第三届地下和空间粒子物理与宇宙物理前沿问题研讨会
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Nuclear $0\nu\beta\beta$ decay within Relativistic Configuration-interaction Density functional theory

Yakun Wang (王亚坤)

Collaborators：Jie Meng (孟杰) , Pengwei Zhao (赵鹏巍)

Nuclear structure and new physics searches

- Properties of neutrinos and dark matters are revealed in experiment by using nuclei

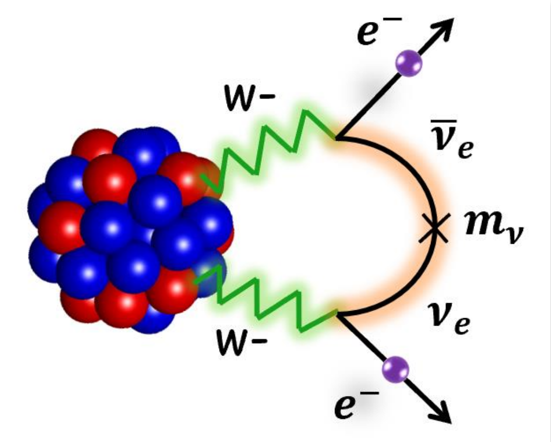
- Nuclear structure aspects are thus involved in the detection of decay half-life and/or differential cross section

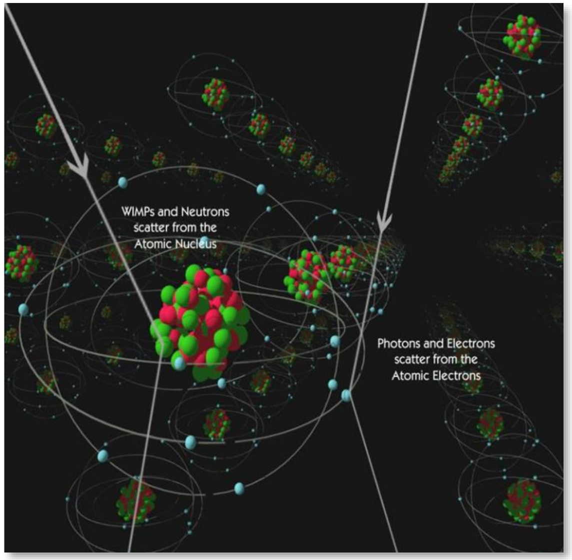
$0\nu\beta\beta$ decay: $[T_{1/2}^{0\nu}]^{-1} \propto |M^{0\nu}|^2 \left| \frac{\langle m_{\beta\beta} \rangle}{m_e} \right|^2$

Dark matter: $\frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}} \propto |\sum_i c_i \zeta_i \mathcal{F}_i|^2$

$M^{0\nu}$: Nuclear matrix element

$\mathcal{F}_i$: Nuclear structure factor





Nuclear $0\nu\beta\beta$ decay

- Violation of **lepton number**
- **Majorana nature** of neutrinos
- **Matter dominance** in the Universe
- Neutrino **mass scale** and **hierarchy**

Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$

Isotopes	$T^{0\nu}_{1/2}$ (yr)	Collaborations
^{48}Ca	$> 5.8 \times 10^{22}$	ELEGANT VI
^{76}Ge	$> 1.8 \times 10^{26}$	GERDA, MAJORANA, CDEX
^{82}Se	$> 3.5 \times 10^{24}$	CUPID-0, NνDEx
^{100}Mo	$> 1.5 \times 10^{24}$	CUPID-Mo
^{130}Te	$> 3.2 \times 10^{25}$	CUORE
^{136}Xe	$> 2.3 \times 10^{26}$	KamLAND-Zen, EXO-200, PandaX
^{150}Nd	$> 2.0 \times 10^{22}$	NEMO-3

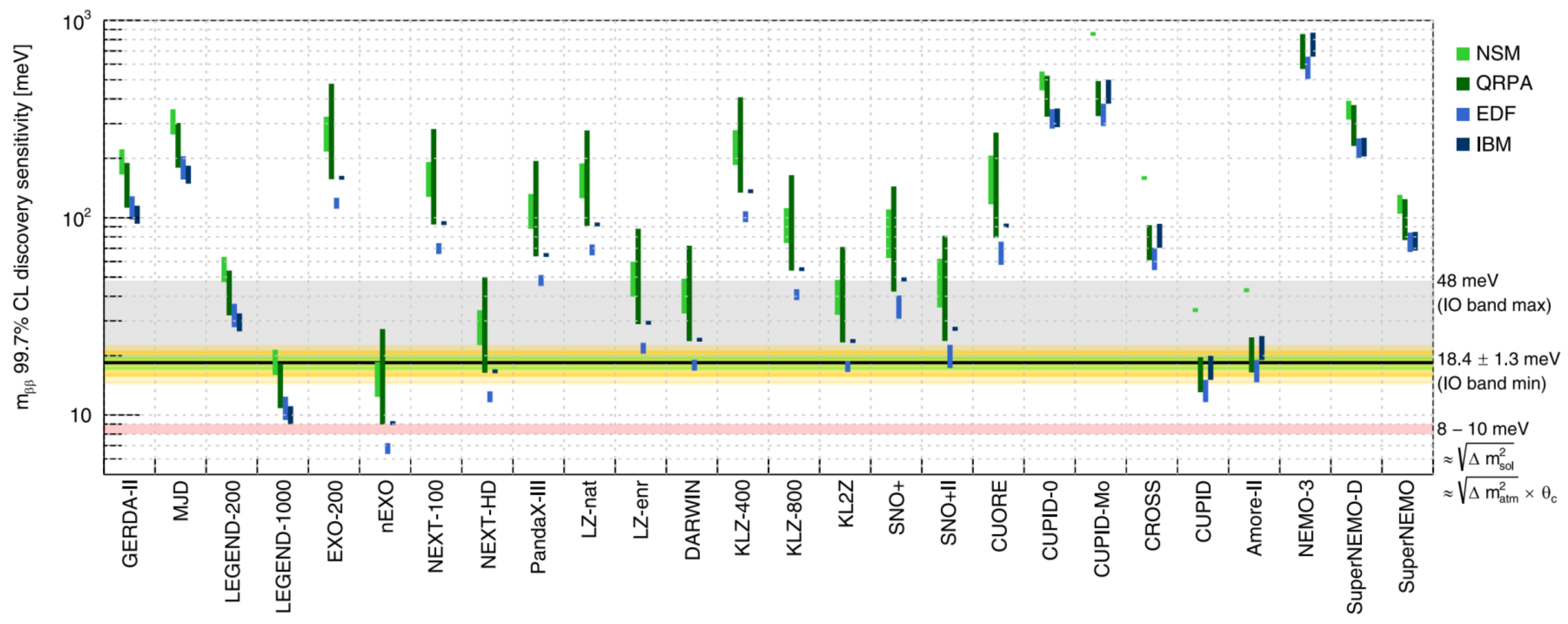
✓ No $0\nu\beta\beta$ -decay signal has been observed so far.

✓ Current limit on the decay half-life ranges from 10^{22} yr to 10^{26} yr.

Agostini, Benato, Detwiler, Menéndez, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

Next-generation experiments: inverted hierarchy

- $0\nu\beta\beta$ decay half-life is $[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}(Q,Z)|M^{0\nu}|^2 \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$
- It is sensitive to the effective neutrino mass $\langle m_{\beta\beta}\rangle$ and hierarchy



Agostini, Benato, Detwiler, Menéndez, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

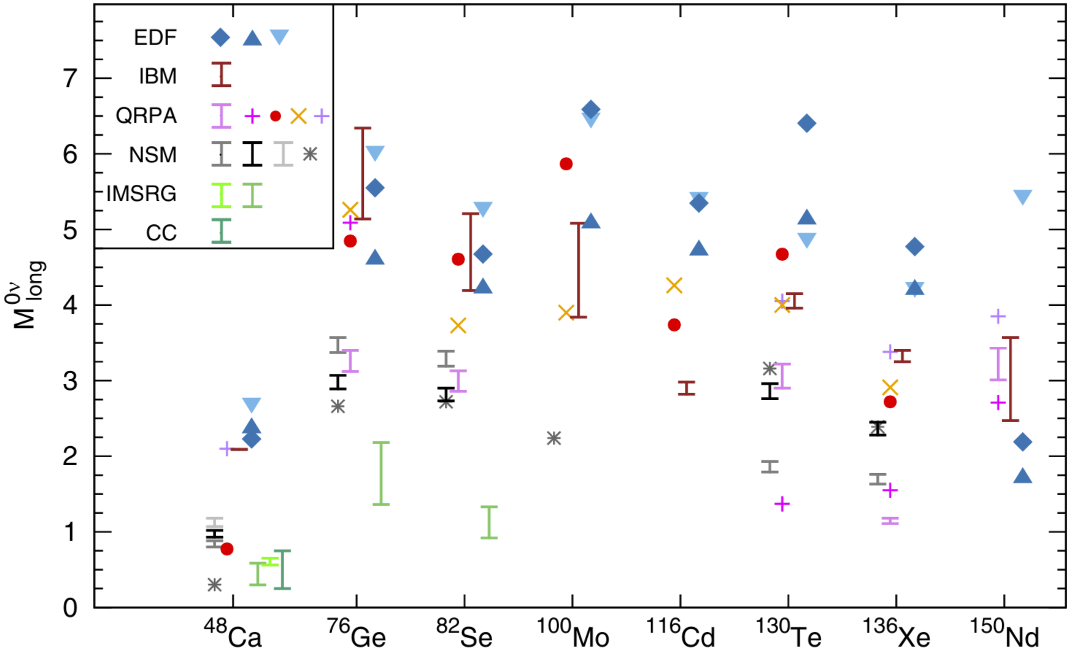
NMEs are needed to make sure the next-generation experiments fully explore inverted hierarchy

