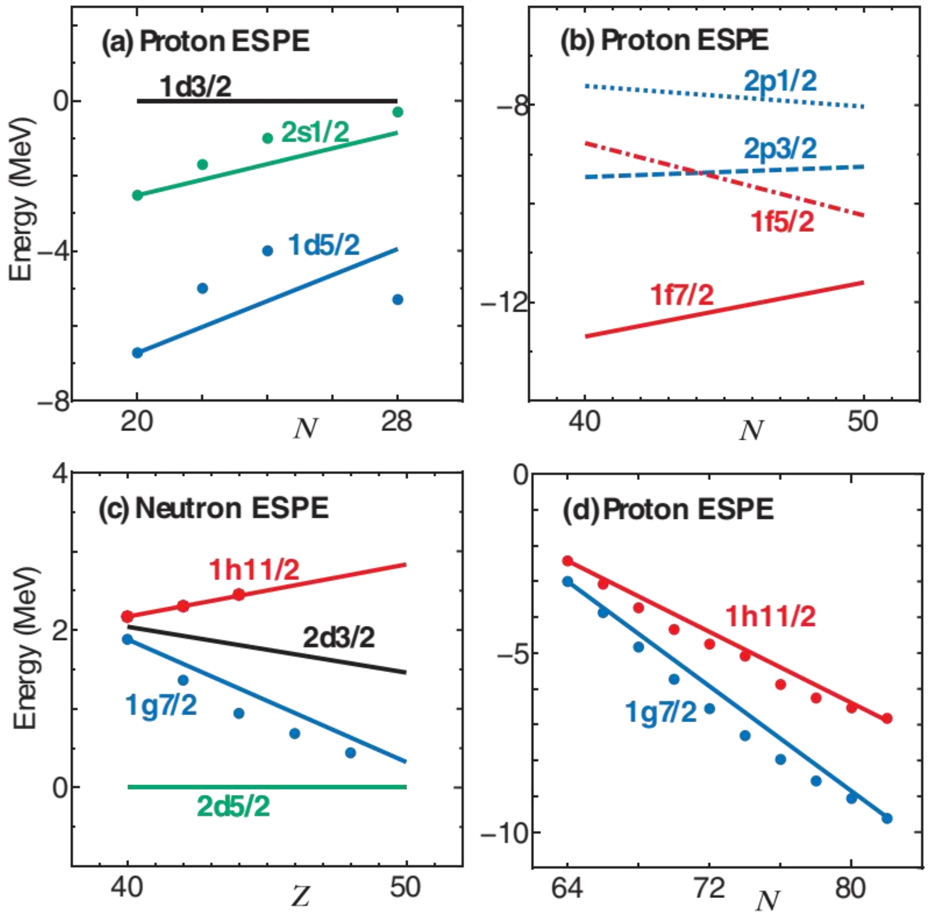
Evaluate the influence of the effective Hamiltonian, e.g., tensor force

The tensor force has a robust effect on the nuclear structure.

The tensor force $V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2)([\vec{s}_1\ \vec{s}_2]^{(2)} \cdot Y^{(2)}) f(r)$





