第三届地下和空间粒子物理与宇宙物理前沿问题研讨会
2024年5月7日-12日， 西昌

Majoron dark matter and the BAU

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Based on works with Ying-quan Peng， Hai-jun Li, Ming-jie Jin and Yue Wang

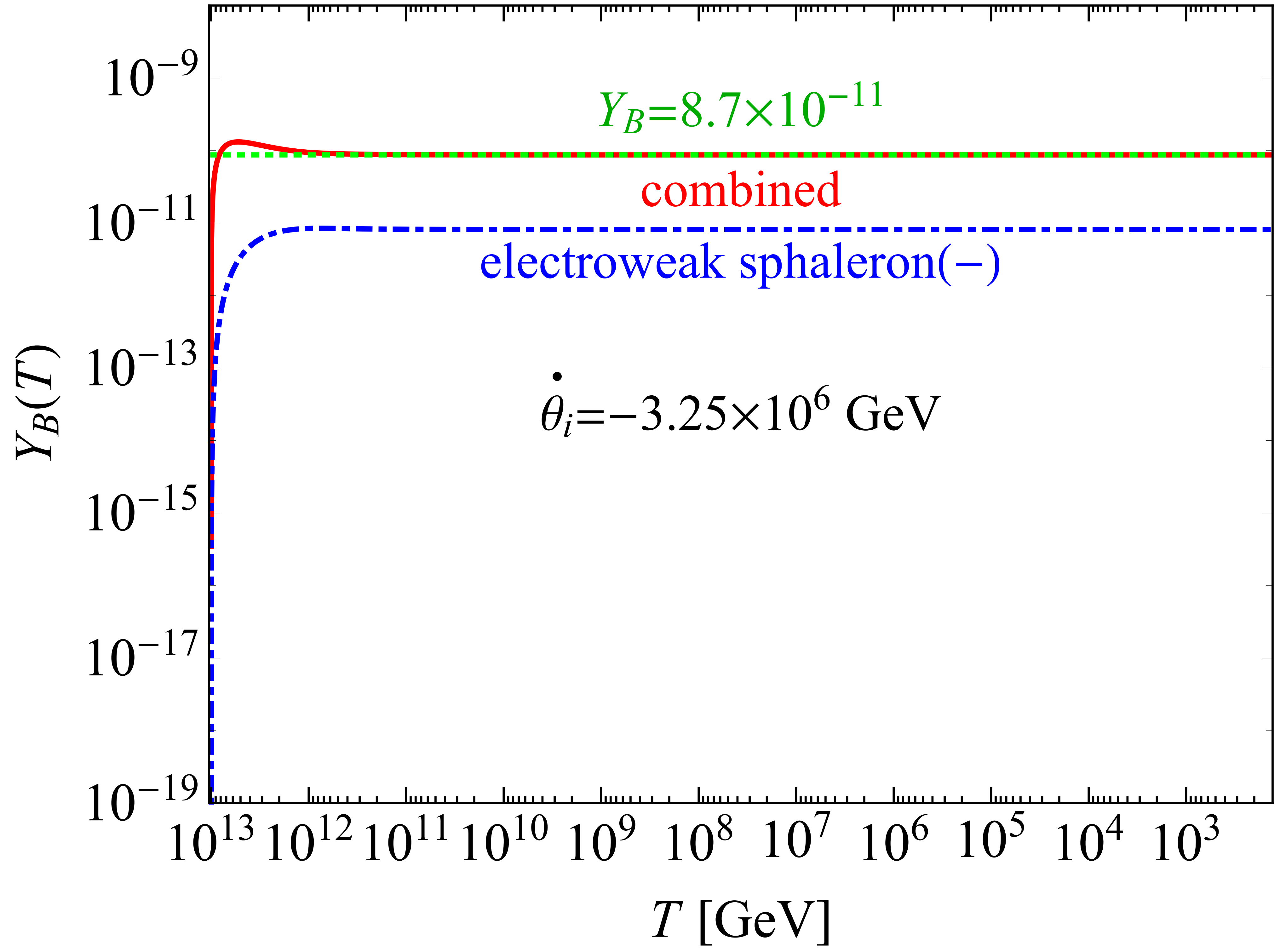
Preview

I: The origin of the ALP mass

There is no well-known mechanism for the mass generation of ALP, except the QCD axion.

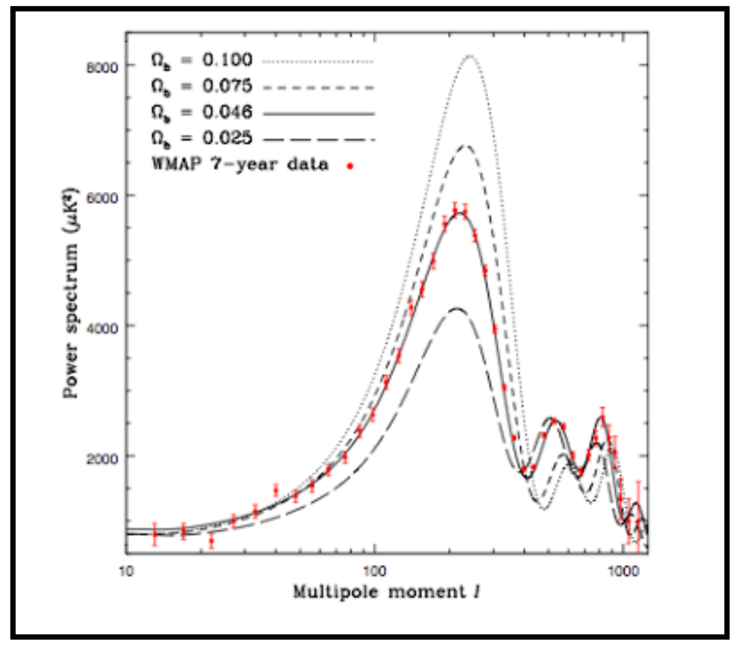
We present a mechanism for the mass generation of Majoron via the type-I and the type-II seesaw mechanism.

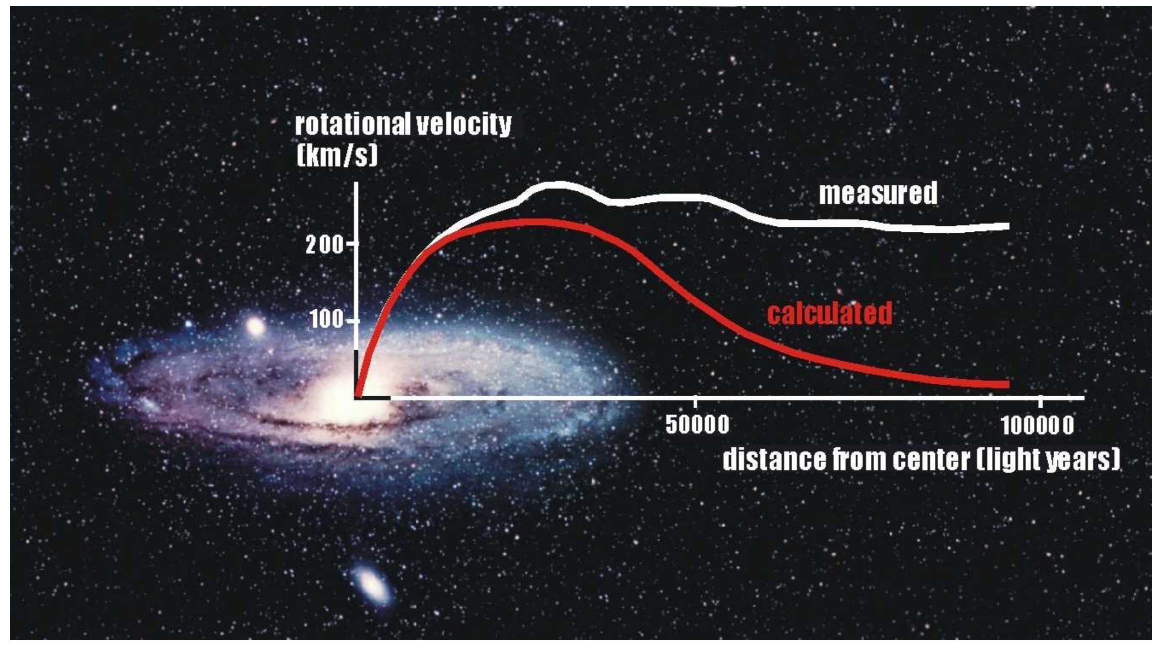
II: The Baryon Asymmetry of the Universe