Outline

- ☐ Present status of the NME prediction
- ☐ ReCD theory
  - ✓ Basic conception of ReCD theory
  - ✓ Test the validity of ReCD theory
  - ✓ Recent progress on modeling $0\nu\beta\beta$ decay
- ☐ Summary

Nuclear Matrix Elements (NMEs)

$$M^{0\nu} = \langle \Psi_f | \hat{O}^{0\nu} | \Psi_i \rangle$$



Agostini, Benato, Detwiler, Menéndez, Vissani, Rev. Mod. Phys. 80, 481 (2008)

Nuclear structure aspects:

- ✓ Pairing correlations
- ✓ Shapes + fluctuations
- ✓ Noncollective collections
- ✓ Model space …

Origin of NME uncertainty:

Missing correlations and the limited model space

Approaches within a full model space and more many-body correlations are highly desirable

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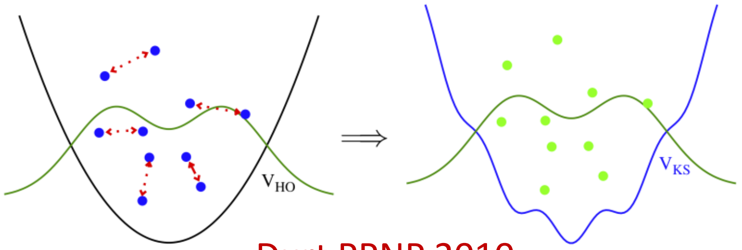
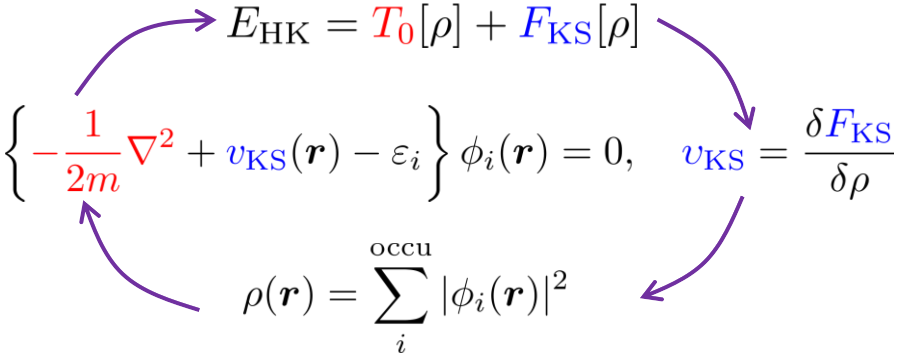
Density functional theory and shell model

Density functional theory (DFT)

The exact ground-state energy of a quantum many-body system is a universal functional of the local density



Durt PPNP 2010



$$E_{\rm HK} = T_0[\rho] + F_{\rm KS}[\rho]$$

$$\left\{-\frac{1}{2m}\nabla^2 + v_{\rm KS}(\boldsymbol{r}) - \varepsilon_i\right\}\phi_i(\boldsymbol{r}) = 0, \qquad v_{\rm KS} = \frac{\delta F_{\rm KS}}{\delta\rho}$$

$$\rho(\boldsymbol{r}) = \sum_i^{\rm occu} |\phi_i(\boldsymbol{r})|^2$$

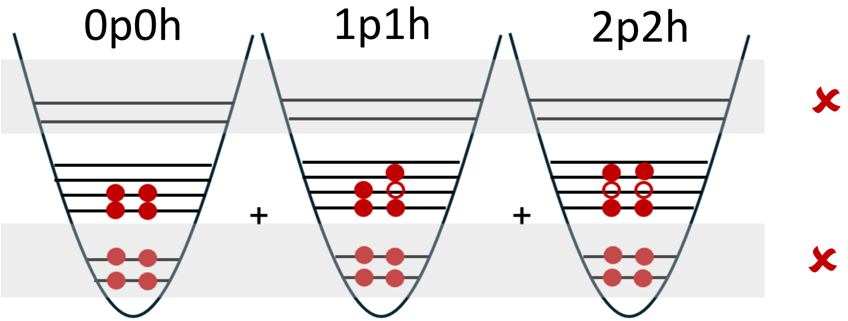
Nuclear shell model (SM)

The full nuclear Hamiltonian in the complete model space is replaced by an effective Hamiltonian in a limited model space

$$\hat{H}|\Psi\rangle = E|\Psi\rangle$$

$$\Downarrow$$

$$\hat{H}_{\rm eff}|\tilde{\Psi}\rangle = E_{\rm eff}|\tilde{\Psi}\rangle$$



Density functional theory and shell model

Nuclear DFT

- **Universal density functional**

  Symmetry broken

  Full model space

  No configuration mixing
- **Applicable for almost all nuclei**
- **Overestimate the NMEs**

Nuclear SM

- **Non-universal effective Hamiltonian**

  No symmetry broken

  Limited model space

  Strong configuration mixing
- **Intractable for deformed heavy nuclei**
- **Underestimate the NMEs**

A model that combines the advantages of nuclear DFT and SM?

Basic ideas of ReCD theory

1. A self-consistent relativistic DFT calculation

State $|\Phi_0\rangle$ with minimum energy in the PES

2. Construction of intrinsic configuration space

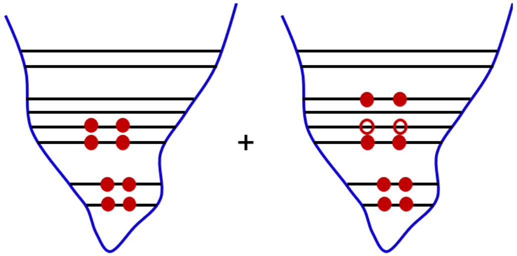
Quasiparticle states on top of $|\Phi_0\rangle$

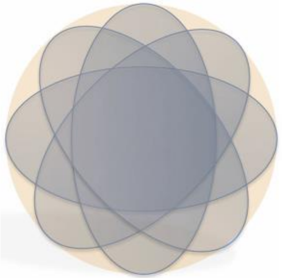
3. Angular momentum projection

Restoration of rotational symmetry

4. Shell model diagonalization

Configuration mixing or interaction based on DFT





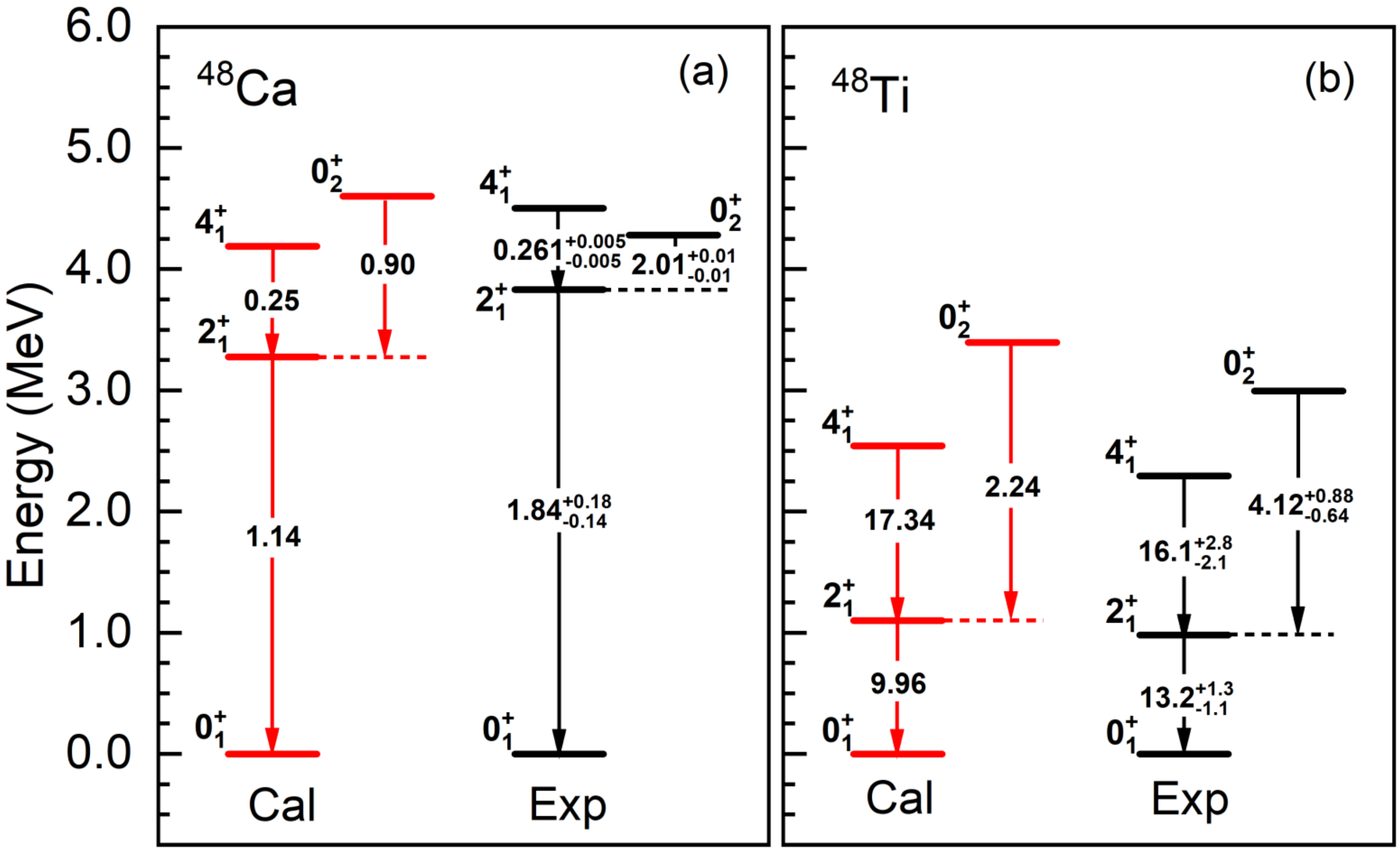
Axial + 2qp for even-even nuclei: P. W. Zhao, P. Ring, J. Meng, PRC 94, 041301(R) (2016)