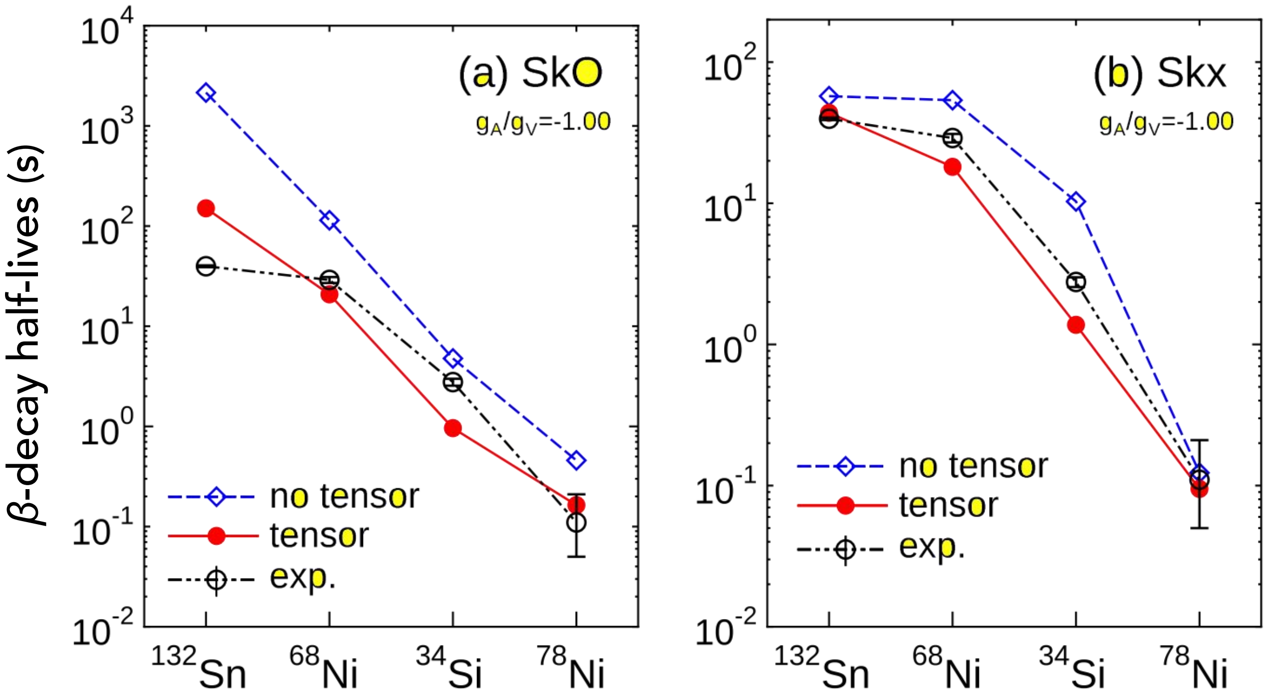


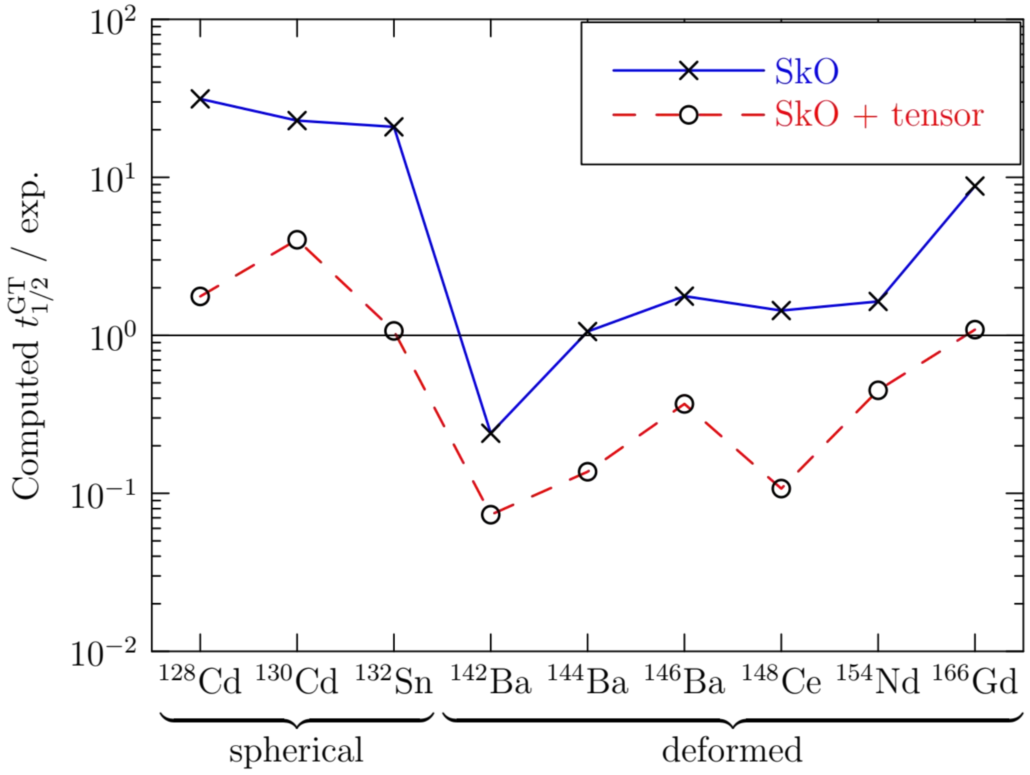
The monopole interaction produced by the tensor force.

