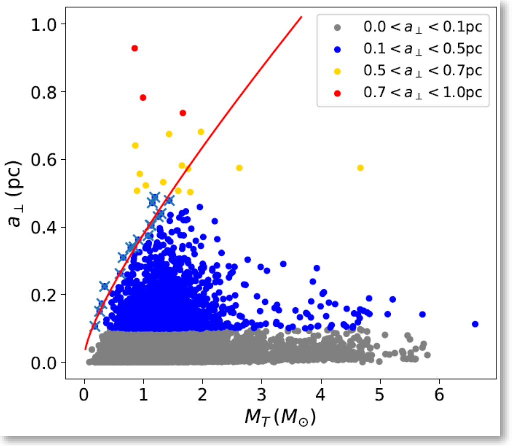
Wide Binary Evaporation by Dark Solitons

Yu Gao (IHEP, CAS)



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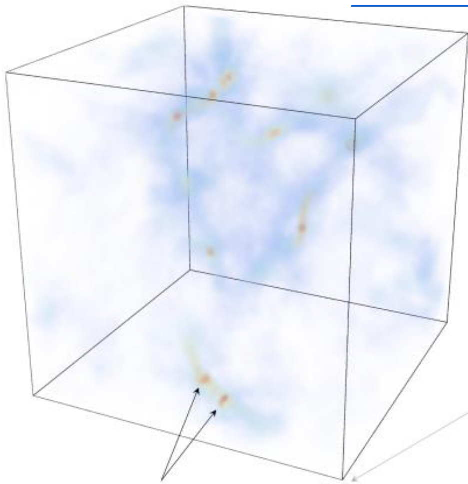
西昌, 2024/05/09

2404.18099

Q. Qiu, Y. Gao, K. Wang, H. Tian, X-M. Yang, Z. Wang

Pic. from 2203.10100

Dark solitons:

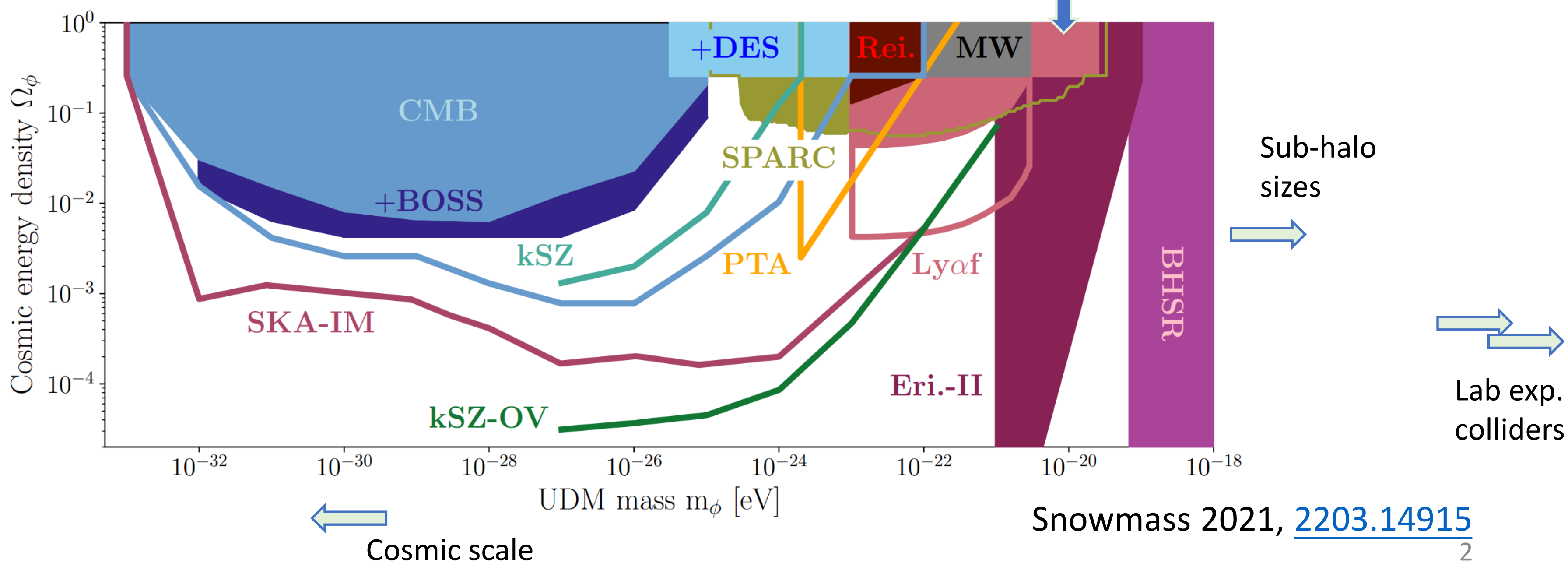


Ultra-light dark matter bosons form localized solutions under gravity or self-interactions.

See Braaten & Zhang, 19' for a nice review

$$i\,\dot{\psi} = -\frac{\nabla^2\psi}{2m} - Gm^2\,\psi\int d^3x'\,\frac{\psi^*(\mathbf{x}')\psi(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{\partial}{\partial\psi^*}V_{nr}(\psi,\psi^*)$$

\`Fuzzy DM', Lam Hui, 1610.08297



Snowmass 2021, 2203.14915

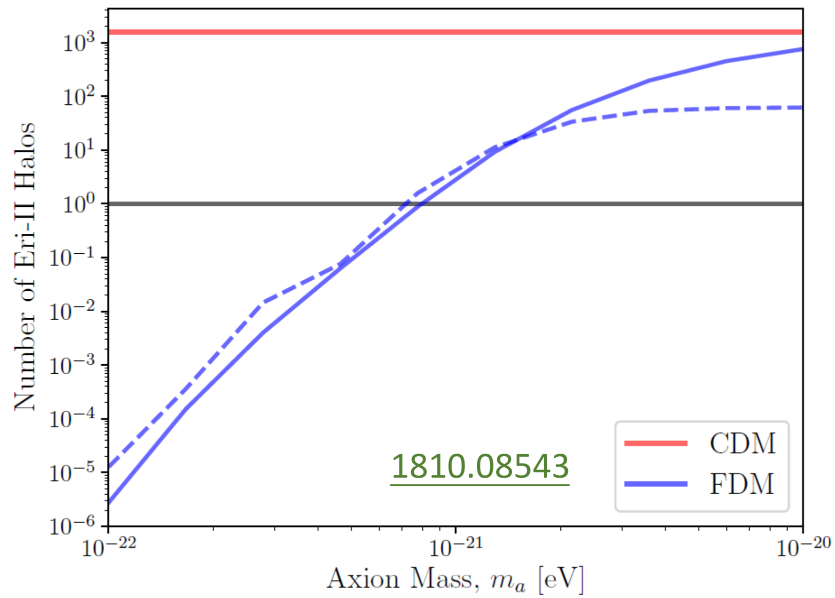
Gravitational interests in dark solitons

>> Preferred as a cored DM halo <<

Galaxy scale dynamics:
Disk thickening, stellar streams
Church, J. P. Ostriker, and P. Mocz, 18'
Amorisco and A. Loeb, 18'
excludes $m < 10^{-22}$ eV

Granularity above the de Broglie wavelength ~ $2\pi/mv$ L.Hui, 16'
exclusion limit $m \rightarrow 10^{-21}$eV

Relaxation of old cluster (dwarf galaxy scale)
exclusion limit $m \rightarrow 10^{-20}$~10^{-19}eV
Bar-Or, Fouvry, and Tremaine, 19'
Marsh and Niemeye, 19'
Wasserman, 19' etc.