Axial + 4qp for even-even nuclei: **Y. K. Wang**, P. W. Zhao, J. Meng, PRC 105, 054311 (2022)

Triaxial + 4qp for even and odd nuclei : **Y. K. Wang**, P. W. Zhao, J. Meng, PLB 848, 138346 (2024)

ReCD theory: the residual two-body interaction and the intrinsic states come from a universal density functional, and no truncation is adopted to the model space.

Description for nuclear spectroscopic properties

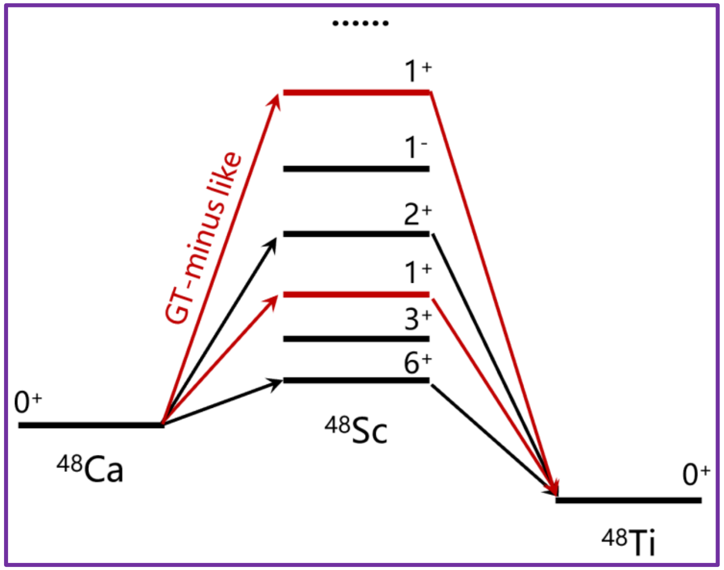


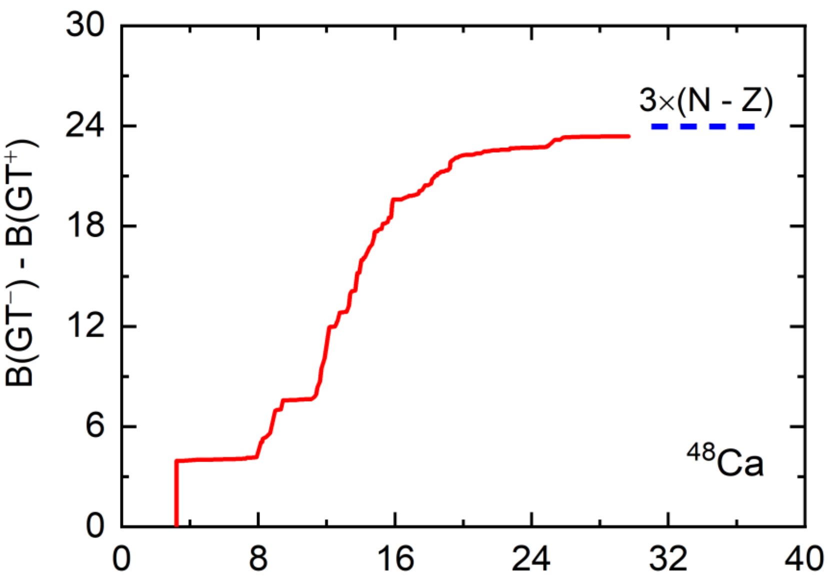
Y. K. Wang, P. W. Zhao, J. Meng, In preparation

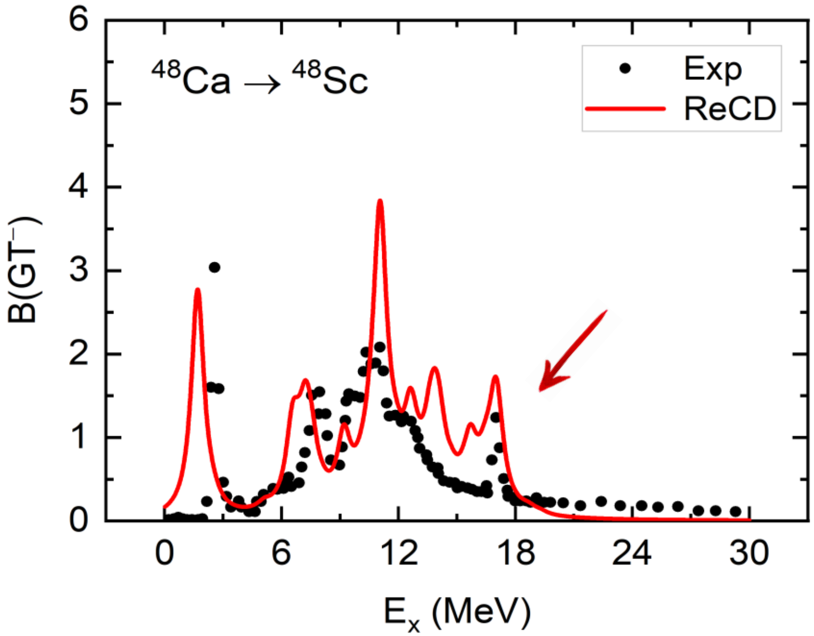
- ☐ The energy spectra and the E2 transition probabilities are reproduced satisfactorily

Description for nuclear spin-isospin transition mode

$$B(GT^-) = \sum_{1_n^+} |\langle 0_i^+|\hat{\boldsymbol{\sigma}}\hat{\tau}^-|1_n^+\rangle|^2, B(GT^+) = \sum_{1_n^+} |\langle 1_n^+|\hat{\boldsymbol{\sigma}}\hat{\tau}^+|0_i^+\rangle|^2$$