T. Otsuka *et al.*, PRL 95, 232502 (2005)
T. Otsuka *et al.*, PRL 105, 012501 (2010)

Evaluate the influence of the effective Hamiltonian, e.g., tensor force

The tensor force has a systematic effect on single-β decay.





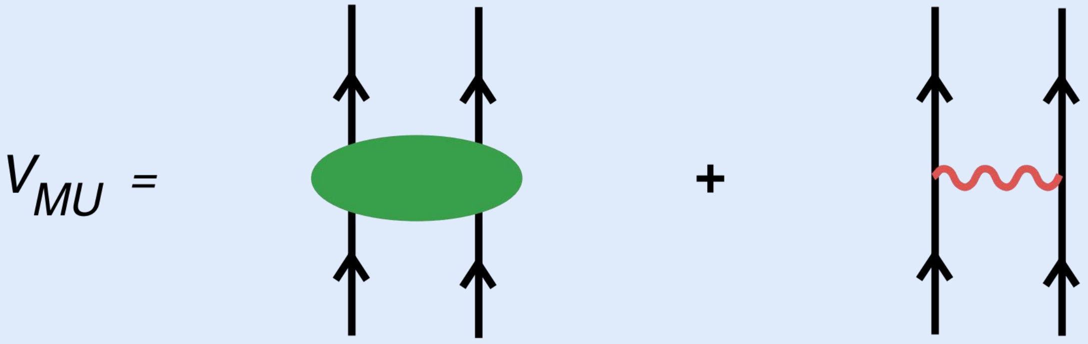
F. Minato and C.L. Bai, PRL 110, 122501 (2013)
F. Minato and C.L. Bai, PRL 116, 089902 (2016)

M. Mustonen *et. al*, PRC 90, 024308 (2014)

Explicit form of the tensor force in effective interactions

(a) central force :
Gaussian
(strongly renormalized)