Sensitivity on bosonic solitons reaches to higher boson mass with smaller & smaller objects.

The tidal relaxation mechanism

Binney & Tremaine, 2008

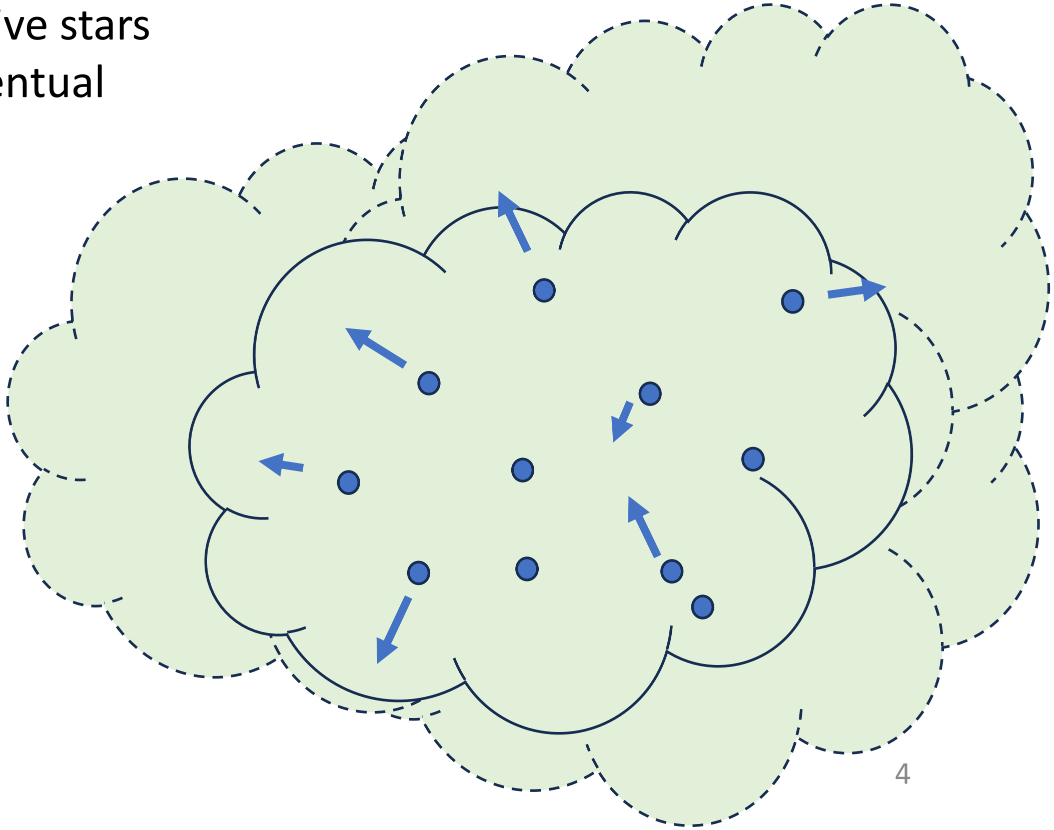
$$t_{\mathrm{r}} \sim \frac{\sigma^3 R^3}{G^2 m^2 N \log N} \sim \frac{N t_{\mathrm{d}}}{\log N}$$

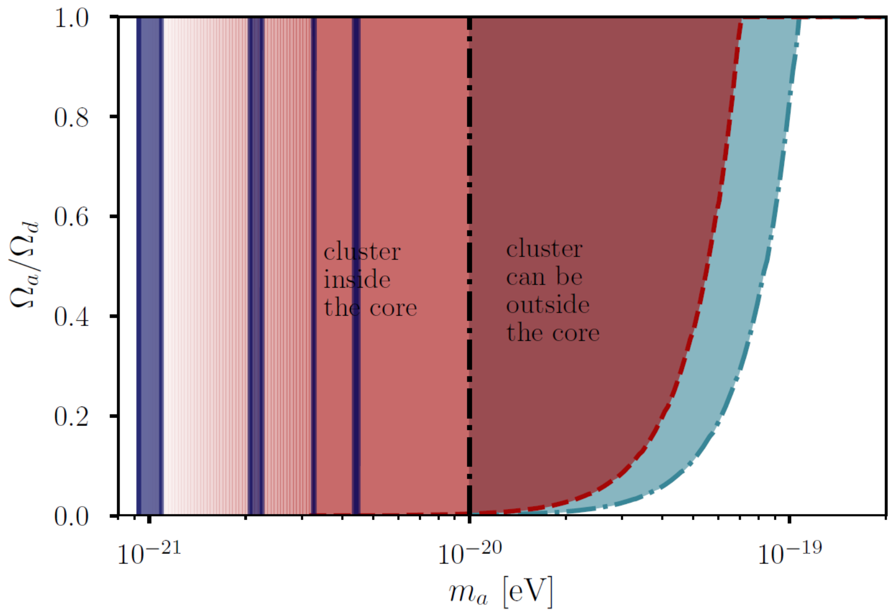
1. Density granularity (by solitons) above its coherent scale creates a randomized, noise-like gravitational potential.

2. Randomized tidal perturbations drive stars away from their orbits, leading to eventual evaporation of the system

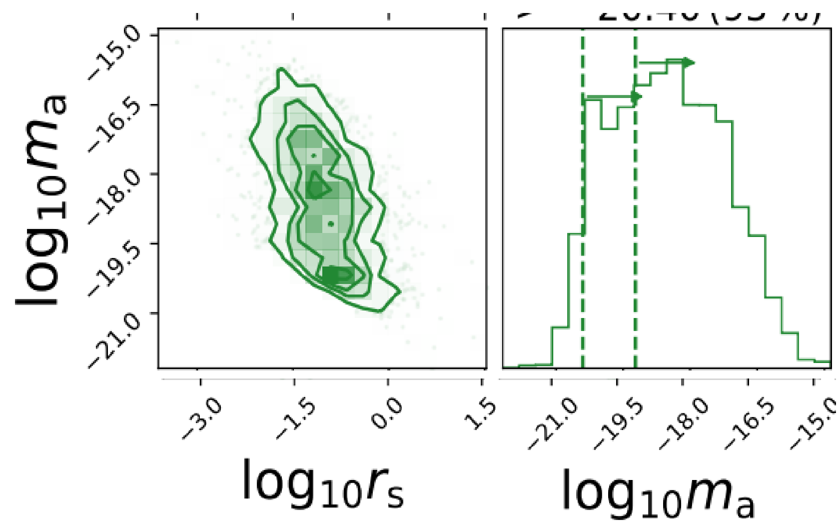
3. Most effective on systems comparable or larger than the granularity size.