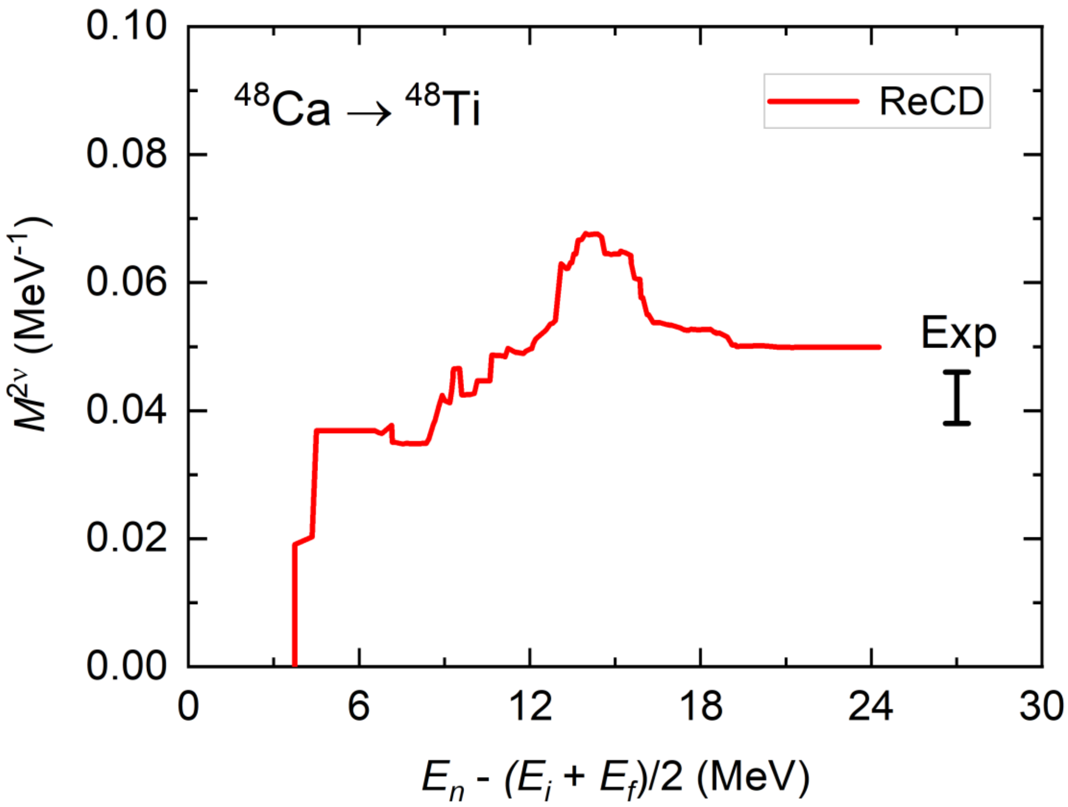




- ☐ The Ikeda sum rule is fulfilled by around 97%.
- ☐ The peak of Gamow-Teller strength at $E_x \approx 17$MeV is also satisfactorily reproduced.

Y. K. Wang, P. W. Zhao, J. Meng, In preparation

Description for nuclear $2\nu\beta\beta$ decay

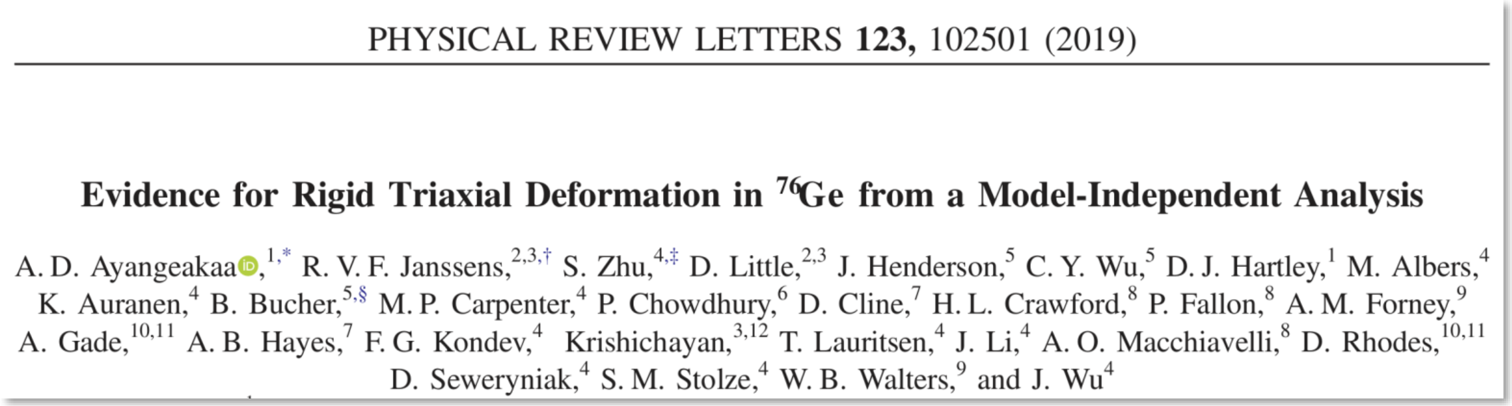


- The $2\nu\beta\beta$ decay NME can be reproduced without introducing the quenching factor to the axial-vector coupling constant g_A

$$M^{2\nu} = (qg_A)^2 \sum_n \frac{\langle f|\sum_a \hat{\boldsymbol{\sigma}}_a \tau_a^+ |1_n^+\rangle\langle 1_n^+|\sum_b \hat{\boldsymbol{\sigma}}_b \tau_b^+ |i\rangle}{E_n - 1/2(E_i + E_f)}$$

Y. K. Wang, P. W. Zhao, J. Meng, In preparation

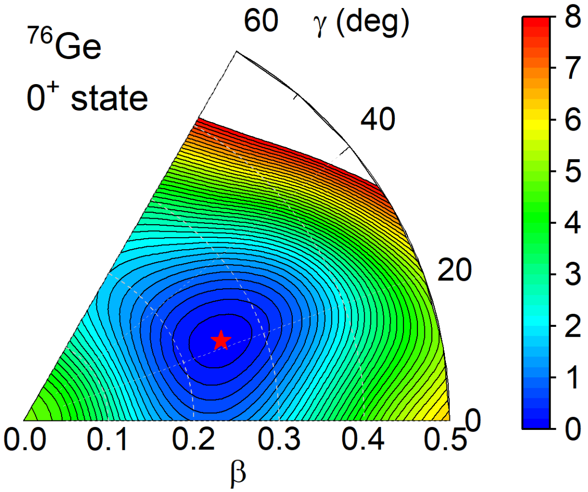
Evaluation of NME: triaxial effects

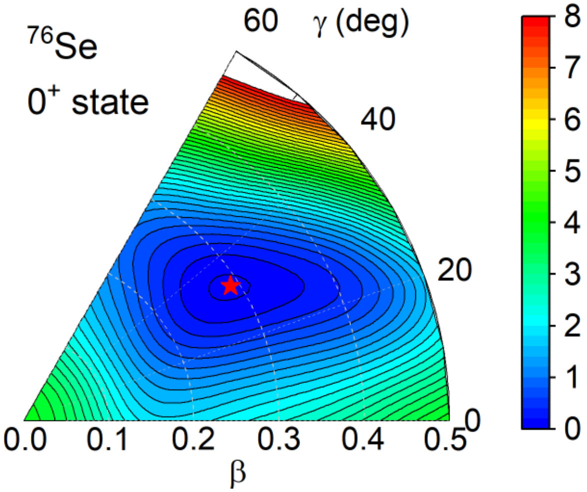


PHYSICAL REVIEW LETTERS **123,** 102501 (2019)

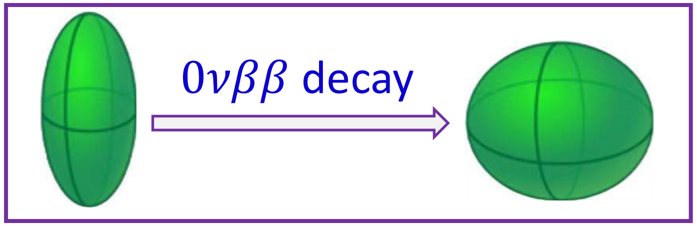
Evidence for Rigid Triaxial Deformation in ^{76}Ge from a Model-Independent Analysis

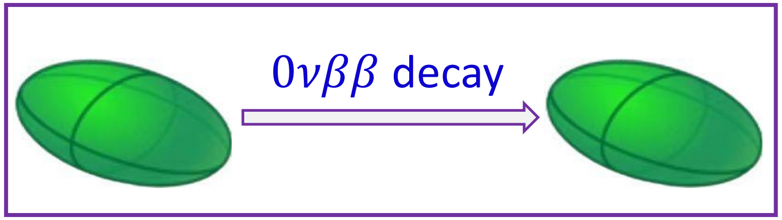
A. D. Ayangeakaa,[1,*] R. V. F. Janssens,[2,3,†] S. Zhu,[4,‡] D. Little,[2,3] J. Henderson,[5] C. Y. Wu,[5] D. J. Hartley,[1] M. Albers,[4] K. Auranen,[4] B. Bucher,[5,§] M. P. Carpenter,[4] P. Chowdhury,[6] D. Cline,[7] H. L. Crawford,[8] P. Fallon,[8] A. M. Forney,[9] A. Gade,[10,11] A. B. Hayes,[7] F. G. Kondev,[4] Krishichayan,[3,12] T. Lauritsen,[4] J. Li,[4] A. O. Macchiavelli,[8] D. Rhodes,[10,11] D. Seweryniak,[4] S. M. Stolze,[4] W. B. Walters,[9] and J. Wu[4]