(b) tensor force :
π + ρ meson
exchange



Diagrams for the V_{MU} interaction

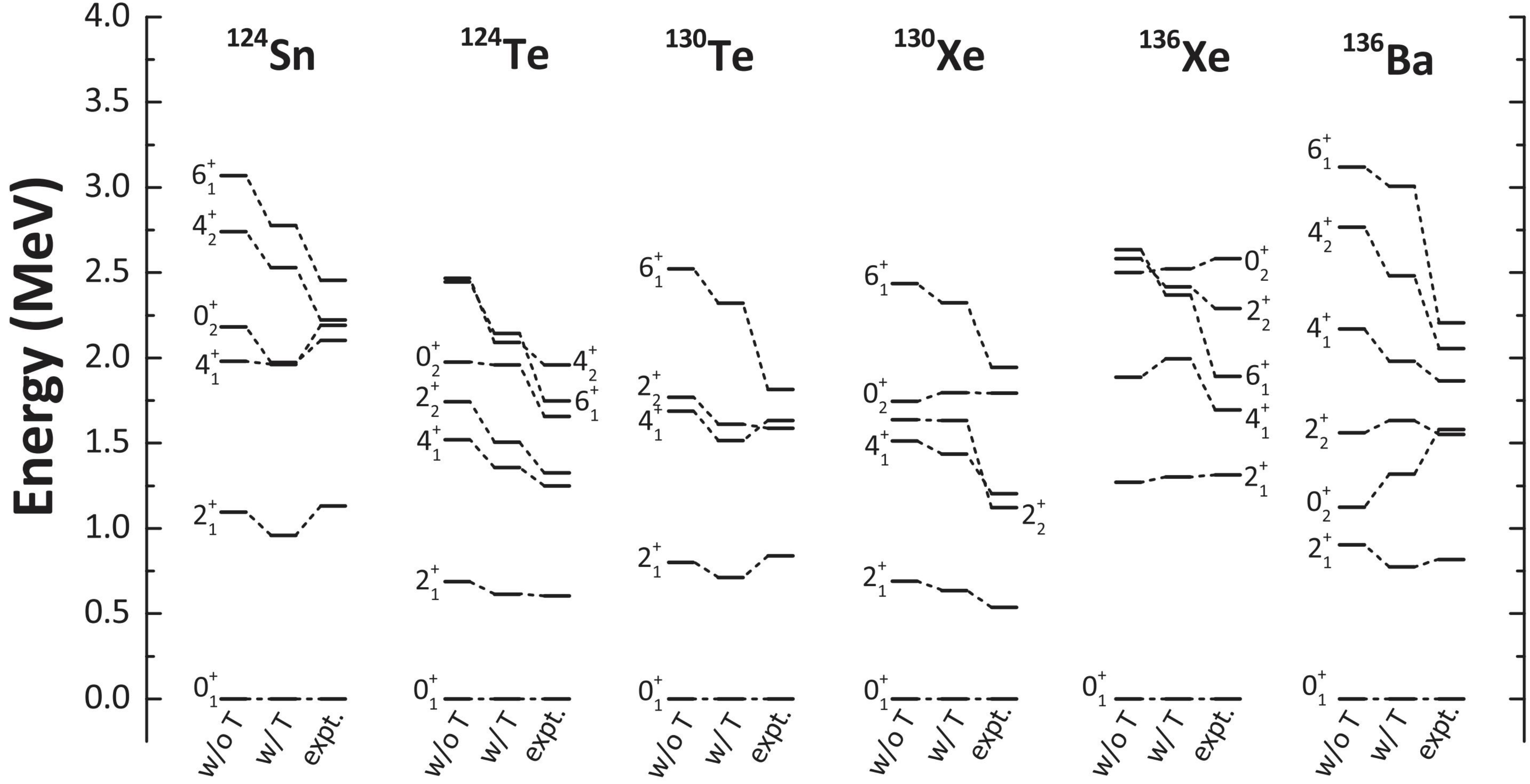
$$V_C(\text{Gauss}) = \sum_{T=0,1.S=0,1} f_{T,S} P_{T,S} exp(-\left(\frac{r}{\mu}\right)^2)$$