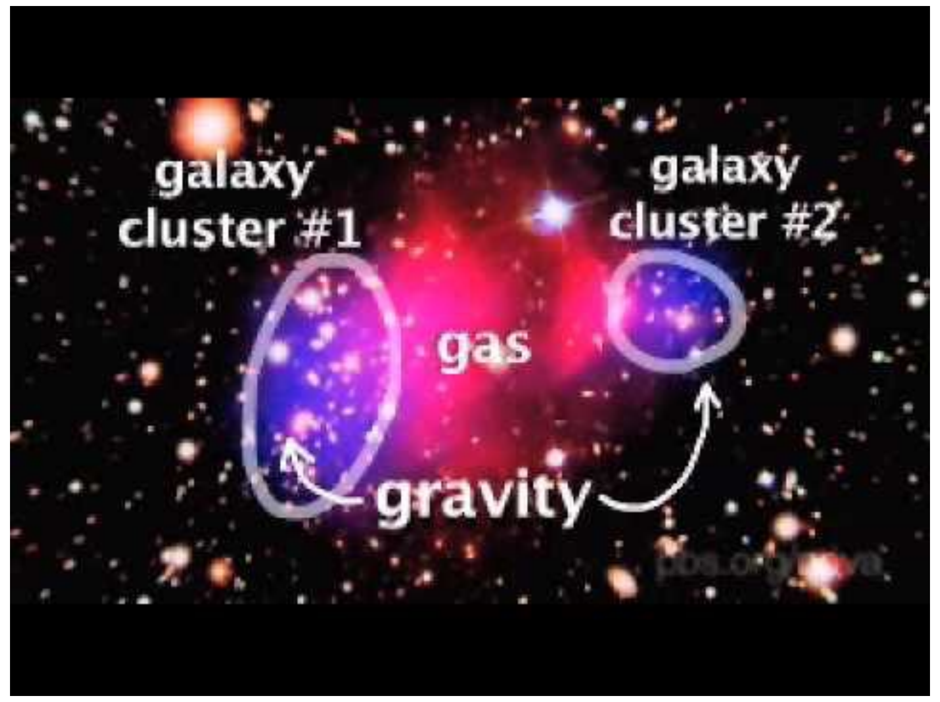


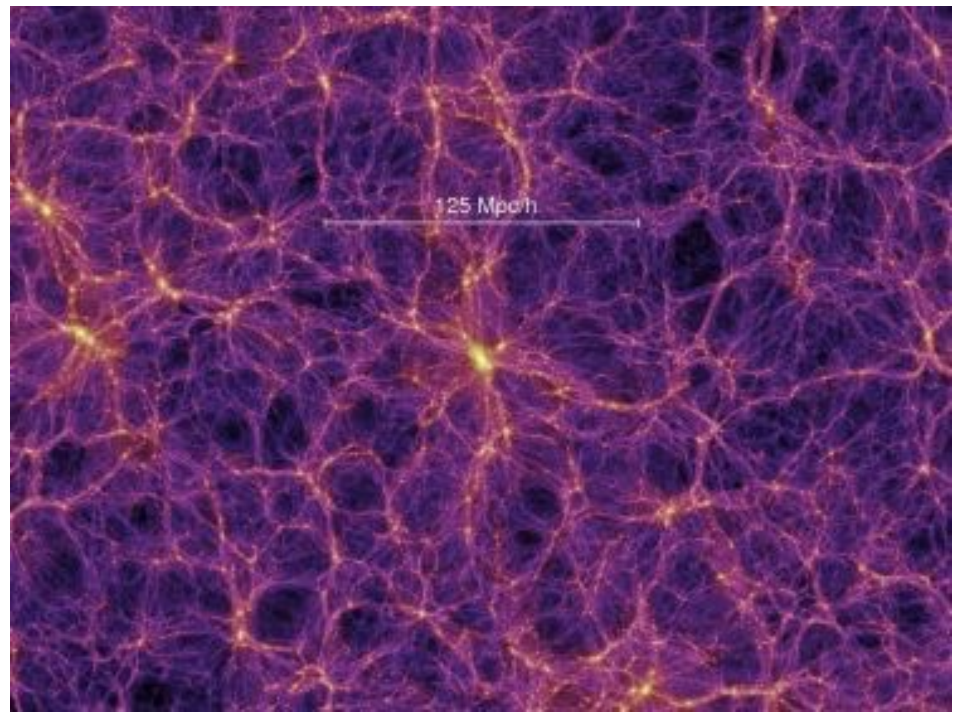
New physics beyond the SM-dark matter

Evidence of dark matter





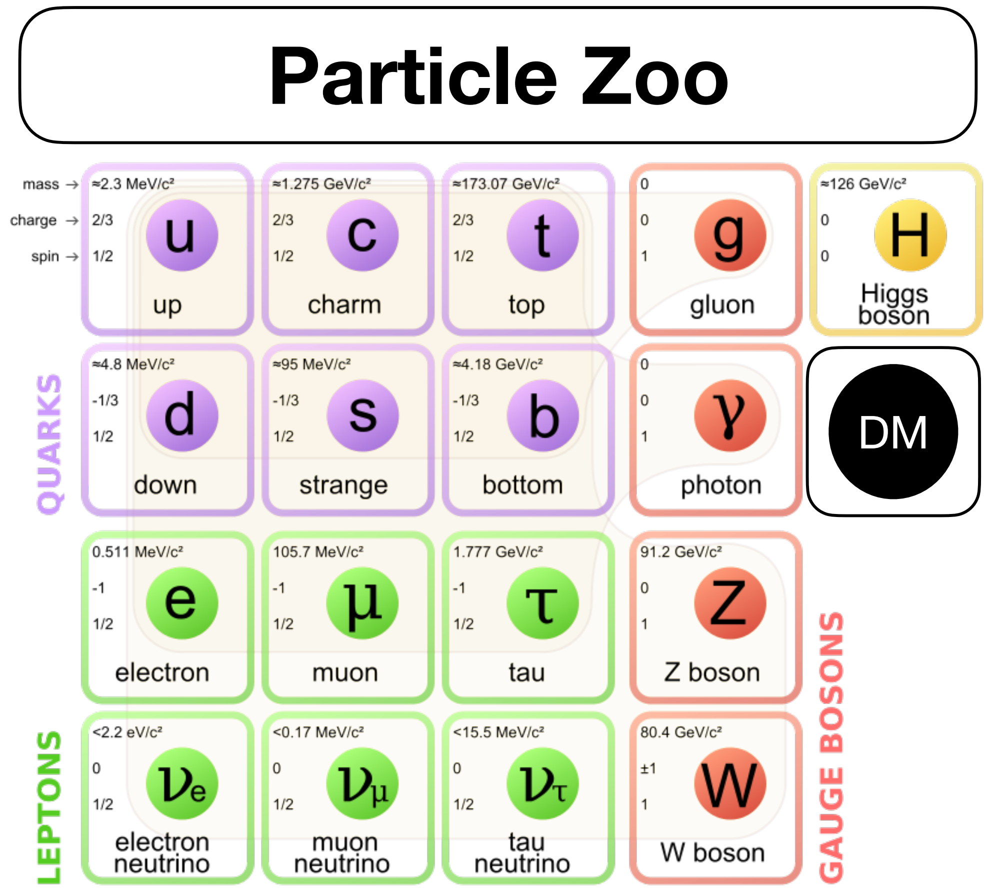


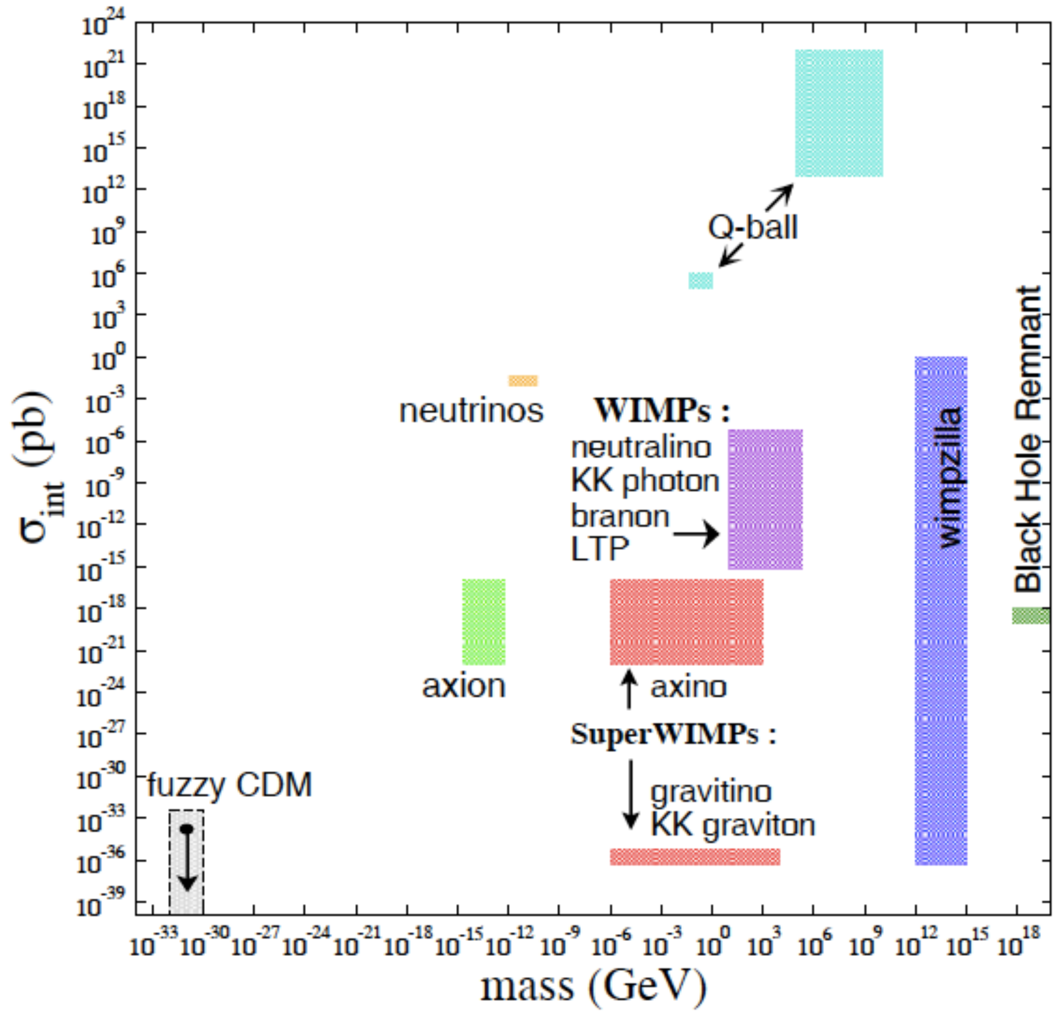


What is dark matter?

~~Mass~~ ~~Spin~~ ~~Interactions~~

Neutral, non-baryonic, weakly interacting particle!