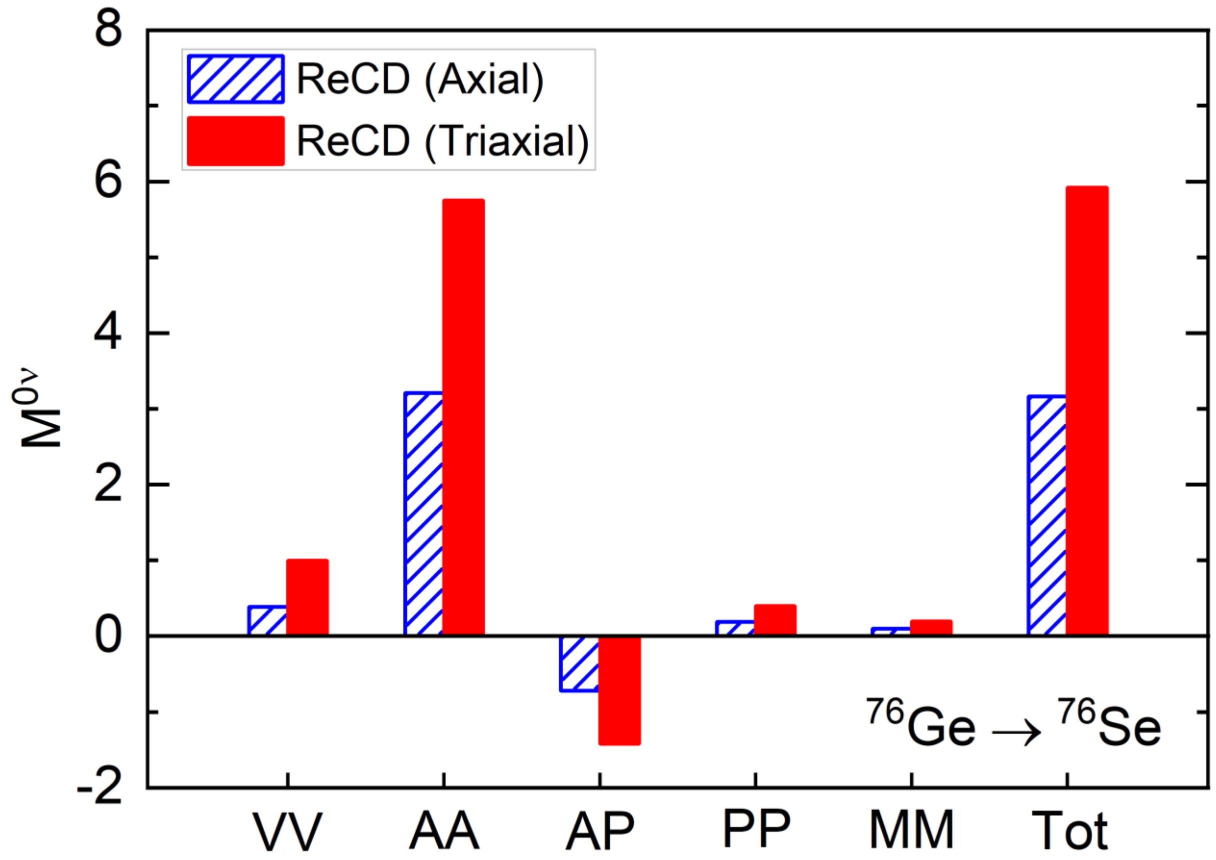


Y. K. Wang, P. W. Zhao, J. Meng, arXiv: 2304. 12009





Evaluation of NME: triaxial effects



$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}(Q_{\beta\beta}, Z)|M^{0\nu}|^2 \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$$

half-life decreases by a **factor of four**

$$\propto \varepsilon \frac{i.a.}{A} Mt$$

Less detector materials are needed

Y. K. Wang, P. W. Zhao, J. Meng, Accepted by Science Bulletin

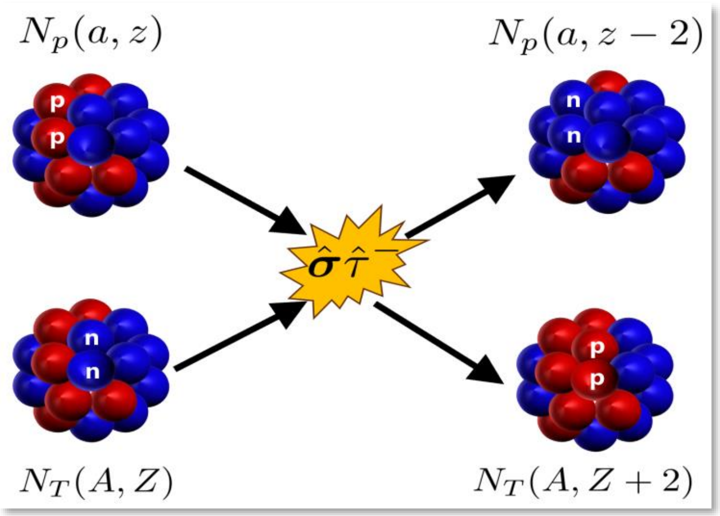
☐ Consideration of triaxial deformation **enhances** the $0\nu\beta\beta$-decay NME **by a factor around two**.

Double Gamow-Teller transition

- DGT cross section can be factorized into two parts: reaction factor, DGT-transition NMEs of initial state and the final state

Santopinto et al., PRC 98, 061601(R) (2018)

⇒ Determining DGT-transition NMEs by the experimental cross section



$$N_T(A, Z) + N_p(a, z) \to N_T(A, Z+2) + N_p(a, z-2)$$

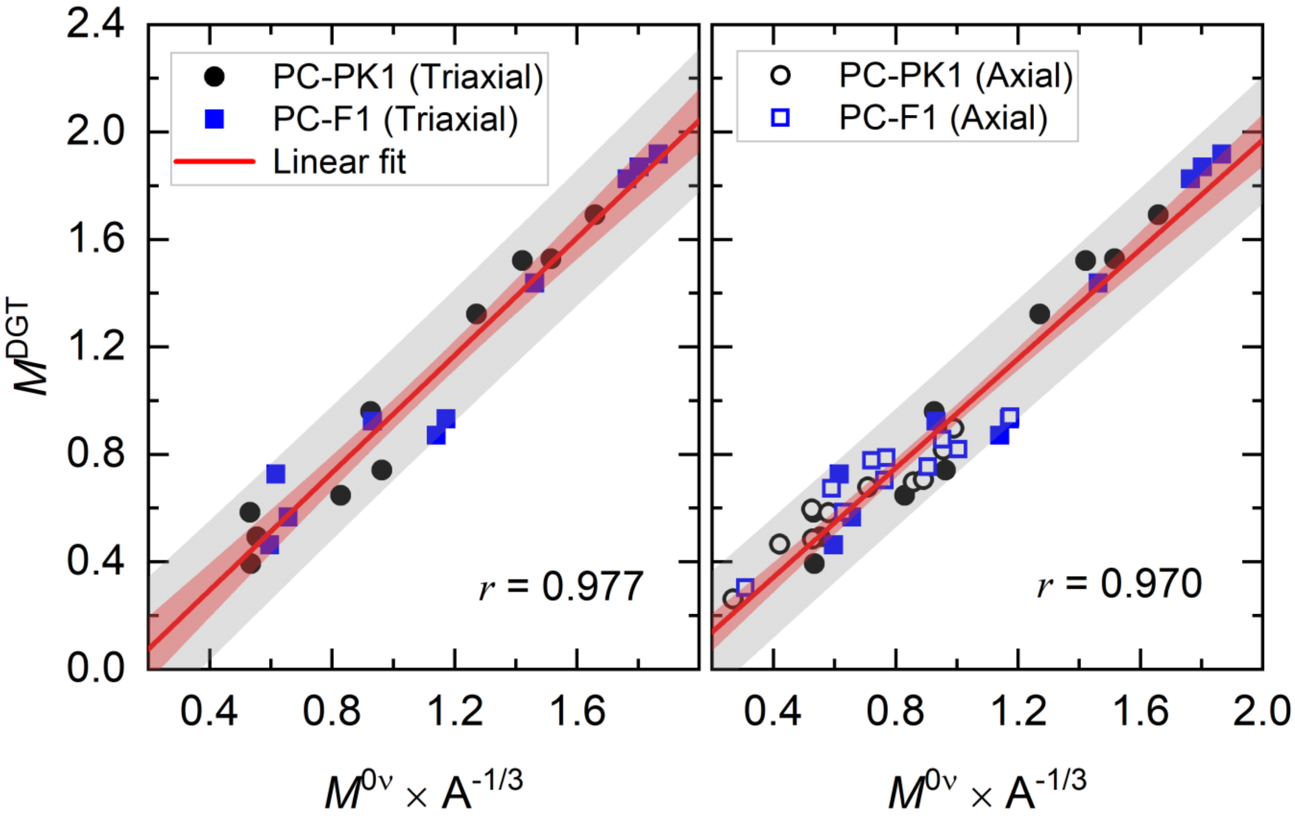
- Same initial and final states, similar decay operator ⇒ correlation between DGT transition and $0\nu\beta\beta$ decay?

Rodríguez et al., PLB 719, 174 (2013), Cappuzzello et al., EPJA 51, 145 (2015)

Constraining the $0\nu\beta\beta$ decay NME by DGT transitions!