$$V_{MU} = V_C(\text{Gauss}) + V_T(\pi + \rho)$$

$$V_{MUC} = V_{MU} + V_{LS}(\text{M3Y})$$

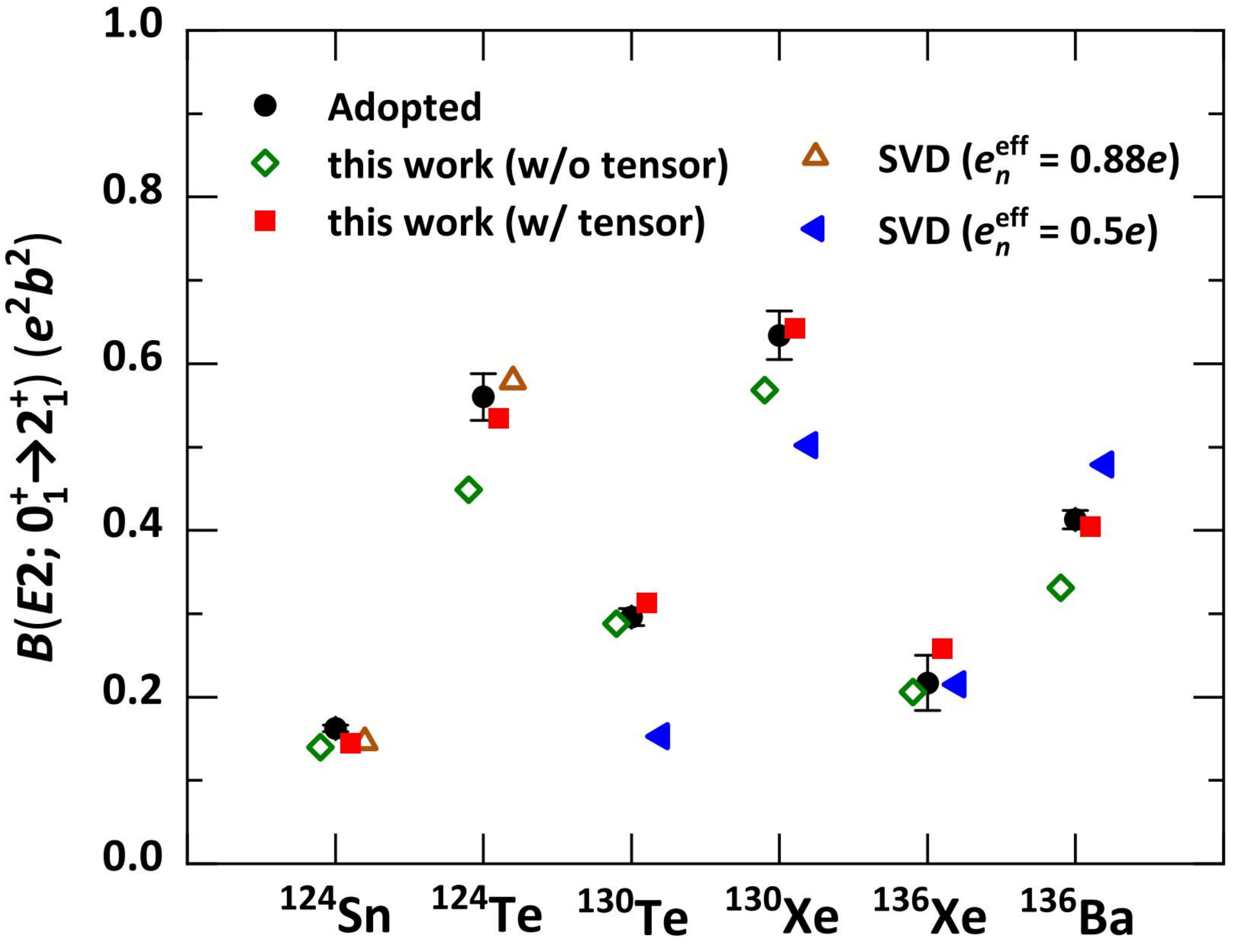
We can investigate the effect by including or excluding the tensor term V_T in V_{MU}

Low-lying spectra given by PGCM



CFJ and C. X. Yuan, in revision

Nuclear structure properties and calculated $0\nu\beta\beta$ NMEs



V_{MU} provides a better description of nuclear structure properties of ^{124}Sn/^{124}Te, ^{130}Te/^{130}Xe, and ^{136}Xe/^{136}Ba