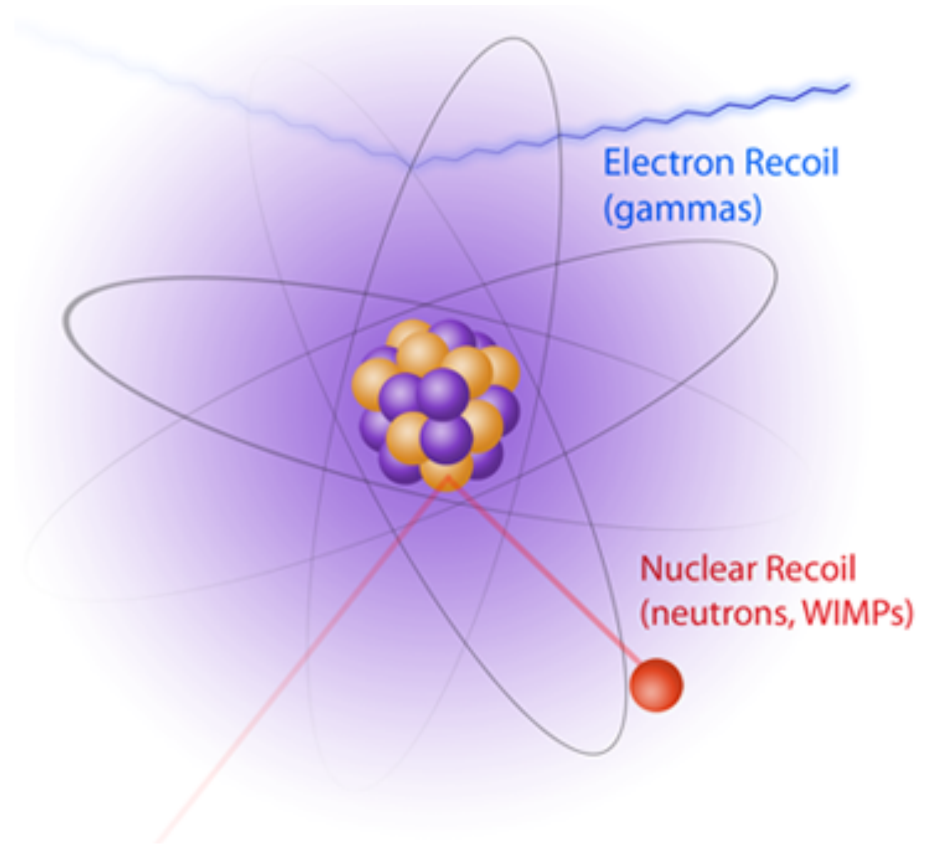


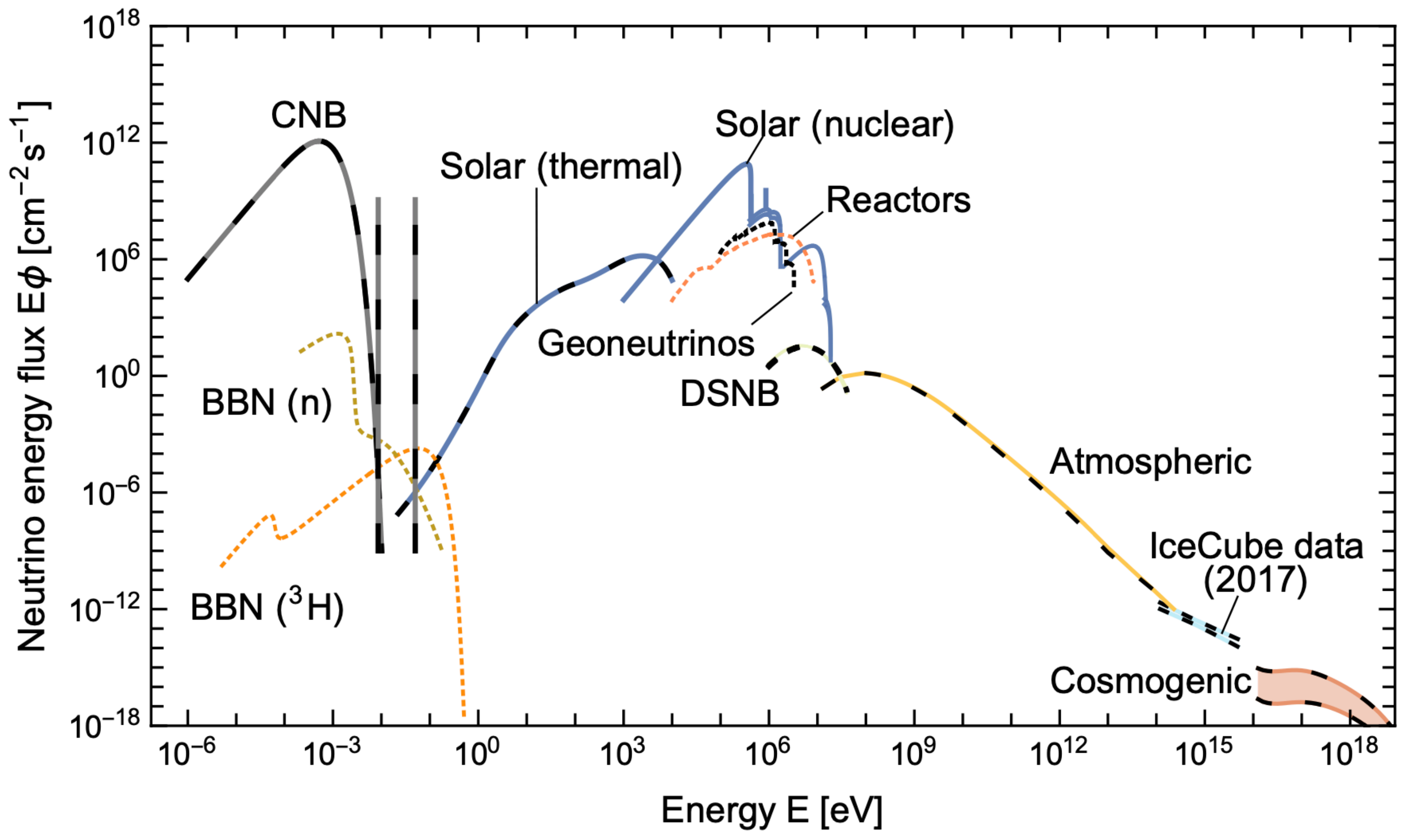


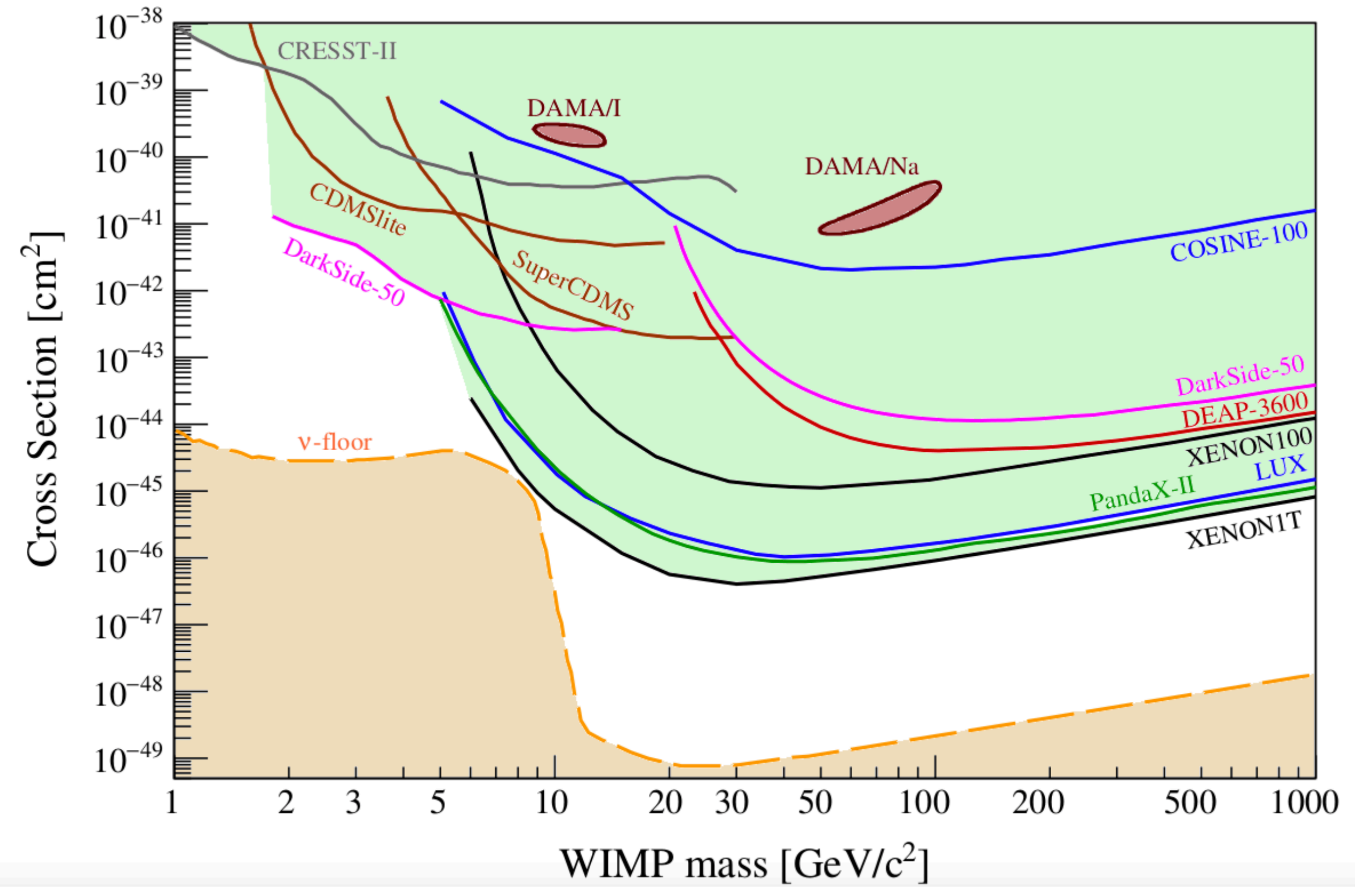
What we concern: construct a DM theory

Where we start from? ν physics

- Properties of neutrinos are similar to these of dark matter
- Neutrino is a **hot** dark matter candidate
- Sterile neutrino is typical **warm/cold** dark matter candidate
- The signal of neutrino in direct detection experiments is similar to that of DM