Shimizu et al., PRL 120, 142502 (2018), Yao et al., PRC 106, 014315 (2022), lv et al., PRC 108, L051304 (2023)

Correlation between $0\nu\beta\beta$ decay and DGT transition



10 nuclei:

^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te, ^{136}Xe

Axial and triaxial deformations

Full model space

- ☐ A strong linear correlation between $0\nu\beta\beta$ decay and DGT transition is demonstrated
- ☐ The linear correlation is robust against nuclear deformations

Y. K. Wang, P. W. Zhao, J. Meng, arXiv: 2403. 06455

Outline

- ☐ Present status of the NME prediction
- ☐ ReCD theory
  - ✓ Basic conception of ReCD theory
  - ✓ Test the validity of ReCD theory
  - ✓ Recent progress on modeling $0\nu\beta\beta$ decay
- ☐ Summary

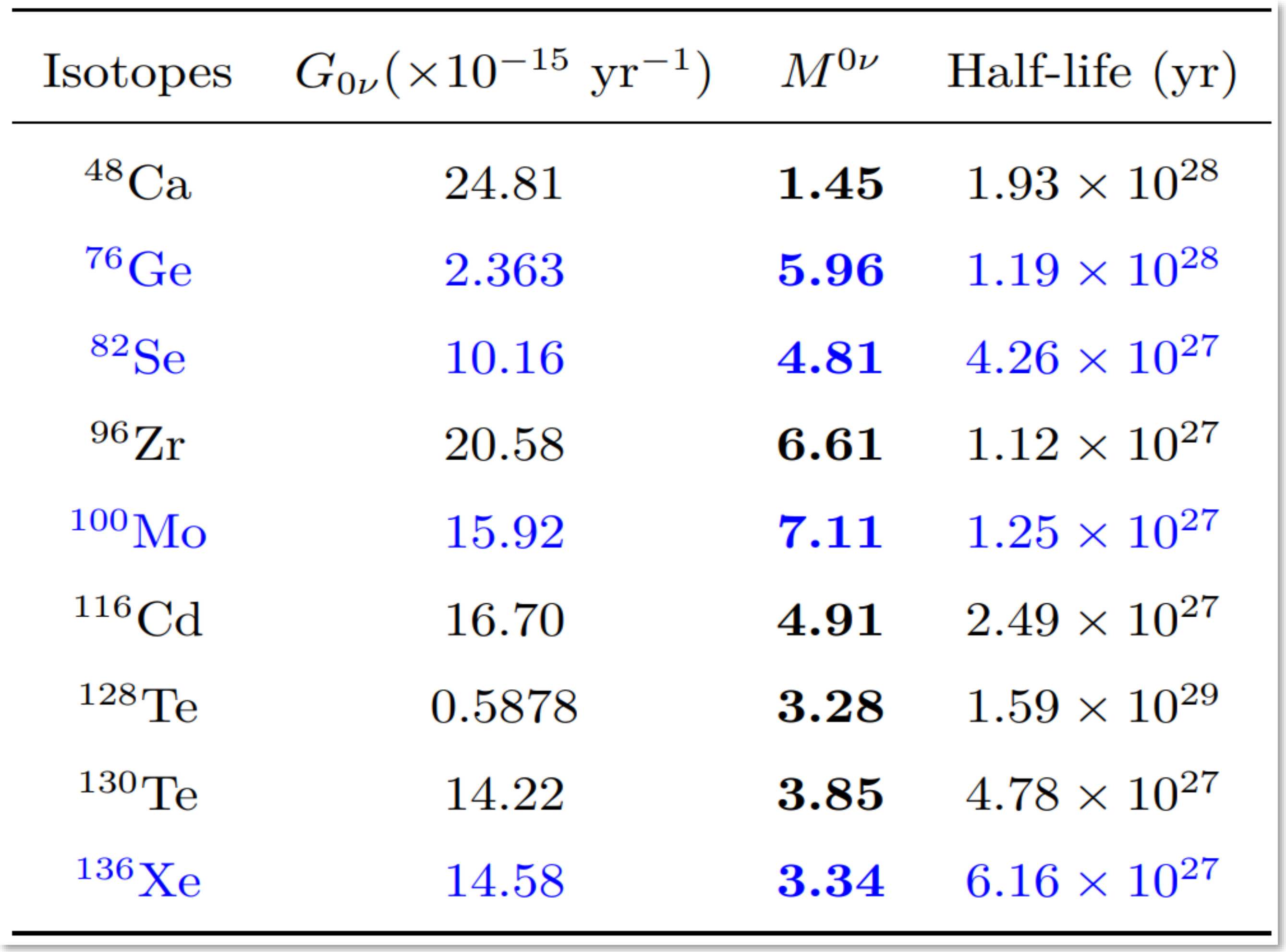
Summary

- The Relativistic Configuration-interaction Density functional theory is developed
- The basic idea of ReCD theory is introduced
- The ReCD theory can reproduce nuclear spectroscopic, spin-isospin excitations, $2\nu\beta\beta$ decay satisfactorily
- The recent progress of ReCD theory on modeling the $0\nu\beta\beta$ decay NMEs are briefly illustrated.

Appendix

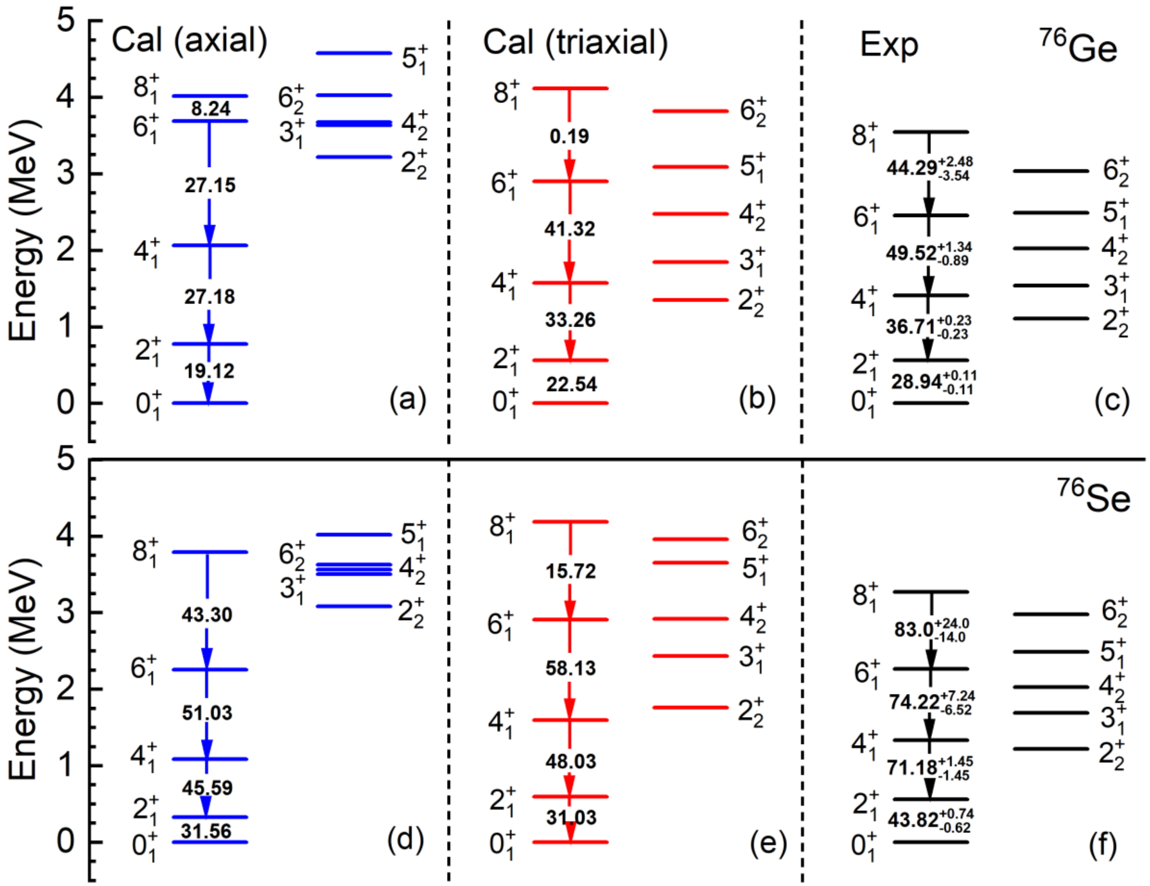
Systematic calculations of $0\nu\beta\beta$ decay