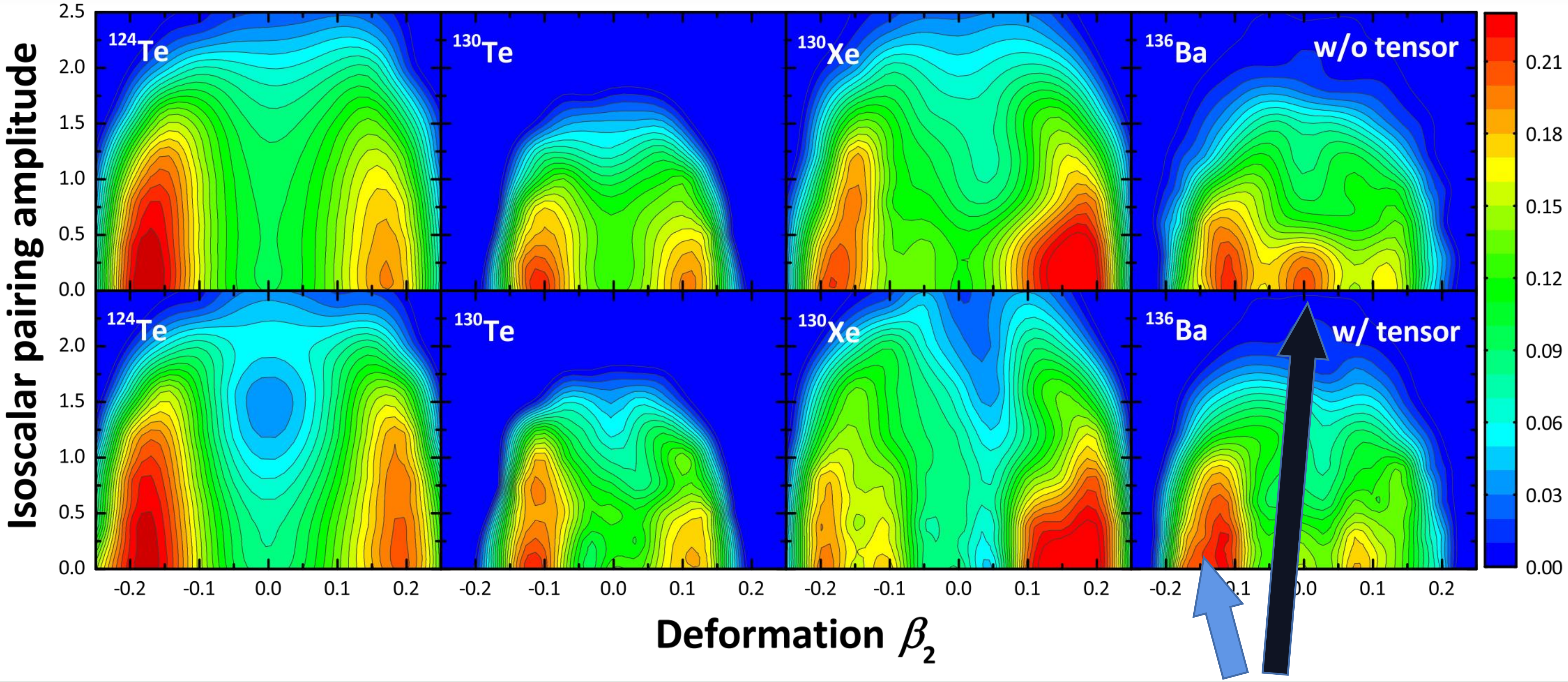
How about $0\nu\beta\beta$ NMEs?

		$M^{0\nu}_{GT}$	$M^{0\nu}_{F}$	$M^{0\nu}_{T}$	$M^{0\nu}$
^{124}Sn	w/o tensor	3.56	−0.64	−0.061	3.91
	w/ tensor	2.65	−0.64	−0.020	3.04
^{130}Te	w/o tensor	4.29	−0.75	−0.064	4.70
	w/ tensor	3.33	−0.65	−0.015	3.73
^{136}Xe	w/o tensor	3.26	−0.44	−0.046	3.49
	w/ tensor	2.17	−0.50	−0.009	2.48