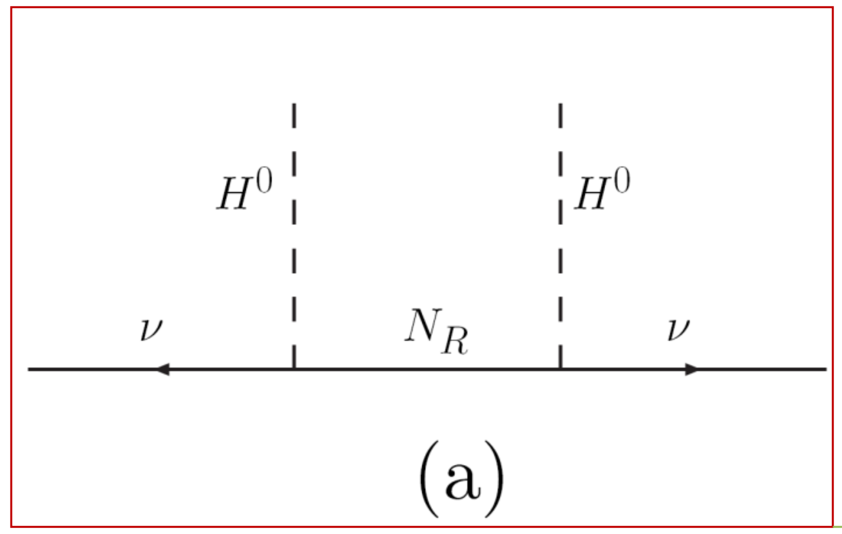






Neutrino mass generations

History： Majorana neutrino mass from the dim-5 Weinberg operator $\kappa_{\alpha\beta}\overline{\ell_L^{\alpha}}\tilde{H}\tilde{H}^T\ell_L^{\beta C}$

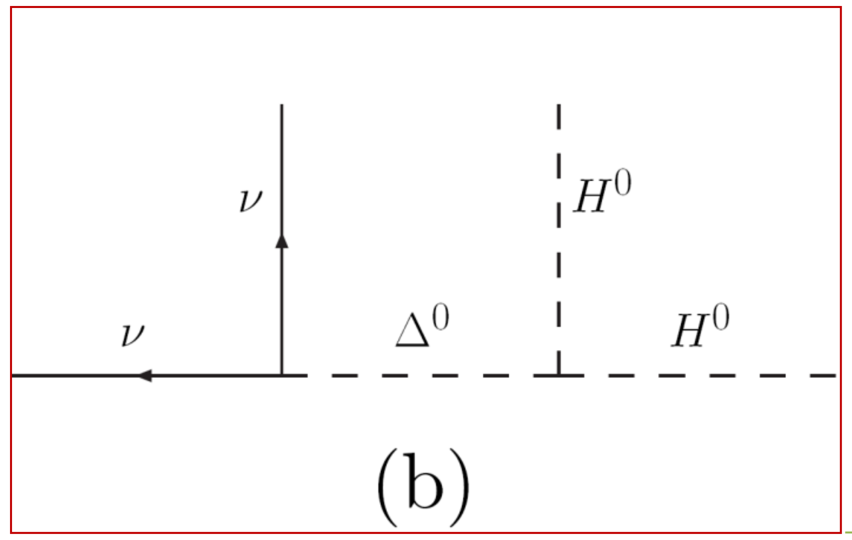


$$-\mathcal{L} = \overline{\ell_L}Y_\nu N_R\tilde{H} + \frac{1}{2}\overline{N_R^C}M_R N_R + \text{h.c.}$$

SU(2)_L fermion singlet

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Minkowski（77）

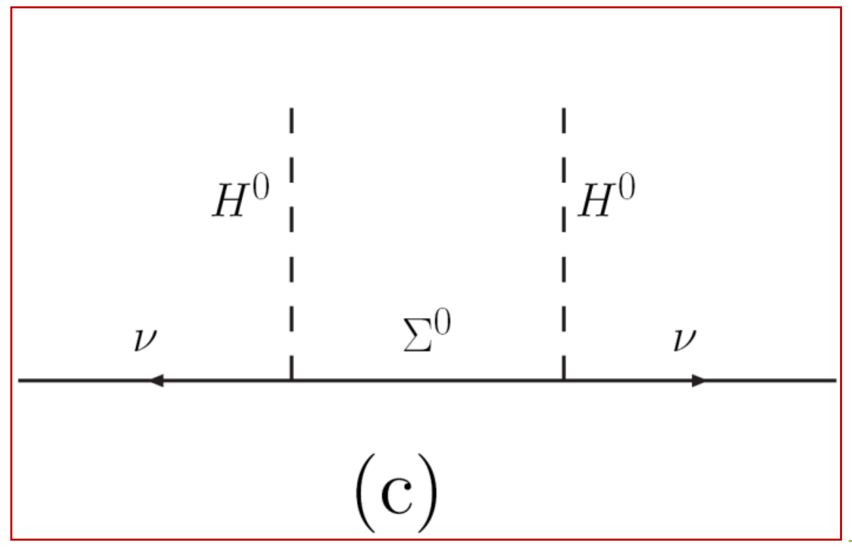


$$-\mathcal{L} = \frac{1}{2}\overline{\ell_L}Y_\Delta\Delta\varepsilon\ell_L^C - \lambda_\Delta M_\Delta H^T\varepsilon\Delta H + \text{h.c.}$$

SU(2)_L scalar triplet

$$M_\nu = Y_\Delta v_\Delta$$

Magg， Wetterich（80）

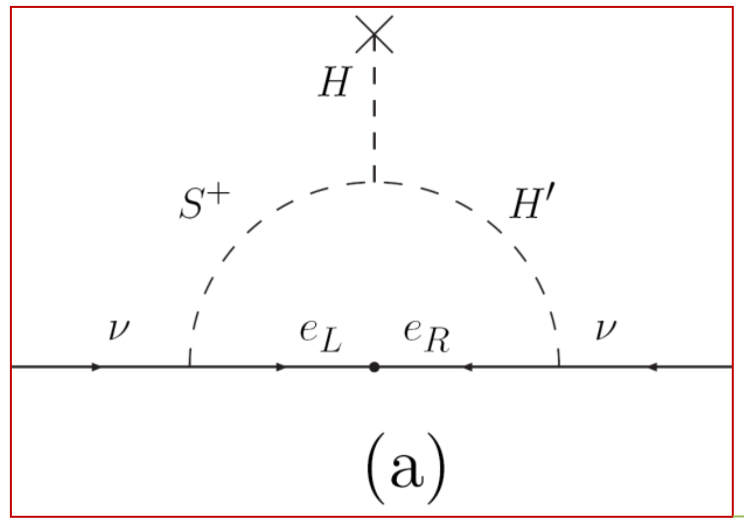


$$-\mathcal{L} = \frac{1}{2}\text{Tr}\left[\Sigma M_\Sigma \Sigma^C\right] + \sqrt{2}\overline{\ell_L}Y_\Sigma\Sigma\tilde{H} + \text{h.c.}$$

SU(2)_L fermion triplet

$$M_\nu = -M_D M_\Sigma^{-1} M_D^T$$

Foot，Lew， He，Joshi（89）

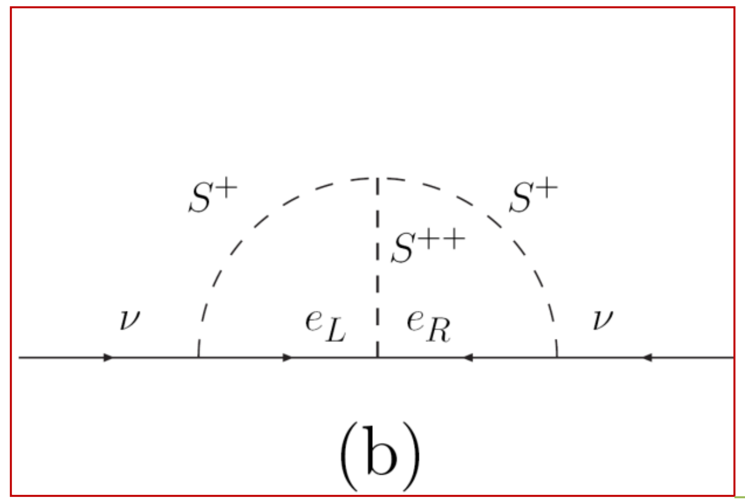


$$-\mathcal{L} = \ell_L^T Y_S\varepsilon\ell_L S^+ - \mu H^T\varepsilon H' S^- + \text{h.c.}$$

Zee（80）

$$(M_\nu)_{\alpha\beta} = A\,(Y_S)_{\alpha\beta}\,(m_\alpha^2 - m_\beta^2)$$

Charged scalar singlet+ doublets

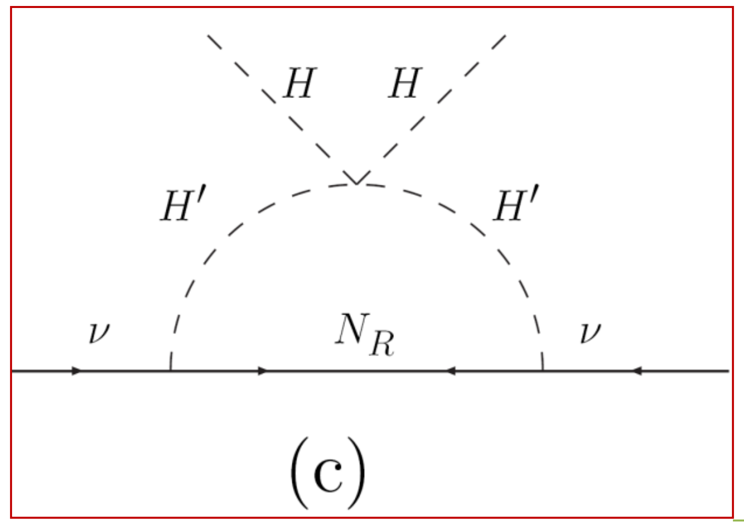


$$-\mathcal{L} = \ell_L^T Y_S\varepsilon\ell_L S^+ + e_R^T F_S e_R S^{++} + \mu S^- S^- S^{++} + \text{h.c.}$$

Babu（88）

$$(M_\nu)_{\alpha\beta} = 8\mu\sum_{\kappa\lambda} m_\kappa m_\lambda\,(Y_S)_{\alpha\kappa}\,(F_S)_{\kappa\lambda}\,(Y_S)_{\lambda\beta}\,I_{\kappa\lambda}$$