Predicted decay half-life for $m_{\beta\beta} = 10$ meV

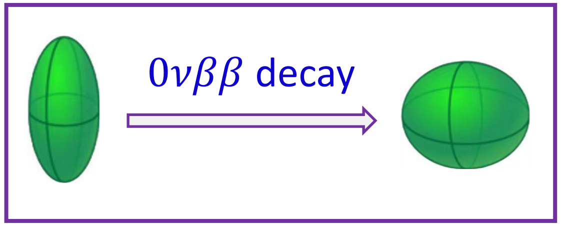


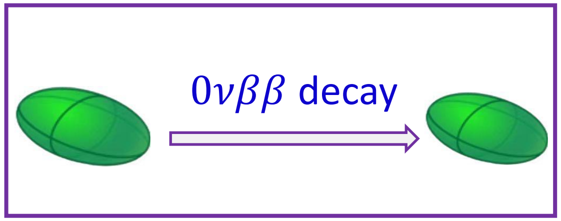
Isotopes	$G_{0\nu}(\times 10^{-15}\ \mathrm{yr}^{-1})$	$M^{0\nu}$	Half-life (yr)
^{48}Ca	24.81	**1.45**	1.93×10^{28}
^{76}Ge	2.363	**5.96**	1.19×10^{28}
^{82}Se	10.16	**4.81**	4.26×10^{27}
^{96}Zr	20.58	**6.61**	1.12×10^{27}
^{100}Mo	15.92	**7.11**	1.25×10^{27}
^{116}Cd	16.70	**4.91**	2.49×10^{27}
^{128}Te	0.5878	**3.28**	1.59×10^{29}
^{130}Te	14.22	**3.85**	4.78×10^{27}
^{136}Xe	14.58	**3.34**	6.16×10^{27}

Advantage of ReCD: triaxial deformation



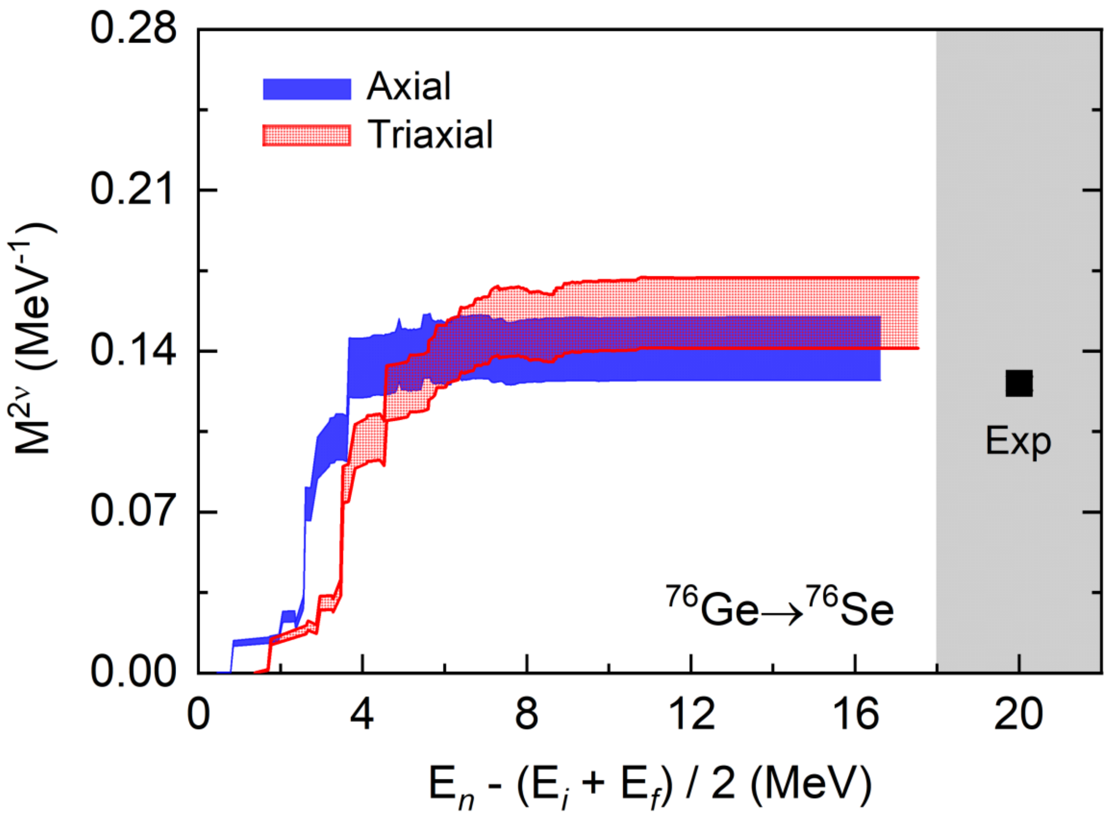
Y. K. Wang, P. W. Zhao, J. Meng, arXiv: 2304. 12009


$0\nu\beta\beta$ decay


$0\nu\beta\beta$ decay

- ☐ Description of the underlying wavefunctions are **significantly improved by triaxial deformation**

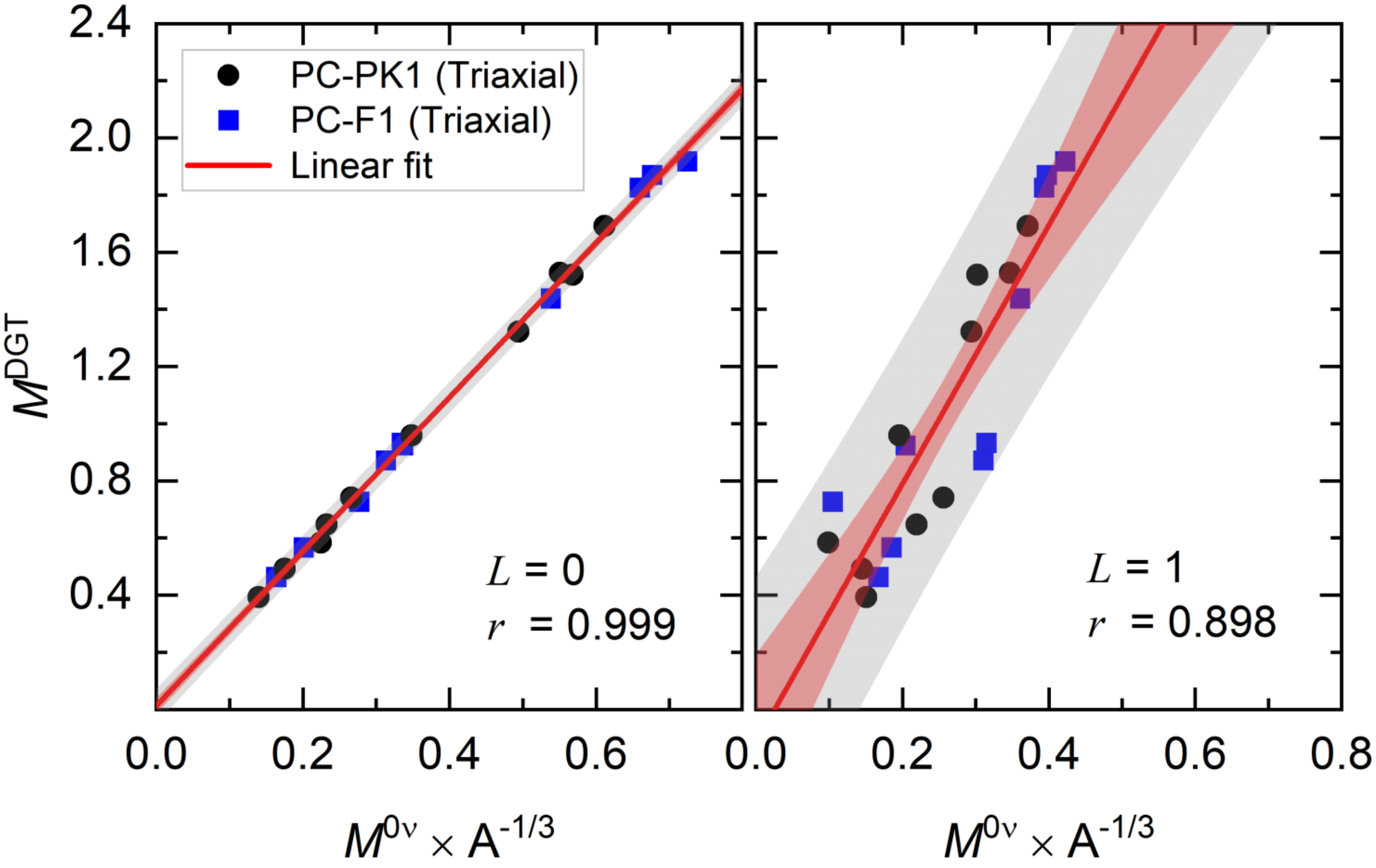
NME of the $2\nu\beta\beta$ decay



$$M^{2\nu} = q^2 \sum_n \frac{\langle \Psi_f | \sum_a \boldsymbol{\sigma}_a \tau_a^+ | \Psi_{1_n^+} \rangle \langle \Psi_{1_n^+} | \sum_b \boldsymbol{\sigma}_b \tau_b^+ | \Psi^i \rangle}{E_n - (E_i + E_f)/2}$$

- ☐ All possible **odd-odd intermediate 1^+ states** needs to be considered (around 200 1^+ states are included).
- ☐ The two-body currents are not considered ⇒ quenching factor ***q*** ranging from **0.68 to 0.77 are adopted** in our calculations.