NMEs are suppressed, ***why?***

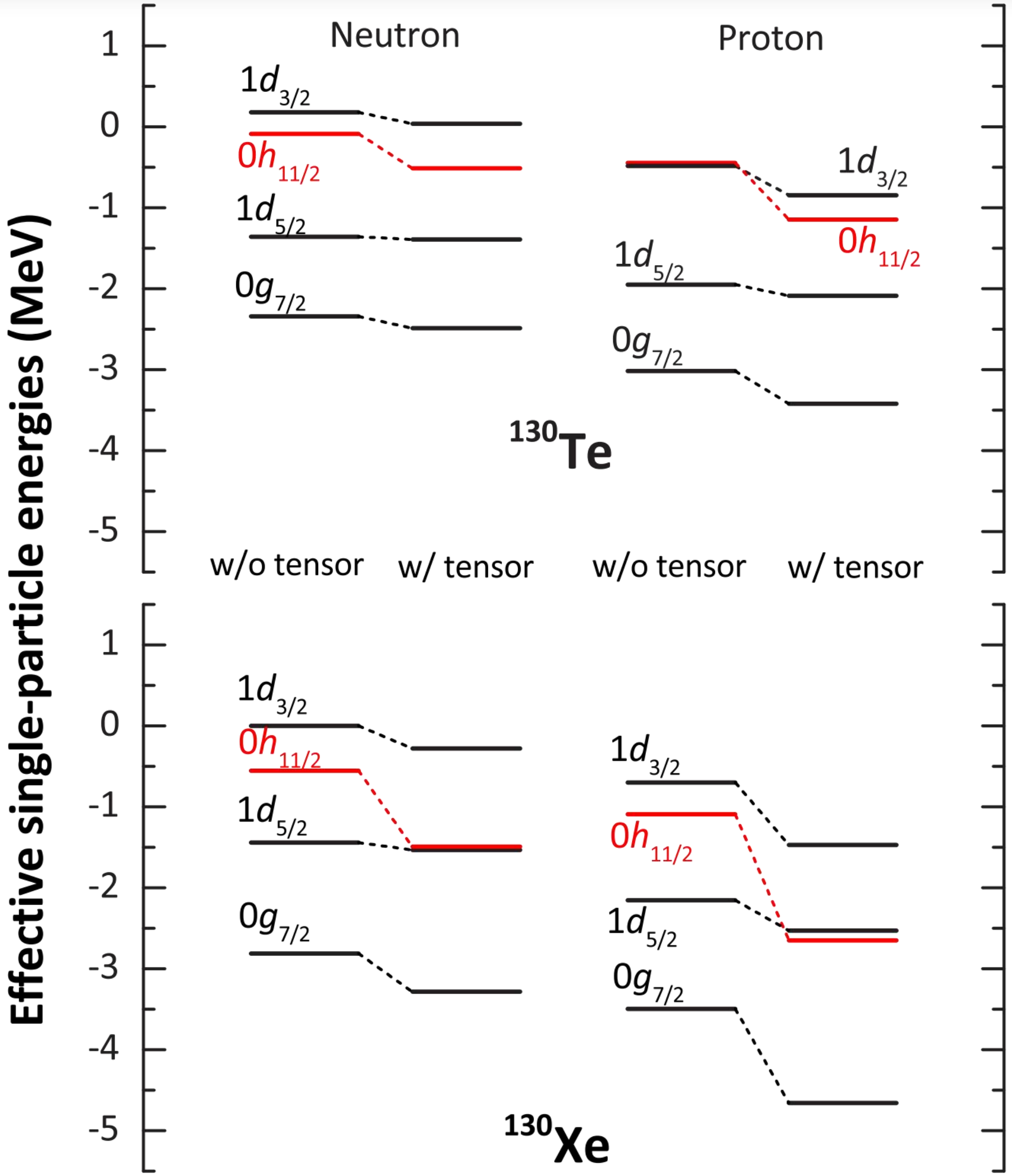
CFJ and C. X. Yuan, in revision

Effect from tensor force on axial deformation



- Enhanced quadrupole deformation, especially in daughter nuclei.
- Enhanced isoscalar pairing: suppression of NMEs.

Effective single-particle energies: change of shell structure



- The neutron and proton $0h_{11/2}$ orbits are shifted most significantly.
- Suppressions are more drastically in daughter nuclei.
  - Attraction between $\pi 0g_{7/2}$ and $\nu 0h_{11/2}$

    More $0g_{7/2}$ protons in daughter nuclei.
  - Repulsion between $\pi 0h_{11/2}$ and $\nu 0h_{11/2}$

    Repulsion between $\pi 0h_{11/2}$ and $\nu 1d_{5/2}$

    Less $1d_{5/2}$ and $0gh_{11/2}$ neutrons in daughter nuclei.
- Both proton and neutron Fermi surface get close to $0h_{11/2}$, **more deformation-driving effects occur in daughter nuclei.**