4. Unlike WIMPs, solitons have a macroscopically significant mass and contribute to relaxation.





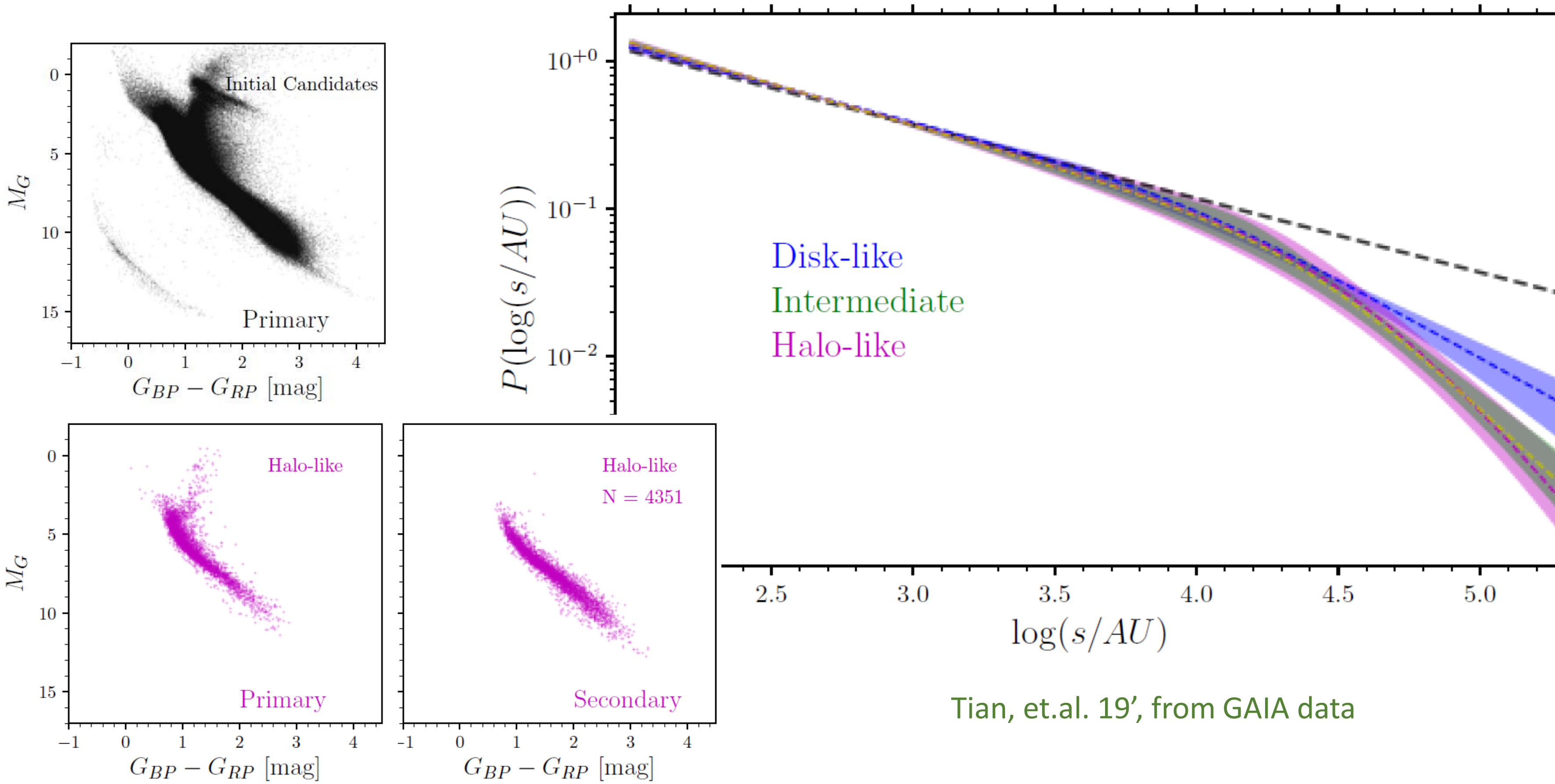
Relaxation constraint from Eridanus II
Marsh and Niemeye, 19'
See `revised constraints', Chiang et.al: 2104.13359



The MUSE-Faint survey (2101.00253)
Quote: "Substantial evidence (Bayes factor $\sim 10^{-0.6}$) for cold dark matter (a cuspy halo) over self-interacting dark matter (a cored halo) and weak evidence (Bayes factor $\sim 10^{-0.4}$) for fuzzy dark matter over cold dark matter…. These limits are equivalent to a fuzzy-dark-matter particle mass $\mathbf{m_a > 4*10^{-20}\ eVc^{-2}}$."

Smaller (<< kpc) objects?

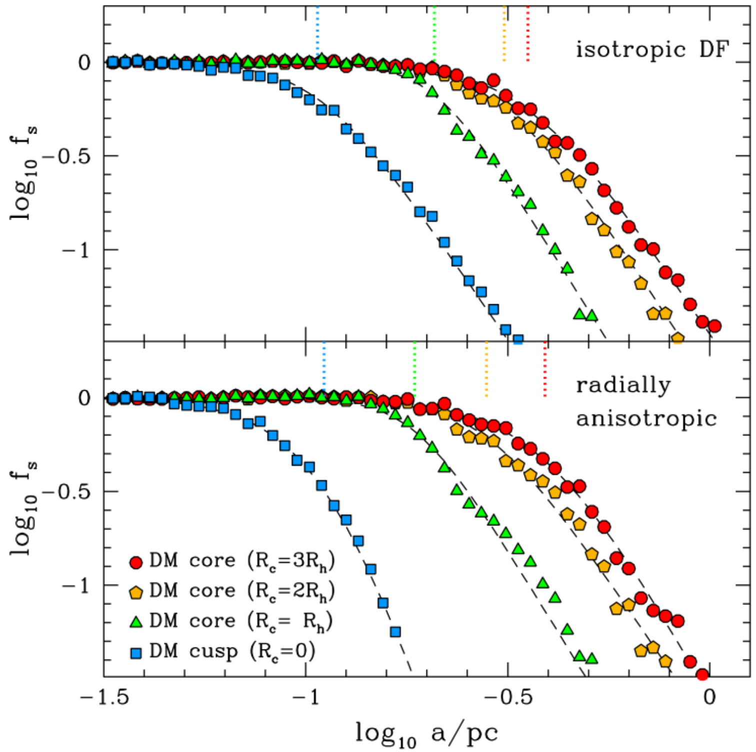
Our Galaxy hosts a population of very wide (up to ~ 1pc) binaries/candidates



Tian, et.al. 19’, from GAIA data

Astrophysical binary system disruptions…

Binary survival under a dwarf-galaxy host halo's tidal field, Peñarrubia, Ludlow, et.al, 16'



Binary evaporation by encountering field stars (with impact parameter cut-off, exclude violent collisions)

$$\tau_{evap.} = \frac{m_1 + m_2}{m_*} \frac{\sigma}{16\sqrt{\pi} G \rho_* a \ln \Lambda}$$

$$\Lambda = \frac{b_{\max} v^2_{\rm typ.}}{G(m + m_*)}$$

-> Can select isolated `halo-like' candidate binaries.