Two charged scalar singlets



$$-\mathcal{L} = \overline{\ell_L}Y_\nu N_R\tilde{H}' + \frac{1}{2}\overline{N_R^C}M_R N_R + \frac{1}{2}\lambda_5(H^\dagger H')^2 + \text{h.c.}$$

Ma （98）

$$(M_\nu)_{\alpha\beta} = \sum_i (Y_\nu)_{\alpha i}M_i^{-1}(Y_\nu)_{\beta i}I(M_i^2/m_0^2)$$

Higgs doublets + Majorana neutrino+ Z2 symmetry

● ● ● Y. Cai, T. Han, T. Li, 1711.02180

Majoron & neutrino mass via type-I seesaw

Type-I seesaw + spontaneous breaking $U(1)_L$ symmetry

$$\mathscr{L}_{\rm BSM} = \left(\partial_\mu \Phi\right)^\dagger (\partial^\mu \Phi) + \mu_\Phi^2 \Phi^\dagger \Phi - \lambda_1 (\Phi^\dagger \Phi)^2 - \lambda_2 (\Phi^\dagger \Phi)(H^\dagger H) - \left[Y_{\rm N} \overline{\ell_L} \tilde{H} N_R + \frac{1}{2} \overline{N_R^C} \left(Y_{\rm M} \Phi + m \right) N_R + {\rm h.c.} \right]$$

LNV term!

$$H = \begin{pmatrix} \phi^+ \\ \dfrac{v_\phi + \phi + i\chi}{\sqrt{2}} \end{pmatrix} \qquad \Phi = \frac{v_s + \tilde{s} + i\tilde{a}}{\sqrt{2}} \qquad \tilde{a}: \quad \textbf{Majoron}$$

Yukawa Interaction $\quad -Y_{\rm N} \overline{\ell_L} \tilde{H} N_R \to M_D = Y_N v / \sqrt{2}$

Key term: $\quad m \overline{N_R^C} N_R + h.c.$ **Quantum Gravity effect!**

Majoron interactions and Majoron mass

Field-dependent phase transformation

$$\ell_L \to e^{-\frac{ia}{2f}}\ell_L \qquad S \to e^{+\frac{ia}{f}}S$$

$$E_R \to e^{-\frac{ia}{2f}}E_R \qquad H \to H$$

$$\mathscr{L} \to \mathscr{L} - \frac{a}{2f}\partial_\mu\left(\overline{\ell_L}\gamma^\mu\ell_L + \overline{E_R}\gamma^\mu E_R\right)$$

$$= \mathscr{L} - \frac{a}{2f}\partial_\mu J^L_\mu$$

$$= \mathscr{L} + \frac{a}{2f}\frac{N_f}{32\pi^2}\left(g^2 W^a_{\mu\nu}\widetilde{W}^{\mu\nu,a} - g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right)$$

$$N_R \to e^{-\frac{ia}{2f}}N_R \longrightarrow \frac{1}{2}e^{-i\theta}\overline{N^C_R}mN_R + h.c.$$

Majoron interactions and Majoron mass

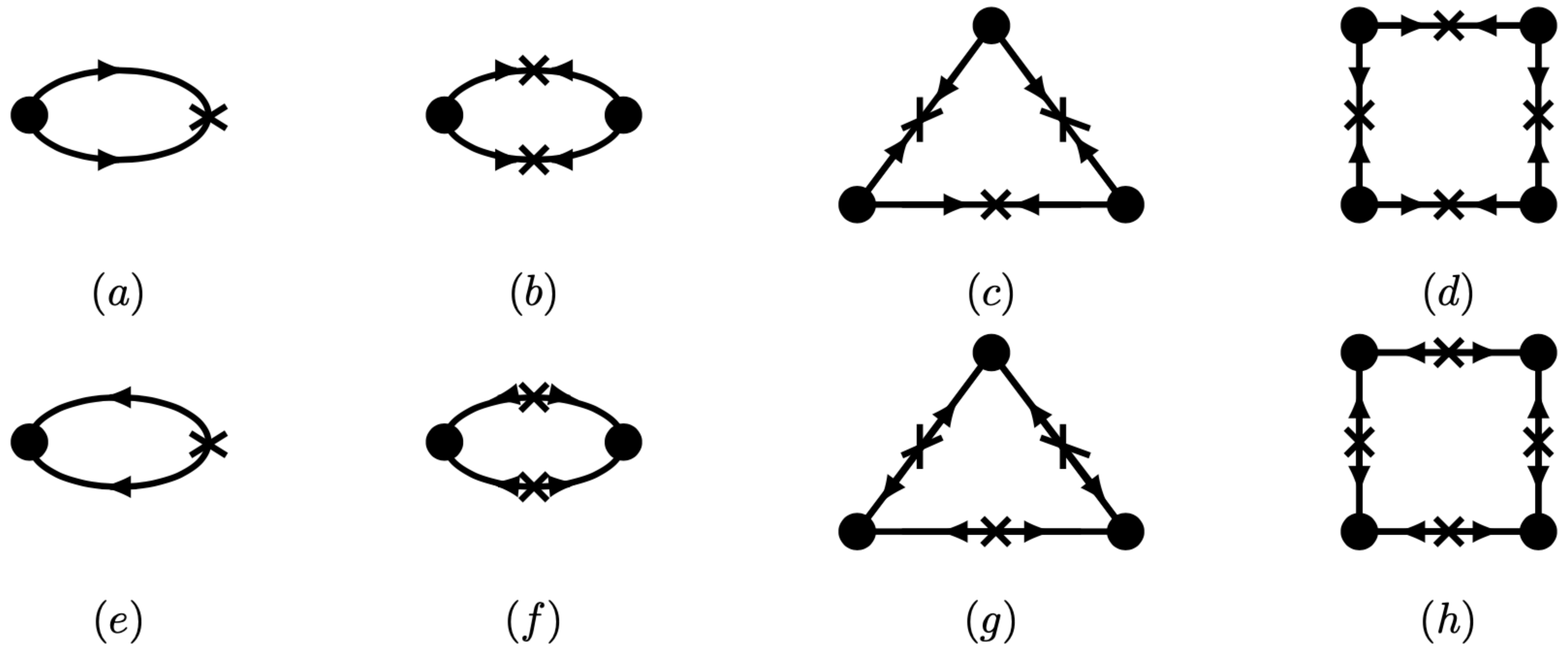
$$\frac{1}{2}e^{-i\theta}\overline{N_R^C}mN_R + h.c. \longrightarrow$$

Mass insertion of right-handed neutrino masses:

Before symmetry breaking: $M = m$

After symmetry breaking: $M = f_a Y_M/\sqrt{2} + m$

$$V_a \sim -\frac{1}{16\pi^2}\sum_{n=1}^{4} a_n \cos n\theta \, .$$

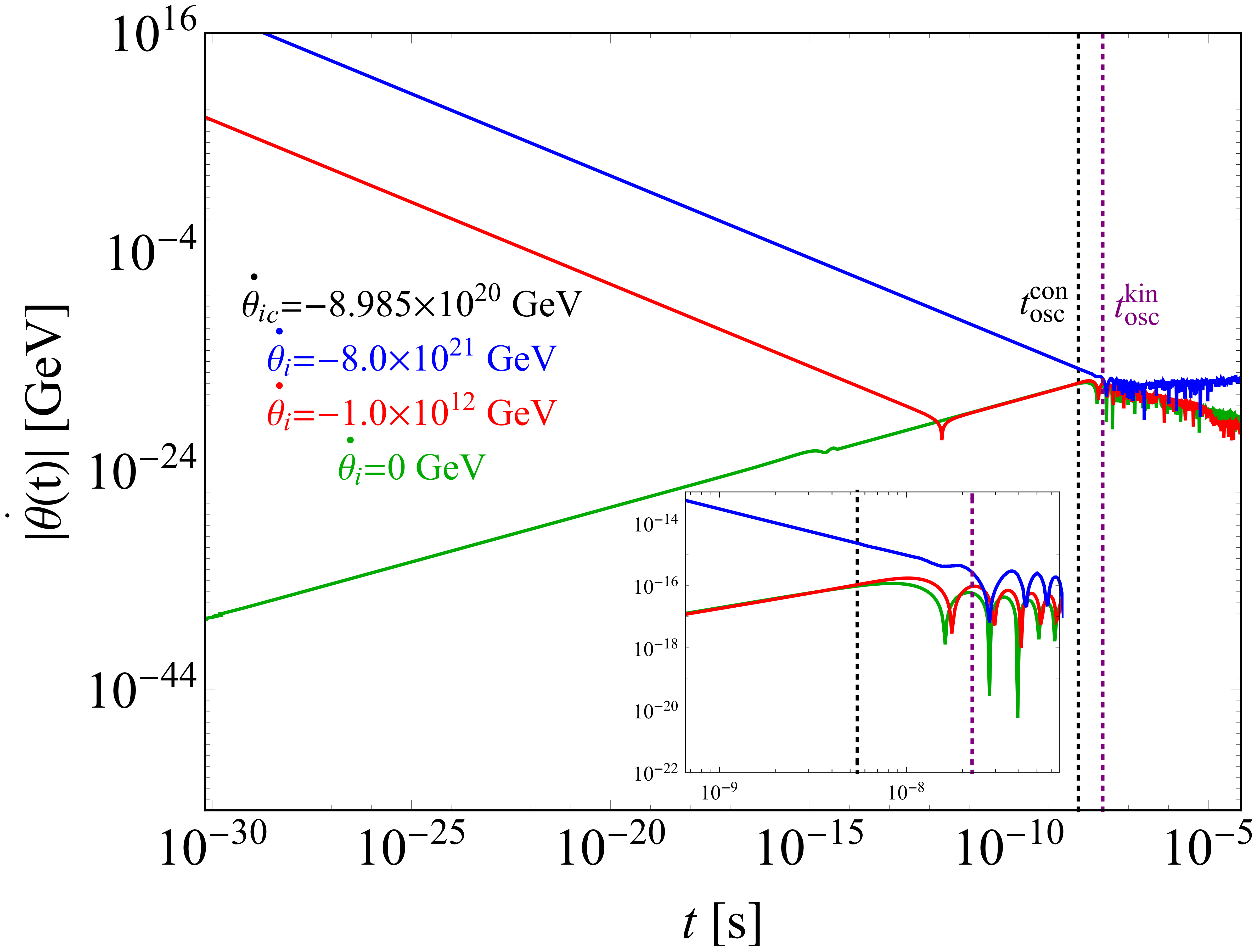


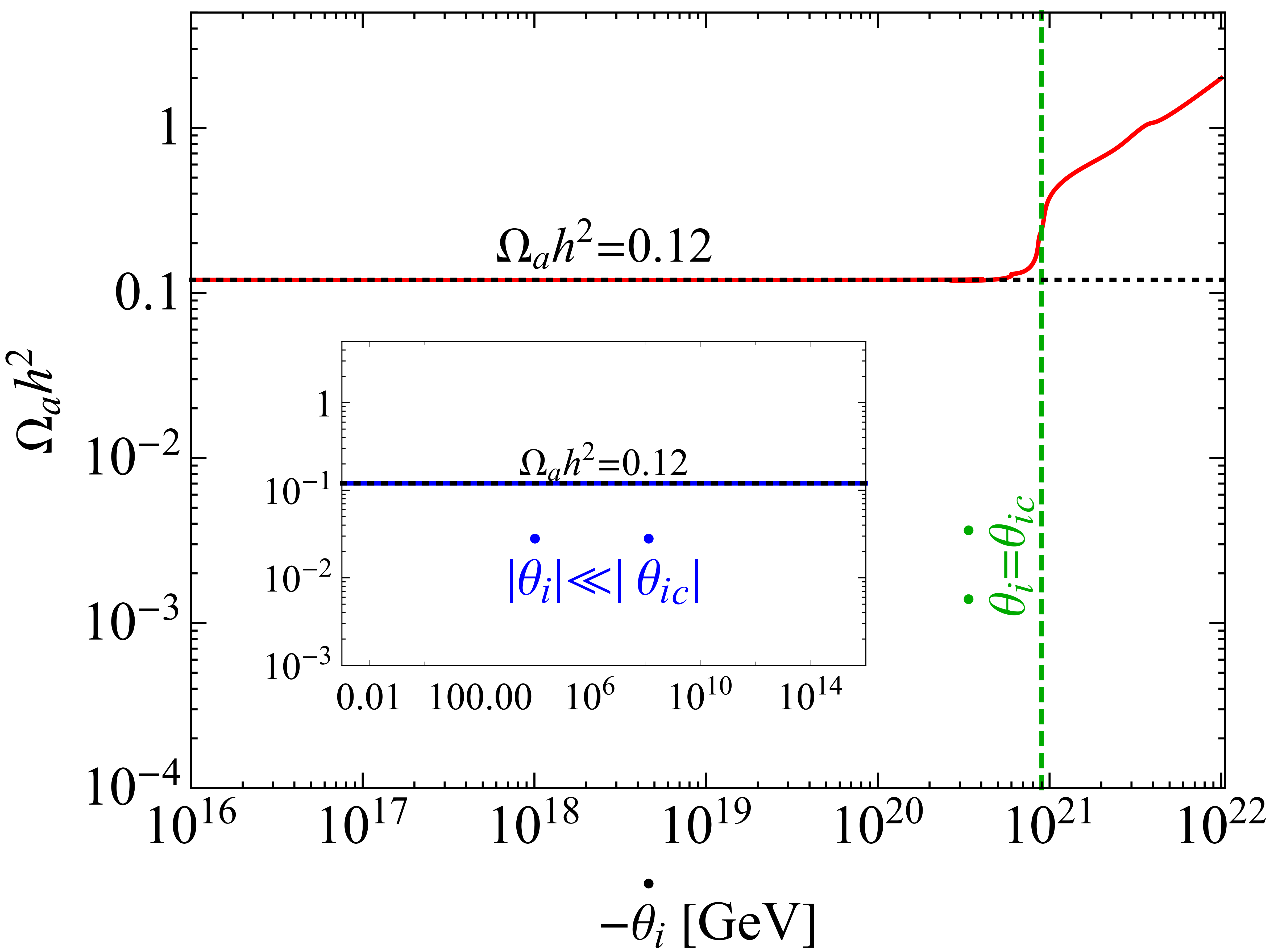
a_1	a_2	a_3	a_4
$mM^3\left(1+\log\frac{M^2}{M_{pl}^2}\right)$	$2m^2M^2\log\frac{M^2}{M_{pl}^2}$	m^3M	m^4

Majoron mass and its relic density

Majoron mass:	$m_a^2 = \frac{1}{f_a^2}\frac{d^2V}{d\theta^2} = \frac{1}{16\pi^2 f_a^2}\left\lvert a_1 + 4a_2 + 9a_3 + 16a_4 \right\rvert .$	
Initial velocity: (From Noether theorem)	$\dot\theta^2 \propto \frac{\mathrm{Tr}[Y_M^4]}{96\pi^2} f_a^2 \cos 4\theta,$	**In the traditional misalignment mechanism** $\dot\theta_i = 0$
EOM	$\ddot\theta + 3H\dot\theta + \frac{1}{f_a^2}\frac{dV_a}{d\theta} = 0,$	**Different oscillation temperature**

Majoron mass and its relic density





Baryon asymmetry of the universe

$m\overline{N_R^C}N_R + h.c.$ → **LNV carried by $\dot{\theta}$** → **Non-zero B-L**

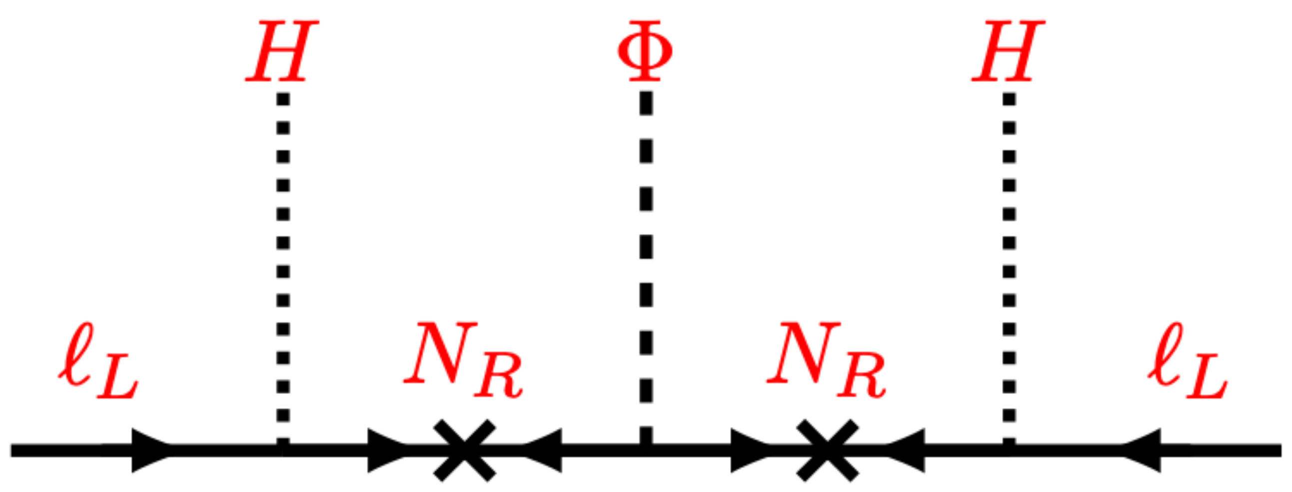
Weinberg operator

Chern-Simons operator

Non-zero B-L → **BAU**

Sehemas

Key:



$$\mathscr{L}_{\rm int} \supset \frac{1}{2M}\frac{a}{f_a}\ell\ell HH,$$

$$\mathscr{L}_{\rm int} \supset \frac{3g^2}{64\pi^2}\frac{a}{f_a}W\widetilde{W}$$

Baryon asymmetry of the universe

Transport equations:

$$-\frac{d}{d\ln T}\left(\frac{\mu_i}{T}\right) = -\frac{1}{g_i}\sum_\alpha n_i^\alpha \frac{\gamma_\alpha}{H}\left[\sum_j n_j^\alpha \left(\frac{\mu_j}{T}\right) - n_S^\alpha \frac{\dot{\theta}(T)}{T}\right],$$