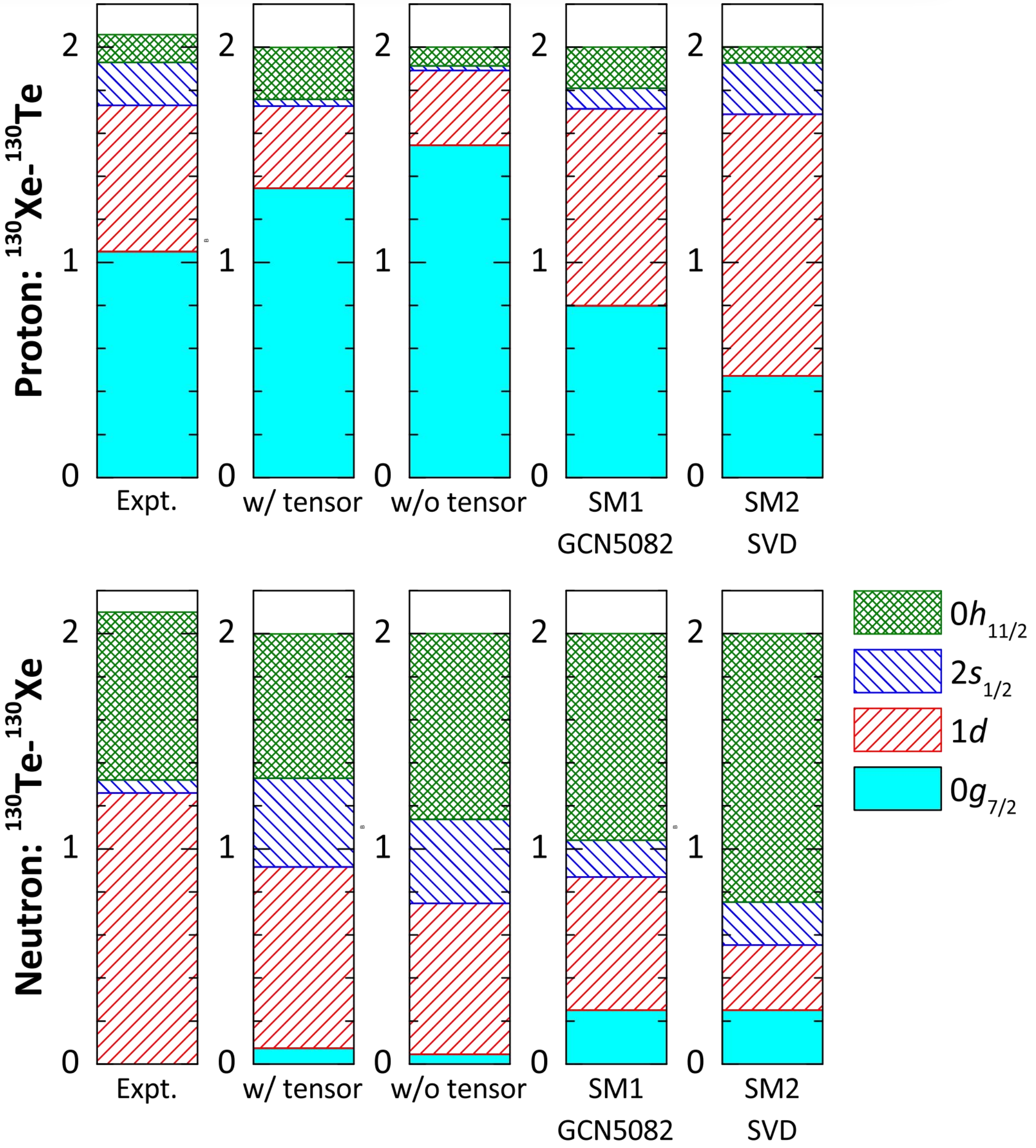
Change of the g.s. nucleon occupancies: a constraint on the NMEs

Why?

It directly determines which neutrons decay and which protons are created in the decay, and how their configurations are re-arranged.

Our calculation reproduces qualitatively the two most important contributions.

Inclusion of tensor force improves the description of the change of the nucleon occupancies.



CFJ and C. X. Yuan, in revision

Summary

- $0\nu\beta\beta$ decay is crucial for determining whether neutrinos are Majorana fermion.
- Hamiltonian-based GCM enables treatment of systems currently **unreachable** by other methods. It can be used to evaluate the effect from aspects of nuclear structure on $0\nu\beta\beta$ NME calculations.
- The tensor force may change the shell structure and hence suppress the $0\nu\beta\beta$ NMEs.

Next Steps from Here…

- Improvement of GCM: more correlations, QRPA-evolved basis.
- Undergoing task: effective Hamiltonian in the larger space from VS-IMSRG method.
  - **Target nuclei: ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{150}Nd…**

Thanks for your attention!