* Newtonian point-point collisions (textbook)

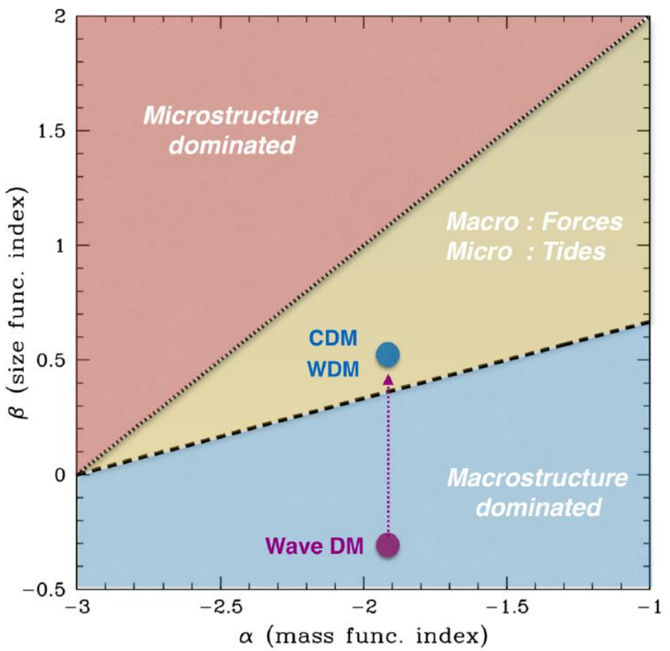
* Analogous to Brownian motion under random noise fluctuations

Smaller solitons, more fluctuation at short scales.

Micro-structure domination of stochastic fluctuations when the mass function is steep

$$\frac{\mathrm{d}^2 n}{\mathrm{d}M\mathrm{d}c} = B_0 \left(\frac{M}{M_0}\right)^{\alpha} \delta\left[c - c_0\left(\frac{M}{M_0}\right)^{\beta}\right]$$

$\beta \geq 1 + \alpha/3$ Peñarrubia, 17'



Smaller solitons with shorter coherent scales than the galaxy's observable subhalos:

* less wavy DM like
* Revealed at the size of wide binaries
* Non-negligible size for binaries (< parsec).

** for binaries, the soliton size matters.

Construction of the potential

An randomly distributed ensemble of solitons:

$$\rho(\vec{x},t) = \sum_i |\varphi(\vec{x}-\vec{x}_i-\vec{v}_i t)|^2 - \langle\rho\rangle$$

We aim at an analytic & accurate result for binary evaporation by soliton's potential.

Soliton profiles are typically solved from S-P equation. Sample ansatzes:

$$\varphi(r) = \begin{cases} \frac{m_s^{\frac{1}{2}}}{(2\pi R^2)^{\frac{3}{4}}} e^{-\frac{r^2}{4R^2}}, & \text{Gaussian [27];} \\ \left(\frac{3m_s}{\pi^3 R^3}\right)^{\frac{1}{2}} \text{sech}\left(\frac{r}{R}\right), & \text{Sech [16];} \\ \left(\frac{m_s}{7\pi R^3}\right)^{\frac{1}{2}} \left(1+\frac{r}{R}\right) e^{-\frac{r}{R}}, & \text{Exponential linear (EL) [16].} \end{cases}$$

normalized so that the density of the scalar field satisfies $\rho(r) \propto \varphi(r)^2$

Correlation functions

Spectra of the two-point correlation functions for the density ρ and gravitational potential Φ

$$\langle \rho(\vec{r},t)\rho(\vec{r'},t')\rangle \equiv C_\rho(\vec{r}-\vec{r'},t-t')$$

$$\langle \Phi(\vec{r},t)\Phi(\vec{r'},t')\rangle \equiv C_\Phi(\vec{r}-\vec{r'},t-t')$$

Fourier trans.

$$C_\rho(\vec{r},t) = \int \frac{d^3k d\omega}{(2\pi)^4} C_\rho(\vec{k},\omega) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

By Poisson Eq. $\nabla^2\Phi = 4\pi G\rho$

$$C_\Phi = 16\pi^2 G^2 k^{-4} C_\rho$$

$$F(\vec{v}) = \frac{\rho_0}{(2\pi\sigma^2)^{\frac{3}{2}}} e^{-\frac{v^2}{2\sigma^2}}$$

Small-sized solitons are more likely virialized so we assume a Maxwellian velocity distribution.

Performing an average over their *v*-distribution

$$C_\rho(\vec{k},\omega) = \frac{1}{m_s}\int d^3r d^3r' d^3v dt \rho(\vec{r})\rho(\vec{r'})F(\vec{v})e^{-i\vec{k}\cdot(\vec{r}+\vec{r'}+\vec{v}t)}e^{i\omega t}$$

$$C_\rho(\vec{k},\omega) = \frac{1}{m_s}\rho^2(\vec{k})\rho_0\sqrt{\frac{2\pi}{k^2\sigma^2}}e^{-\frac{\omega^2}{2k^2\sigma^2}}$$

$\rho(k)$ is the F.T. of the soliton profile

Density corr. function with soliton profile and Maximillian velocity distribution

$$C_{\rho,\mathrm{Gauss}}(\vec{k},\omega) = m_s \rho_0 \sqrt{\frac{2\pi}{k^2\sigma^2}} e^{-\frac{\omega^2}{2k^2\sigma^2}} e^{-k^2 R^2},$$

$$C_{\rho,\mathrm{Sech}}(\vec{k},\omega) = \frac{9 m_s}{\pi^2 k^2 R^2} \left[-2 + \pi k R \coth\left(\frac{\pi k R}{2}\right)\right]^2 \mathrm{csch}^2\left(\frac{\pi k R}{2}\right) \rho_0 \sqrt{\frac{2\pi}{k^2\sigma^2}} e^{-\frac{\omega^2}{2k^2\sigma^2}},$$

$$C_{\rho,\mathrm{EL}}(\vec{k},\omega) = \frac{4096 m_s}{49} \frac{(28 + k^2 R^2)^2}{(4 + k^2 R^2)^8} \rho_0 \sqrt{\frac{2\pi}{k^2\sigma^2}} e^{-\frac{\omega^2}{2k^2\sigma^2}}.$$

Time variance: relative motion btw (1) binary/solitons, (2) solitons in the bkg

Assuming $\omega \approx k \cdot v$, these functions (1) approach to a noise spectrum on large scales; (2) suppressed for $k \cdot R \gg 1$

Evaporation: increments of relative motion's energy

For binary evaporation, the two-body system can be reduced to a single-body problem (plus some input from its COM motion)

$$\Delta E = \mu \vec{v_r} \cdot \Delta \vec{v_r} + \frac{1}{2}\mu (\Delta \vec{v_r})^2$$

$\nabla \Phi$ drives the CM's random walk (can be small compared to $v_{cm} \sim 10^{-3} c$)

Tidal acceleration should be in proportional to $\partial \wedge \partial \Phi$

$$\Delta \vec{v}_r = \Delta \vec{v}_1 - \Delta \vec{v}_2,$$

$$\frac{\langle \Delta \vec{v_r}^2 \rangle}{T} = \frac{1}{T}\left(\langle \Delta \vec{v}_1^2 \rangle + \langle \Delta \vec{v}_2^2 \rangle - 2 \langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle \right)$$

$$\frac{\langle \Delta E \rangle}{T} = \mu \frac{\vec{v}_r \cdot \langle \Delta \vec{v}_r \rangle}{T} + \frac{1}{2}\mu \left(\frac{\langle \Delta \vec{v}_1^2 \rangle}{T} + \frac{\langle \Delta \vec{v}_2^2 \rangle}{T} - \frac{2 \langle \Delta \vec{v}_1 \cdot \Delta \vec{v}_2 \rangle}{T} \right)$$

$\langle \; \rangle$: spectral averaging over the soliton ensemble.

1/T : time averages… should also take account of the binary's Keplerian motion.