Decomposition of the NMEs

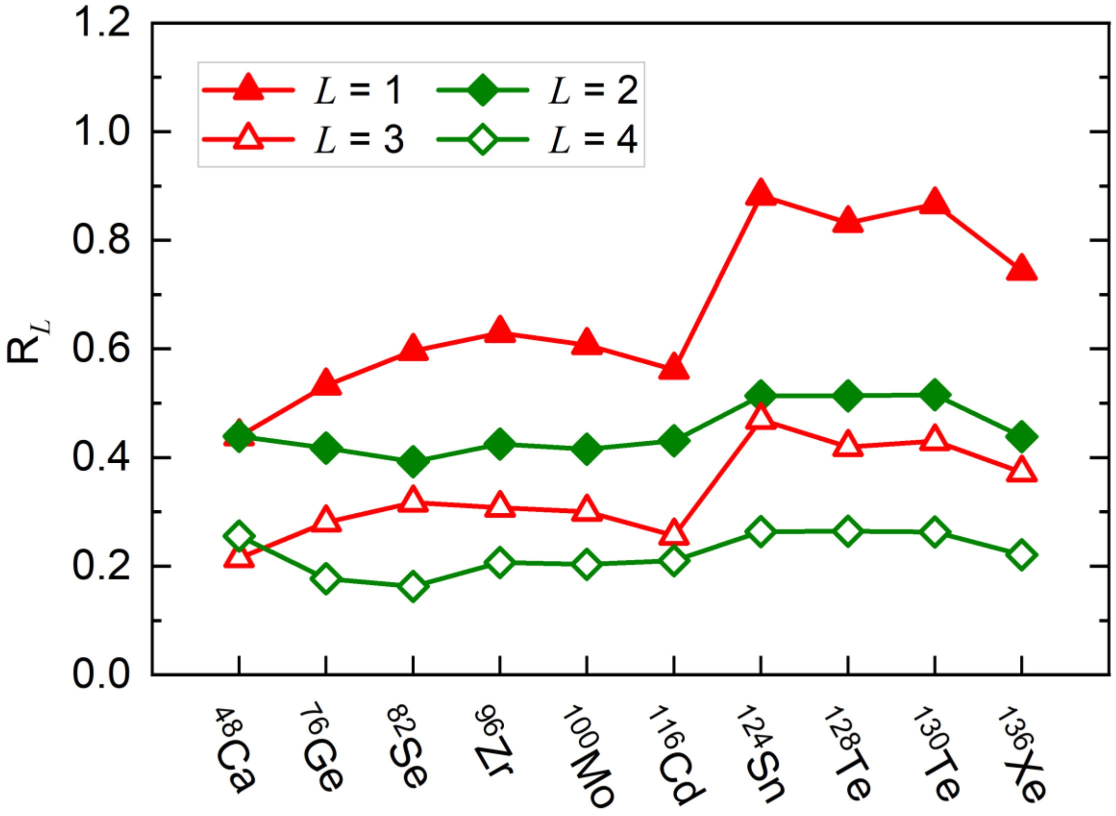


$$\hat{O}^{0\nu} = \sum_L \hat{O}^{0\nu}_L$$

$$M^{0\nu} = \sum_L M^{0\nu}_L$$

☐ The **leading order term $M^{0\nu}_{L=0}$** correlated strongly with M^{DGT}, while the correlation between $M^{0\nu}_{L=1}$ and M^{DGT} is much weaker.

Contributions from higher-order terms



$$R_L = M^{0\nu}_L / M^{0\nu}_{L=0}$$

- ☐ $R_2 \approx 0.45, R_4 \approx 0.20$, and they are independent on the decay candidates
- ☐ Consideration of higher-order terms with even L would not worsen the correlation
- ☐ Contributions from NMEs with odd L are generally smaller than those from the NMEs with even L

The origin of the linear correlation

- The $0\nu\beta\beta$-decay operator contains five terms

$$\hat{O}^{0\nu} = \hat{O}^{0\nu}_{VV} + \hat{O}^{0\nu}_{AA} + \hat{O}^{0\nu}_{AP} + \hat{O}^{0\nu}_{PP} + \hat{O}^{0\nu}_{MM}$$

- Decay operator in *AA* coupling channel

$$\hat{O}^{0\nu}_{AA} = \sum_{1234} \langle 13 | \mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 | 24 \rangle \hat{d}^\dagger_1 \hat{d}^\dagger_3 \hat{c}_4 \hat{c}_2, \quad |1\rangle \equiv |n_1 l_1 j_1 m_1\rangle$$

Neutrino potential in coordinate space

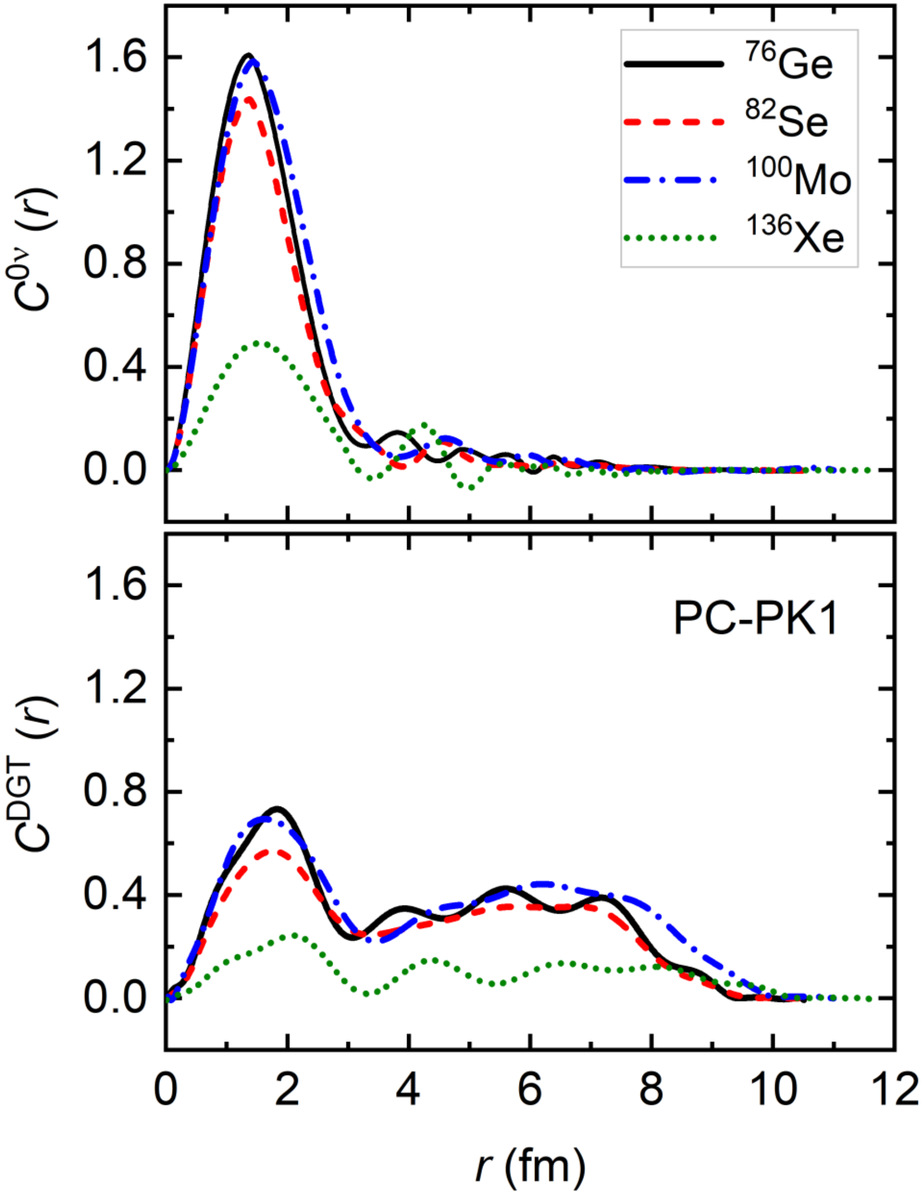
$$\mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \int \frac{d\boldsymbol{q}}{(2\pi)^3} H(\boldsymbol{q}) e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}$$

Multipole expansion for plane waves $e^{\pm i\boldsymbol{p}\cdot\boldsymbol{r}}$ by spherical harmonics

$$e^{i\boldsymbol{q}\cdot\boldsymbol{r}} = 4\pi \sum_{LM} i^L j_L(qr) Y^*_{LM}(\hat{\boldsymbol{q}}) Y_{LM}(\hat{\boldsymbol{r}})$$

$$\mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{2}{\pi} \int q^2 dq H(q) \sum_{LM} [j_L(qr_1) Y_{LM}(\hat{\boldsymbol{r}}_1)][j_L(qr_2) Y^*_{LM}(\hat{\boldsymbol{r}}_2)] = \sum_L \mathcal{O}^{AA}_L(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

NME distributions in coordinate space



$$M^\alpha = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 C^\alpha(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

$$\Big\Downarrow \quad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2);\ \boldsymbol{r} = (\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$M^\alpha = \int dr C^\alpha(r)$$

- The short-range character is observed for $0\nu\beta\beta$ decay, but not for DGT transition
- The explanation that the linear correlation originates from the dominant short-range character in both transitions is thus not support