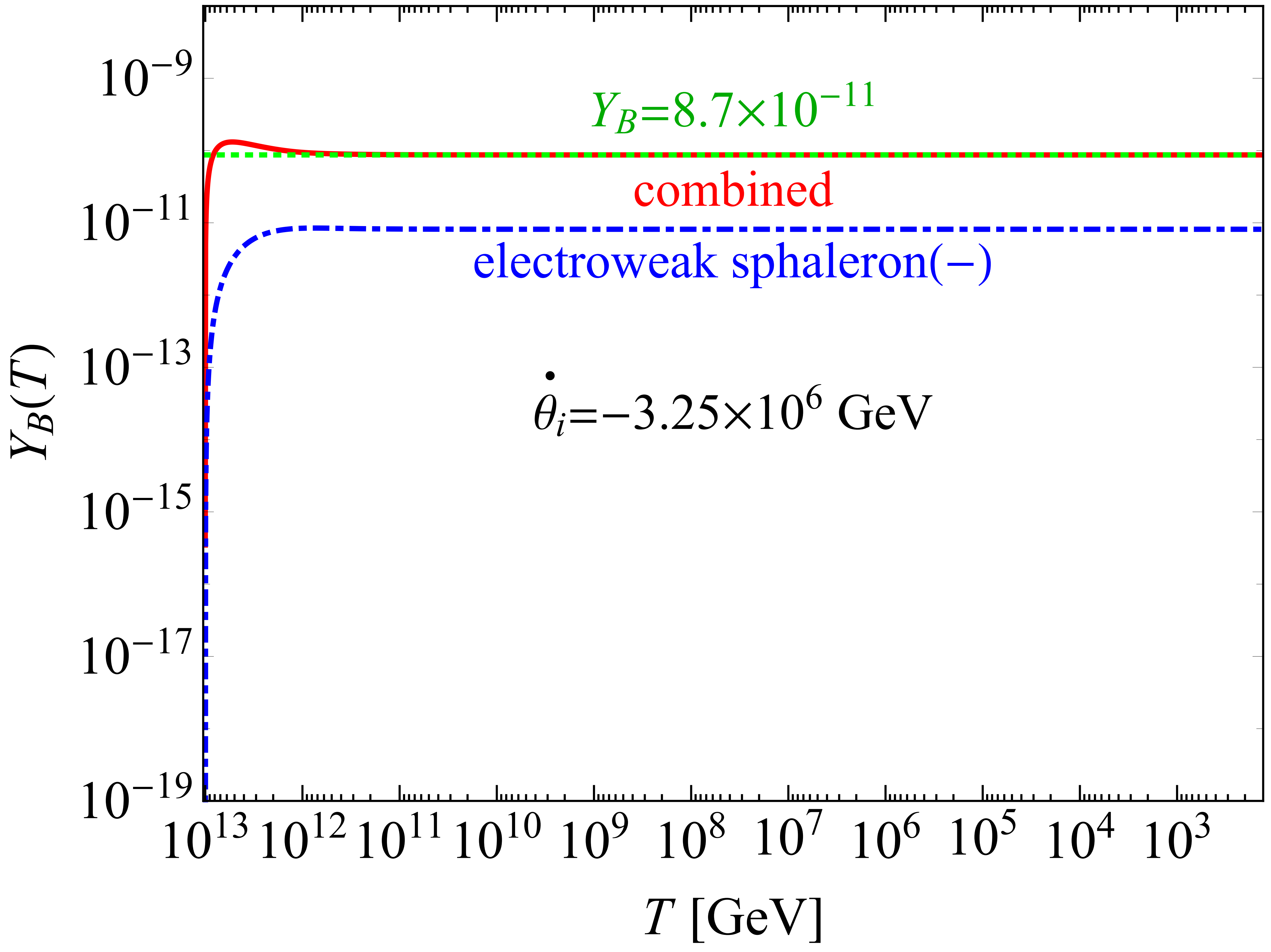
Source term:

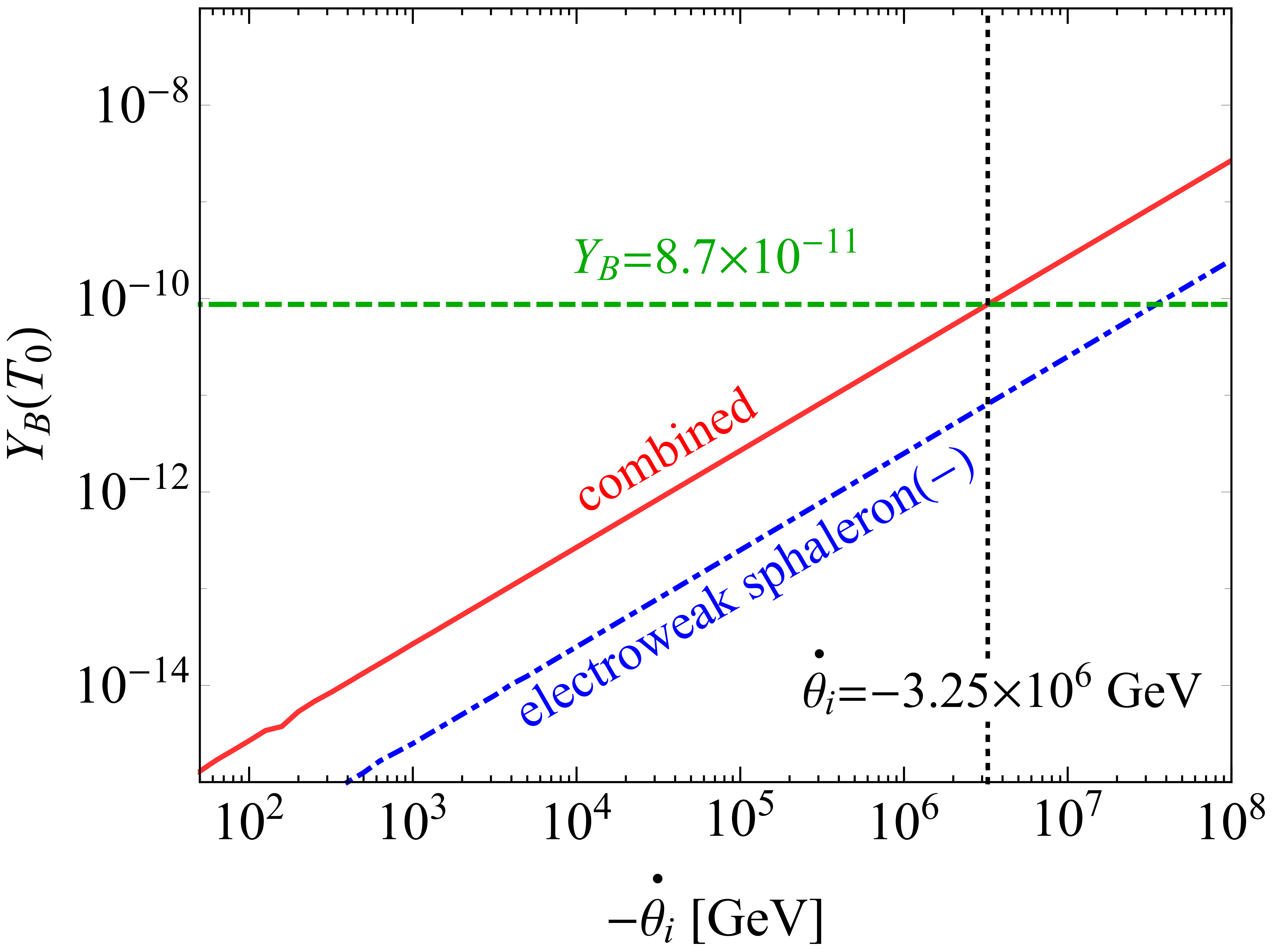
$$\left(n_S^{WS}, n_S^{W_{12}}, n_S^{W_3}, n_S^{SS}, n_S^{Y_\tau}, n_S^{Y_t}, n_S^{Y_b}\right) = \left(\frac{3}{2},1,1,0,0,0,0\right).$$

Weinberg operator decoupling temperature:

$$T_{\rm W} \simeq 6\times 10^{12}\,{\rm GeV}\times\left(\frac{0.05\,{\rm eV}}{m_\nu}\right)^2.$$

Baryon asymmetry of the universe





Majoron & neutrino mass via type-II seesaw

Type-II seesaw + spontaneous breaking $U(1)_L$ symmetry

LNV term!

$$V(S,\Phi,\Delta) = V(\Phi,\Delta) - \mu_S^2(S^\dagger S) + \lambda_6(S^\dagger S)^2$$
$$+\lambda_7(S^\dagger S)(\Phi^\dagger\Phi) + \lambda_8(S^\dagger S)\mathrm{Tr}(\Delta^\dagger\Delta) + \mu\Phi^T i\tau_2\Delta^\dagger\Phi + \lambda S\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.},$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v_\phi + \phi + i\chi}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_\Delta + \delta + i\xi}{\sqrt{2}} & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \qquad S = \frac{v_s + \tilde{s} + i\tilde{a}}{\sqrt{2}}$$

$\tilde{a}$: **Majoron**

Yukawa Interaction $\quad -\mathscr{L}_\Delta = Y_{\alpha\beta}\overline{\ell_L^{\alpha C}} i\sigma^2\Delta\ell_L^\beta + h.c.$

Key term: $\quad \mu\Phi^T i\sigma^2\Delta\Phi + h.c.$

Majoron & neutrino mass via type-II seesaw

Gauge boson masses

$$m_W^2 = \frac{g^2}{4}\left(v_\phi^2 + 2v_\Delta^2\right)\,, \quad m_Z^2 = \frac{g^2}{4\cos^2\theta_W}\left(v_\phi^2 + 4v_\Delta^2\right)\,. \qquad \rho \equiv \frac{m_W^2}{m_Z^2\cos^2\theta_W} = \frac{1+\frac{2v_\Delta^2}{v_\phi^2}}{1+\frac{4v_\Delta^2}{v_\phi^2}}\,.$$

Scalar mixings and masses

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \mathscr{R}(\beta)\begin{pmatrix} \phi^\pm \\ \Delta^\pm \end{pmatrix}\,, \quad \begin{pmatrix} G \\ A \\ a \end{pmatrix} = \mathscr{V}(\beta_1', \beta_2', \beta_3')\begin{pmatrix} \chi \\ \xi \\ \tilde{a} \end{pmatrix}\,, \quad \begin{pmatrix} h \\ H \\ s \end{pmatrix} = \mathscr{U}(\alpha_1, \alpha_2, \alpha_3)\begin{pmatrix} \phi \\ \delta \\ \tilde{s} \end{pmatrix}\,,$$

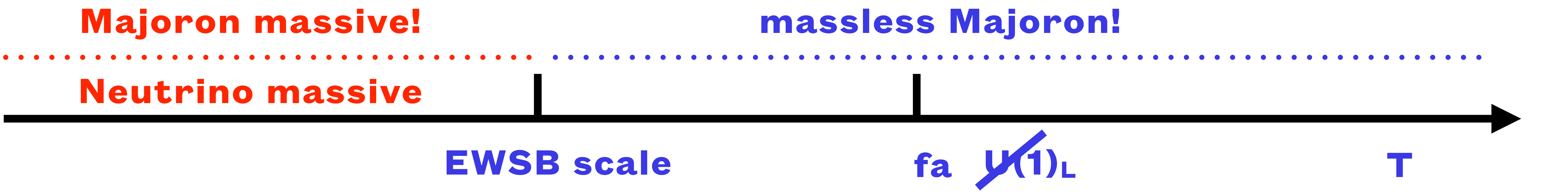
Mixing angl for pseudo-scalars

Majoron gets non-zero mass from the mixing!