`Fast' orbit: compact binaries

$2\pi/\omega_b \ll \lambda_{\rm DM}/v_c$

Binary's orbital period is smaller than that of soliton variations, Orbital average 1/T before performing ensemble average.

1st order: zero-result

$$(\partial_t \vec{v}_r)\cdot\vec{v}_r = 2a\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}\left[(\partial\wedge\partial)\Phi(\vec{x}_{CM},t)\right]\cdot\vec{v}_r$$

$$\approx (\vec{r}_1-\vec{r}_2)\left[(\partial\wedge\partial)\Phi(\vec{x}_{CM},t)\right]\cdot\vec{v}_r$$

$$\dot{E} = \partial_t(\frac{1}{2}\mu\vec{v}^2) = \mu(\partial_t\vec{v}_r)\cdot\vec{v}_r$$

$$\dot{E} = \mu\int\frac{d\vec{k}d\omega}{(2\pi)^4}(\vec{k}\cdot\vec{v}_r)\vec{k}\cdot(\vec{r}_1-\vec{r}_2)\tilde{\Phi}(k,\omega)e^{i\vec{k}\cdot\vec{x}_{CM}} \Longrightarrow \langle\dot{E}\rangle = \frac{1}{T}\int_0^{T=\frac{2\pi}{\omega}}\dot{E}dt = 0$$

Vectors to the odd power vanish in binary orbit average

2nd order derivative:

$$\frac{\partial\dot{E}}{\partial t} = \mu\left(\langle\ddot{\vec{v}}\cdot\vec{v}\rangle + \langle\dot{\vec{v}}\cdot\dot{\vec{v}}\rangle\right) \Longrightarrow \mu\int k^2(k^2+\omega^2)\frac{\pi}{4}(a^2+b^2)\tilde{C}_\Phi(\vec{k},\omega)\frac{d\vec{k}d\omega}{(2\pi)^4}$$

Non-zero after orbital and angular averages: Contribution at 2nd (& higher) orders

Slow orbit: wide binaries

$$\frac{\lambda_{\rm DM}}{v} \ll T \ll \frac{2\pi}{\omega_b}$$

For wide binaries, $v_r / v_{cm} \sim 10^{-3}$. For a longer orbit period than the time scale of potential variations, one can take the ensemble average first, and leave the orbital parameters as constants.

Velocity change driven by the potential's gradient

$$\Delta\vec{v} = -i\int_0^T dt \int \frac{\vec{k}d^3k d\omega}{(2\pi)^4}\Phi(\vec{k},\omega)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

To include the gravitational potential variations, expand out r(t) that travels across the DM background

$$\vec{r}(t) \approx \vec{r}_0 + \vec{v}_0 t + \int_0^t ds(t-s)\dot{\vec{v}}(s)$$

$$\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right] = \exp\left[i\vec{k}\cdot\left(\vec{r}_0+\vec{v}_0 t + \int_0^t d\tau(t-\tau)\dot{\vec{v}}(\vec{r}_0+\vec{v}_0\tau,\tau)\right) - i\omega t\right]$$

$$\approx e^{i\vec{k}\cdot(\vec{r}_0+\vec{v}_0 t)-i\omega t}\left[1 + i\vec{k}\cdot\int_0^t d\tau(t-\tau)\dot{\vec{v}}(\vec{r}_0+\vec{v}_0\tau,\tau)\right]$$

The first term will vanish after the ensemble average.
The only contribution comes from the second term

Expand out the equation

$$\Delta\vec{v} = i\int_0^T dt\int_0^t d\tau(t-\tau)\int\frac{\vec{k}d^3kd\omega}{(2\pi)^4}\int\frac{(\vec{k}\cdot\vec{k'})d^3k'd\omega'}{(2\pi)^4}\Phi(\vec{k},\omega)\Phi^*(\vec{k'},\omega')e^{i\vec{k}\cdot(\vec{r_0}+\vec{v_0}t)-i\omega t}e^{-i\vec{k'}\cdot(\vec{r_0}+\vec{v_0}\tau)+i\omega'\tau}$$

And make use of the relation:

$$\left\langle\Phi(\vec{k},\omega)\Phi^*(\vec{k'},\omega')\right\rangle = (2\pi)^4C_\Phi(\vec{k},\omega)\delta^3(\vec{k}-\vec{k'})\delta(\omega-\omega')$$

We can make it an integral over the correlation function.

$$\langle\Delta\vec{v}\rangle = \int_0^T dt\int_0^t d\tau\int\frac{d^3kd\omega}{(2\pi)^4}\vec{k}^2C_\Phi(\vec{k},\omega)\frac{\partial}{\partial\vec{v_0}}e^{i(\vec{k}\cdot\vec{v_0}-\omega)(t-\tau)}$$

The 1st order `diffusion' coefficient:

$$D[\Delta\vec{v}] = \frac{\langle\Delta\vec{v}\rangle}{T} = -\frac{1}{2}\int\frac{\vec{k}d^3kd\omega}{(2\pi)^3}\vec{k}^2C_\Phi(\vec{k},\omega)K'_T(\omega-\vec{k}\cdot\vec{v_0})$$

where $$K'_T(\omega) = \frac{\omega T\sin(\omega T)-2[1-\cos(\omega T)]}{\pi\omega^3 T}$$

For binary evaporation, we repeat for the relative velocity $\Delta\vec{v}_r = \Delta\vec{v}_1 - \Delta\vec{v}_2$

1st term in energy increment:

$$\frac{\vec{v_r}\cdot\langle\Delta\vec{v_r}\rangle}{T} = -\frac{1}{2}\int\frac{(\vec{k}\cdot\vec{v_r})\vec{k}^2 d^3k d\omega}{(2\pi)^3}C_\Phi(\vec{k},\omega)\left(K'_T(\omega-\vec{k}\cdot\vec{v}_1) - K'_T(\omega-\vec{k}\cdot\vec{v}_2)\right)$$

$$= -\frac{1}{2}\int\frac{(\vec{k}\cdot\vec{v_r})\vec{k}^2 d^3k d\omega}{(2\pi)^3}C_\Phi(\vec{k},\omega)\left(\delta'(\omega-\vec{k}\cdot\vec{v}_1) - \delta'(\omega-\vec{k}\cdot\vec{v}_2)\right)$$

For $v_1 \approx v_2 \approx v_{CM}$ and $|v_1 - v_2| \ll v_{CM}$, this contribution is suppressed by v_r / v_{CM}

The evaporation will be dominated by the 2nd order $(\Delta v)^2$ term.

2nd term in energy increment: $\frac{1}{2}\mu\left(\frac{\langle\Delta\vec{v}_1^2\rangle}{T}+\frac{\langle\Delta\vec{v}_2^2\rangle}{T}-\frac{2\langle\Delta\vec{v}_1\cdot\Delta\vec{v}_2\rangle}{T}\right)$

After some algebra…

$$\frac{\langle\Delta\vec{v}_1^2\rangle}{T}=\int\frac{\vec{k}^2d^3k}{(2\pi)^3}C_\Phi(\vec{k},\vec{k}\cdot\vec{v}_1)$$

$$\frac{\langle\Delta\vec{v}_2^2\rangle}{T}=\int\frac{\vec{k}^2d^3k}{(2\pi)^3}C_\Phi(\vec{k},\vec{k}\cdot\vec{v}_2)$$

$$\frac{\langle\Delta\vec{v}_1\cdot\Delta\vec{v}_2\rangle}{T}=\int\frac{\vec{k}^2d^3k}{(2\pi)^3}C_\Phi(\vec{k},\vec{k}\cdot\vec{v_c})\cos[\vec{k}\cdot(\vec{r}_1-\vec{r}_2)]$$

[Use approximation]

$v_c \gg v_r,\ v_1\approx v_2\approx v_c$

Finally, the tidal heating rate:

Remember $|r_1-r_2| \sim 2a$, soliton size becomes relevant.

$$\frac{\langle\Delta E\rangle}{T}=\sqrt{\frac{2}{\pi}}\frac{\mu\rho_0G^2}{m_s\sigma}\int\frac{d^3k}{k^3}\rho^2(\vec{k})e^{-\frac{(\vec{k}\cdot\vec{v}_c)^2}{2k^2\sigma^2}}2\left(1-\cos\left[\vec{k}\cdot(\vec{r}_1-\vec{r}_2)\right]\right)$$

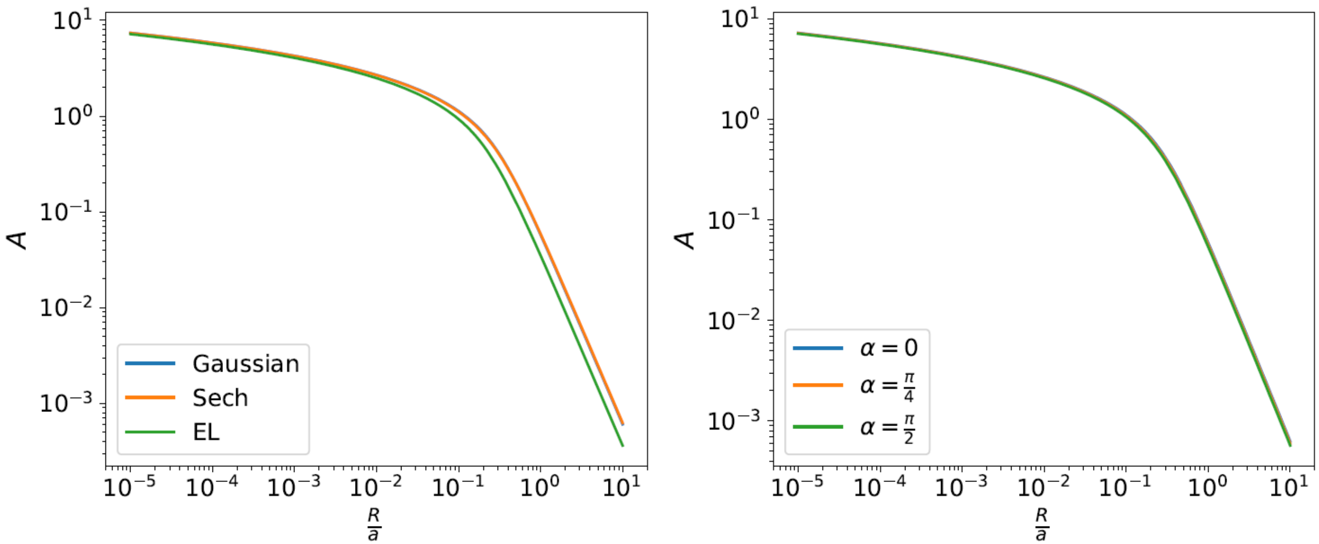
This convolution factor takes care of scale dependence and reveals the difference btw the 2-body relative motion and the Brownian walk of a single-point particle (COM) under tidal perturbations.

Evaporation time scale

$$\frac{\langle \Delta E \rangle}{T} = \frac{8\pi\mu\rho_0 G^2 m_s}{v_c} A\left(r_x, r_y, r_z, R, \frac{v_c}{\sigma}\right)$$

Re-define our integrals and make a similar-looking to the classical formula

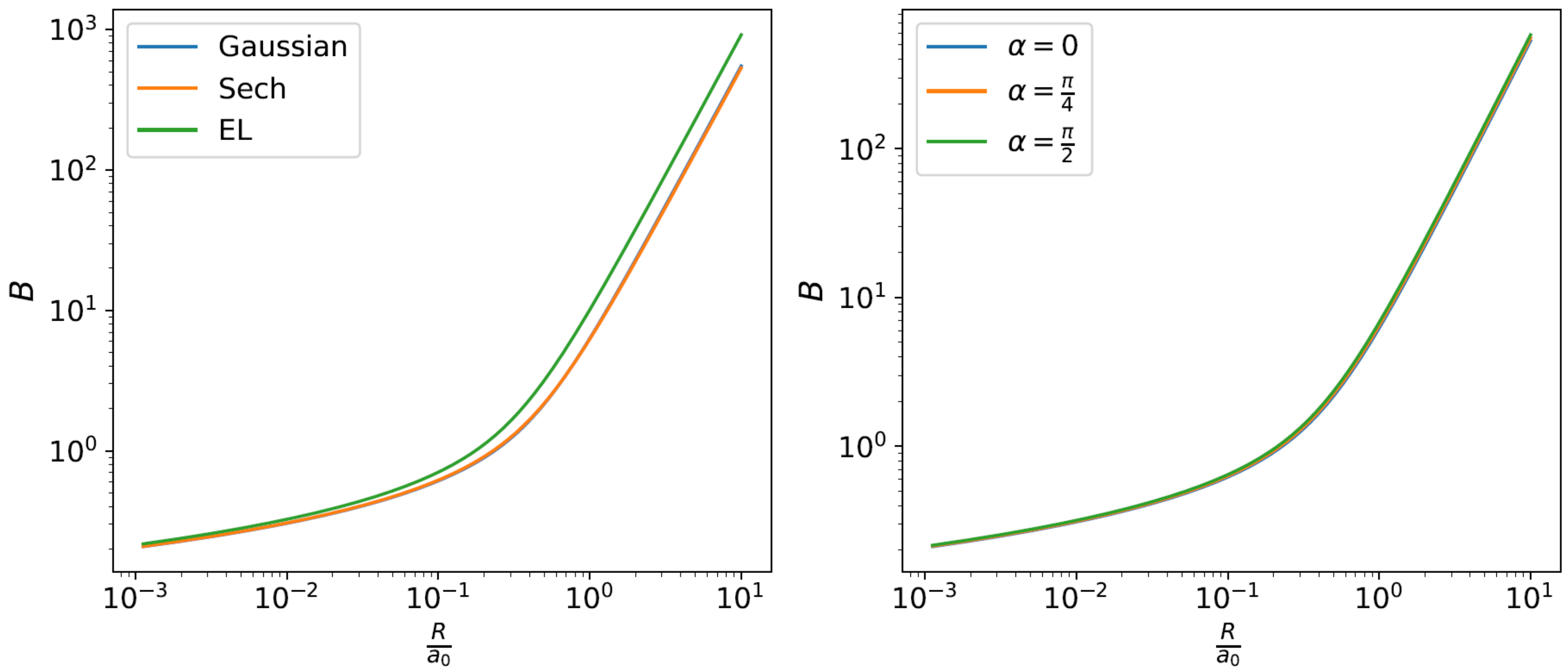
The relaxation time: $$t_d = \frac{|E_0|}{\left(\frac{dE}{dt}\right)_0} \int_0^1 \frac{dx}{A\left(\frac{R}{a_0}x, \frac{v_c}{\sigma}\right)}$$



$$A = \frac{1}{\sqrt{2\pi}} \frac{v_c}{\sigma} \int_0^{+\infty} \frac{dk}{k} \frac{\rho^2(k)}{m_s^2} \int_{-1}^{1} dx e^{-\frac{v_c^2 x^2}{2\sigma^2}} \left[1 - J_0\left(k\sqrt{r_x^2 + r_y^2}\sqrt{1-x^2}\right)\cos(k r_z x)\right]$$

$$t_d = 14.3\,\mathrm{Gyr}\left(\frac{M_T}{0.5\,M_\odot}\right)\left(\frac{a_0}{0.1\,\mathrm{pc}}\right)^{-1}\left(\frac{v_c}{200\,\mathrm{km/s}}\right)\left(\frac{m_s}{30\,M_\odot}\right)^{-1}\left(\frac{\rho_0}{0.4\,\mathrm{GeV/cm^3}}\right)^{-1} B\left(\frac{R}{a_0},\frac{v_c}{\sigma}\right)$$

+ some dependence on the orbital plane orientation and soliton profiles.



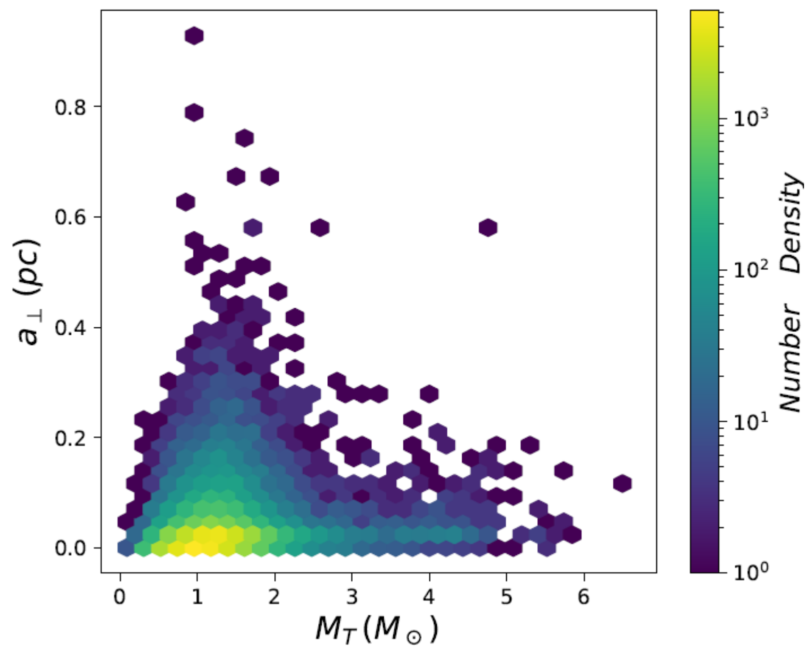
GAIA's halo-like wide binaries

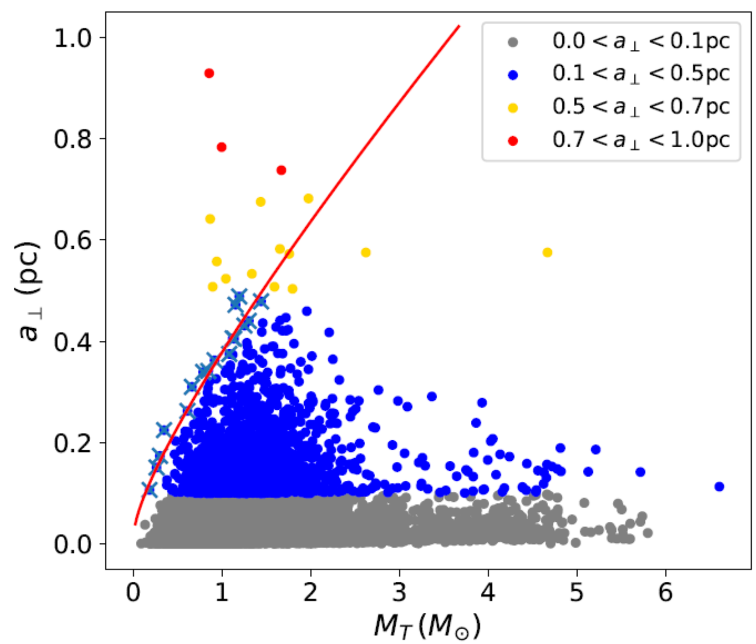
El-Badry and H.-W. Rix, Mon. Not. Roy. Astron. Soc. 480, 4884 (2018)

- Select old binary candidates with large tangential velocity
- Select candidates with a low probability of being aligned by chance
- Veto candidates with close companions, many neighbors (N<2)
- Exclude candidates containing white dwarves

2000+ candidates pass selection cuts out of 62990 from GAIA data

Gaia Collaboration, Astronomy & Astrophysics 649, A9 (2021), 2012.02036.





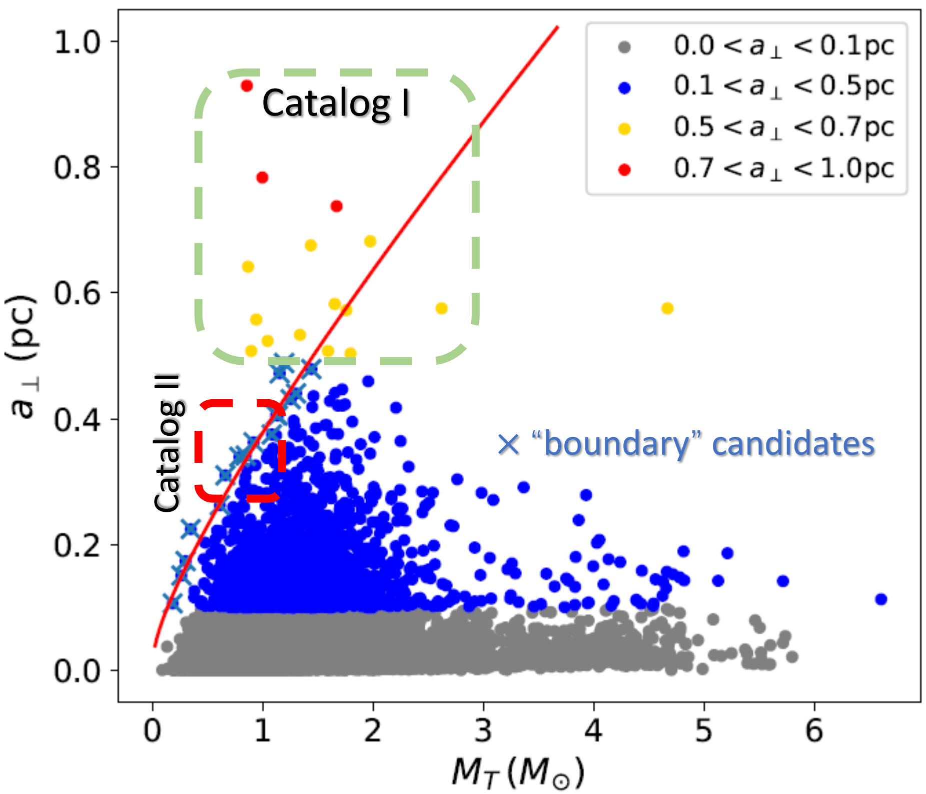
Select high-prob. candidates with large separation and relatively lower mass.

Require their average evaporation to be longer than 10 Gyr

$$\langle t_d \rangle \equiv \frac{1}{N} \sum_i t_{d,i} \; < 10 \text{ Gyr},$$

Source id1	Source id2	parallax1	parallax2	g mag1	g mag2	R chance align	M_1	M_2	M_T	$a_\perp$(pc)
1312689344512158848	1312737894822499968	3.375	3.310	12.07	17.21	0.000996	0.950	0.483	1.432	0.675
6644959785879883776	6644776515331203840	2.007	2.354	17.85	18.00	0.0462	0.440	0.412	0.851	0.929
2305945096292235648	2305945538674043392	2.366	2.316	15.74	17.30	1.53e-09	0.518	0.373	0.891	0.508
2127864001174217088	2127863726296352256	1.370	1.363	13.64	15.60	0.0357	0.924	0.741	1.665	0.737
577970351704355072	580975626220823296	3.117	3.021	16.35	17.47	0.0850	0.484	0.452	0.937	0.557
1401312283813377536	1401310698969746944	1.244	1.234	16.97	18.92	0.0113	0.631	0.409	1.040	0.523
1559537092292382720	1559533965556190848	1.209	1.224	13.63	15.03	0.00142	1.117	0.854	1.971	0.682
5476416420063651840	5476421406528047104	1.204	1.214	13.66	15.46	0.0834	1.016	0.775	1.791	0.503
4004141698745047040	4004029857796571136	5.104	5.100	14.09	16.07	0.00492	0.580	0.412	0.992	0.783
6779722291827283456	6779724009814201984	1.575	1.579	17.72	18.79	0.00712	0.484	0.378	0.862	0.641
3594791561220458496	3594797539814936832	1.065	1.069	14.44	16.31	0.0763	0.917	0.731	1.649	0.582
3871814958946253312	3871818601078520192	1.449	1.499	15.66	17.13	0.0188	0.676	0.657	1.333	0.533
2379971950014879360	2379995177198014976	1.604	1.588	14.21	16.58	0.0270	0.876	0.712	1.588	0.507
6826022069340212864	6826040868412655872	2.379	2.373	11.76	14.05	0.0987	1.016	0.738	1.754	0.572
5798275535462480768	5798276325736369024	1.247	1.261	13.32	13.76	0.000536	1.443	1.176	2.619	0.575

TABLE I: (Catalog I). High probability halo-like wide binaries with $a_\perp > 0.5\,\text{pc}$ and $M_T < 3\,M_\odot$.



- Binaries shift left-ward over time under tidal perturbations
- A t_d=10 Gyr curve and its soliton configuration (m_s, R) is ***disfavored*** if lots of candidates reside on its left side
- (red line) on the boundary of populated region corresponds to t_d= 10 Gyr with (m_s=9.3$m_\odot$, R=0.03pc)

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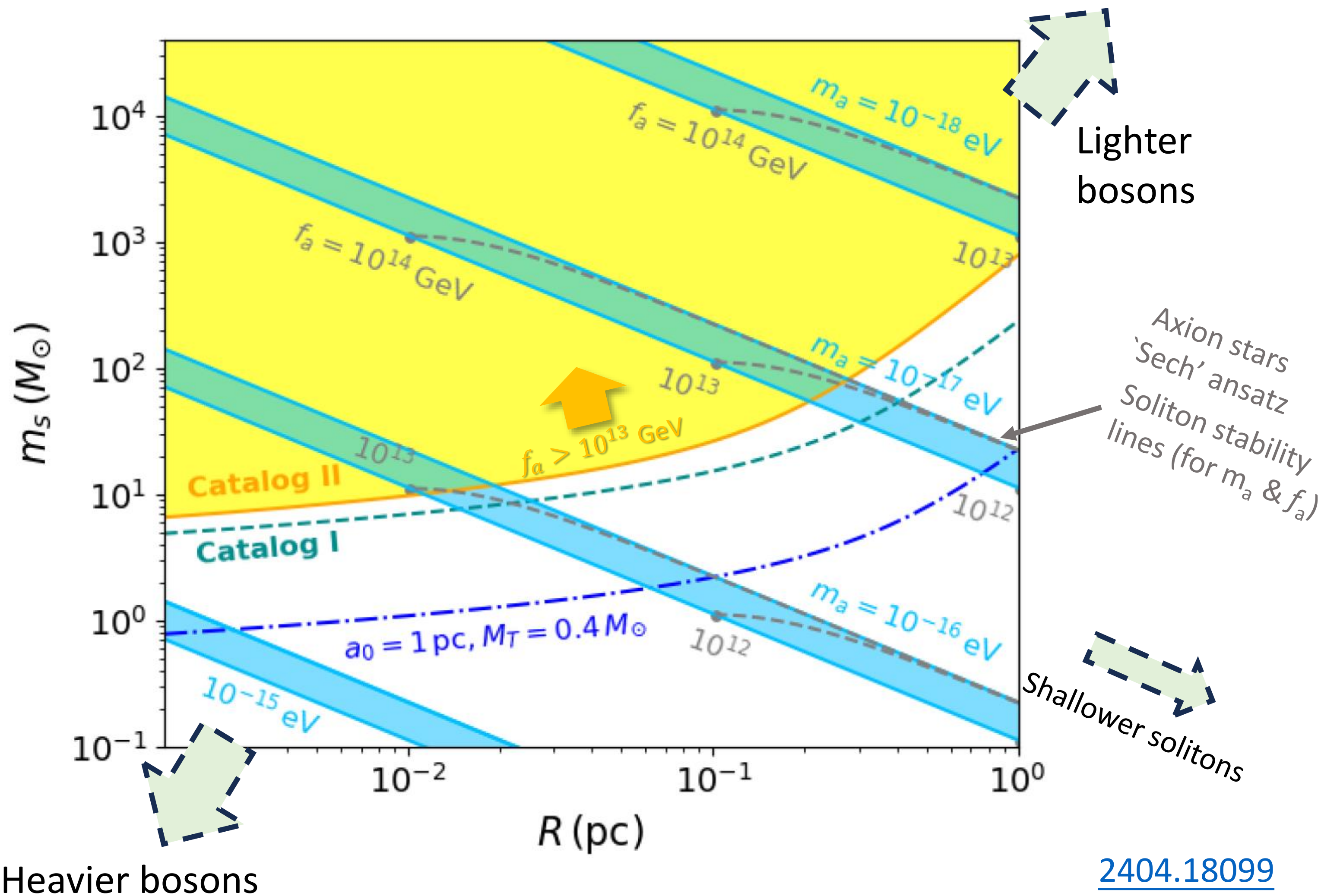
Catalog I & II limits for $t_d = 10^{10}$ yr under solitons' tidal perturbation.
(Blue bands) solutions for ALPs with potential: $V(a) \sim -m_a^2\, f_a^2 \cos(a/f_a)$



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Take home messages

- We provide an *analytic* treatment of binary evaporation by randomly distributed, spatially extended objects. An analytic form factor takes care of scale dependence in the density profile.
- GAIA data contain some *2000+ high-probability, halo-like, a>0.1 pc candidates*. Their survival yields observation sensitivity to tidal disruption from DM.
- For axion stars, sensitivity extends into heavier ALP mass range $10^{-17} < m_a < 10^{-15}$ eV.