$$\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}\,, \quad \tan\beta_1' = \frac{2v_\Delta}{v_\phi}\,, \quad \tan\beta_2' = 0\,, \quad \tan 2\beta_3' = \frac{2\lambda v_\Delta v_s v_\phi\sqrt{v_\phi^2 + 4v_\Delta^2}}{v_\phi^2\left(-\lambda v_\Delta^2 + \lambda v_s^2 + \sqrt{2}\mu v_s\right) + 4v_\Delta^2 v_s\left(\sqrt{2}\mu + \lambda v_s\right)}\,.$$

Majoron & neutrino mass via type-II seesaw

Sequential breaking of various symmetries



Temperature of Universe

$$(m_\nu)_{\alpha\beta} = y_{\alpha\beta} v_\Delta / \sqrt{2} \, .$$

$$m_a^2 = \frac{\sqrt{2}\mu v_\phi^2 v_\Delta (v_\phi^2 + 4v_\Delta^2)}{2v_\phi^2(v_\Delta^2 + v_s^2) + 8v_\Delta^2 v_s^2} \simeq \frac{\mu v_\phi^2 v_\Delta}{\sqrt{2} v_s^2} \, ,$$

For experts of axion physics

Majoron mass should arise from cosine like potential!

$$\ell_L \to e^{-\frac{ia}{2f}}\ell_L \qquad S \to e^{+\frac{ia}{f}}S$$

$$E_R \to e^{-\frac{ia}{2f}}E_R \qquad \Delta \to e^{-\frac{ia}{f}}\Delta$$

$$H \to H$$

$$-\mathscr{L}_{\rm int} \supset \mu e^{i\frac{a}{f_a}}\Phi^T i\tau_2 \Delta^\dagger \Phi + {\rm h.c.}.$$

After electroweak symmetry breaking

$$-\mathscr{L}_{\rm int} \supset \frac{\mu v_\Phi^2 v_\Delta}{\sqrt{2}} \cos\left(\frac{a}{f_a}\right)$$

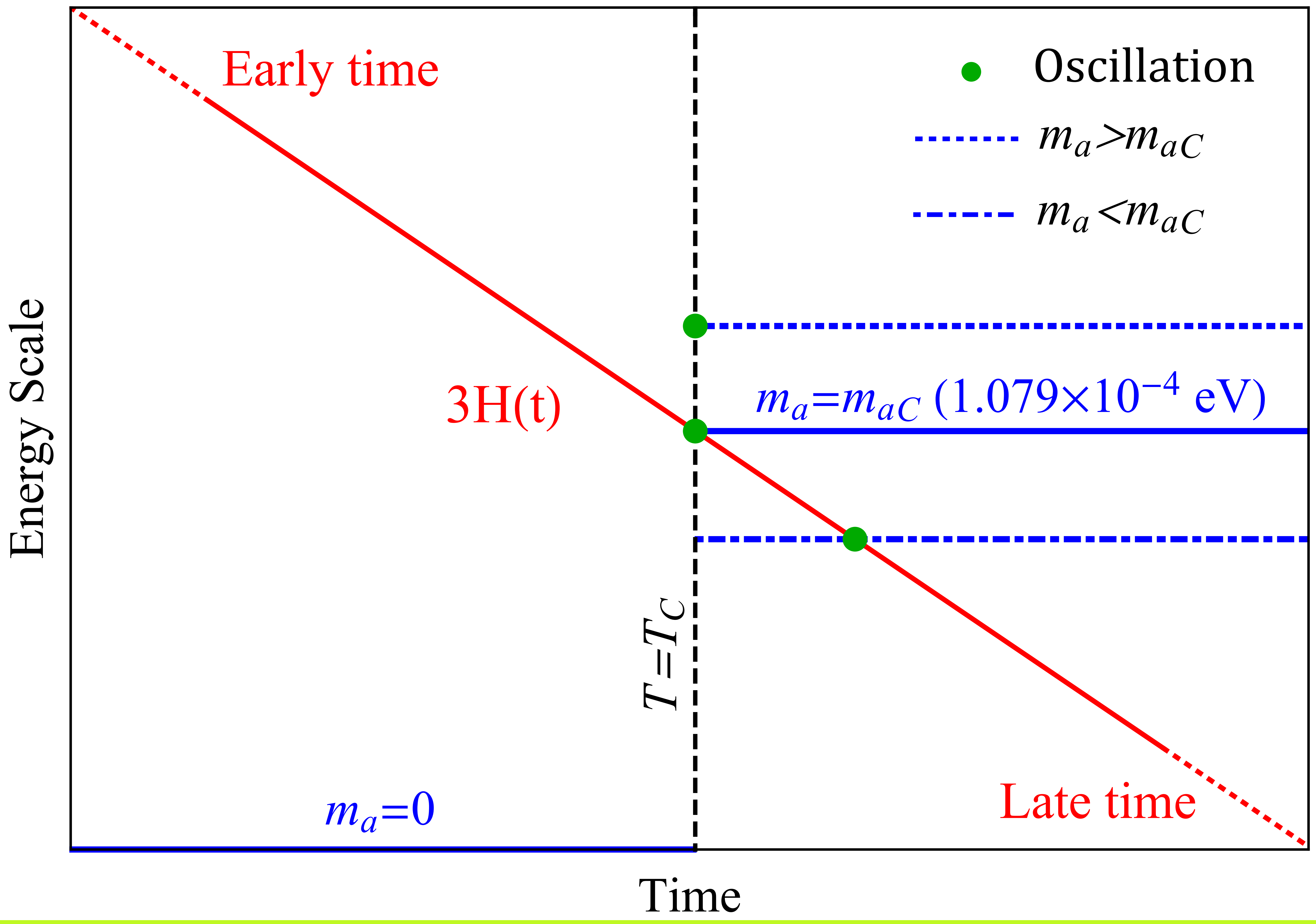
Non-zero Majoron mass

Majoron DM—oscillation time

$$m_a^2(T) = \begin{cases} \dfrac{\mu v_\phi^2(T) v_\Delta(T)}{\sqrt{2} f_a^2}\,, & T \le T_C \\ 0\,, & T > T_C \end{cases}$$

$$T_{\rm osc} = \begin{cases} T_*\,, & m_a < m_{aC} \\ T_C\,, & m_a \ge m_{aC} \end{cases}$$

$$m_{aC} = 1.079 \times 10^{-4}\,\text{eV}$$



Majoron DM—simulations

Equation of motion

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$

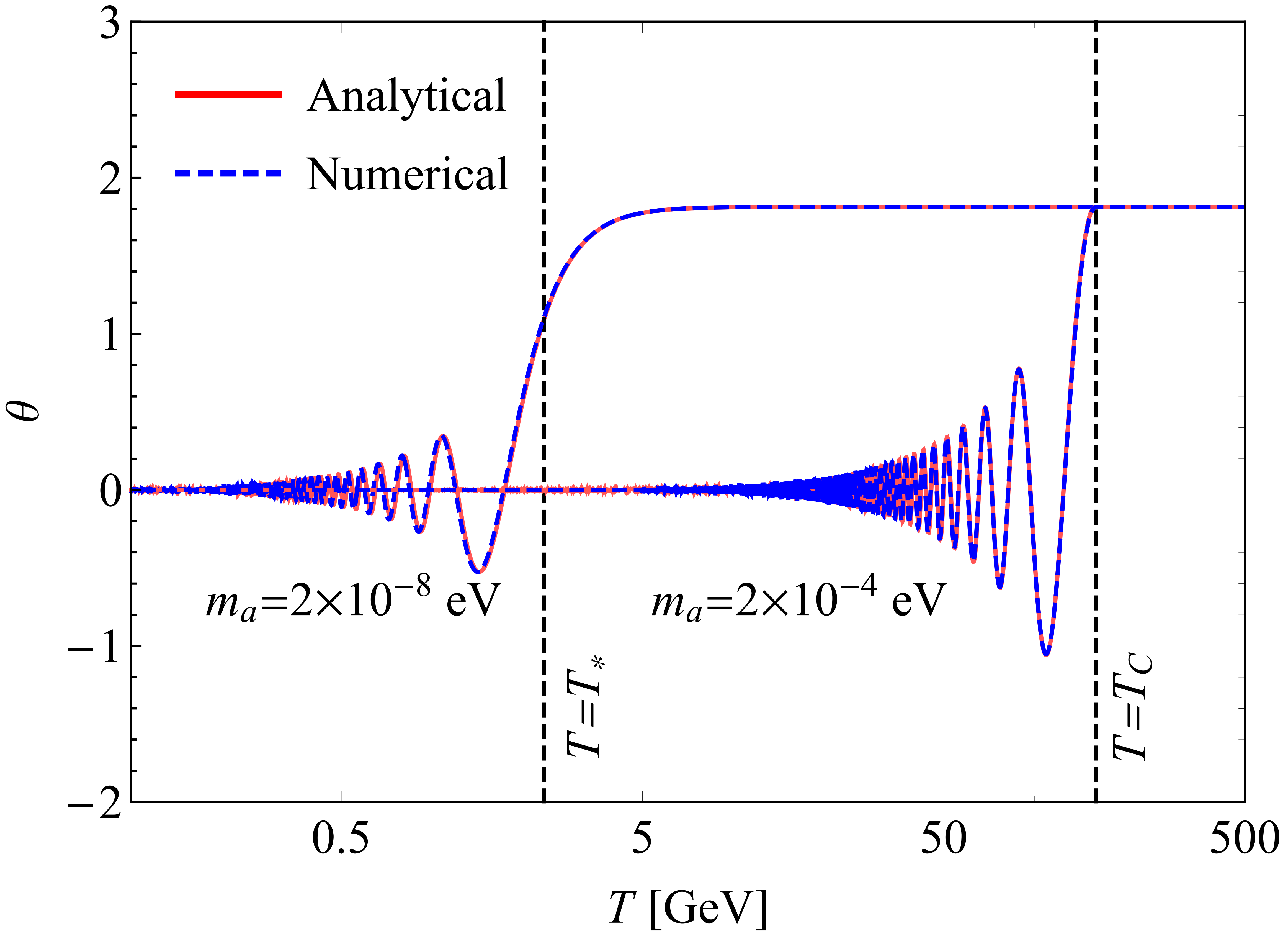
Analytical results

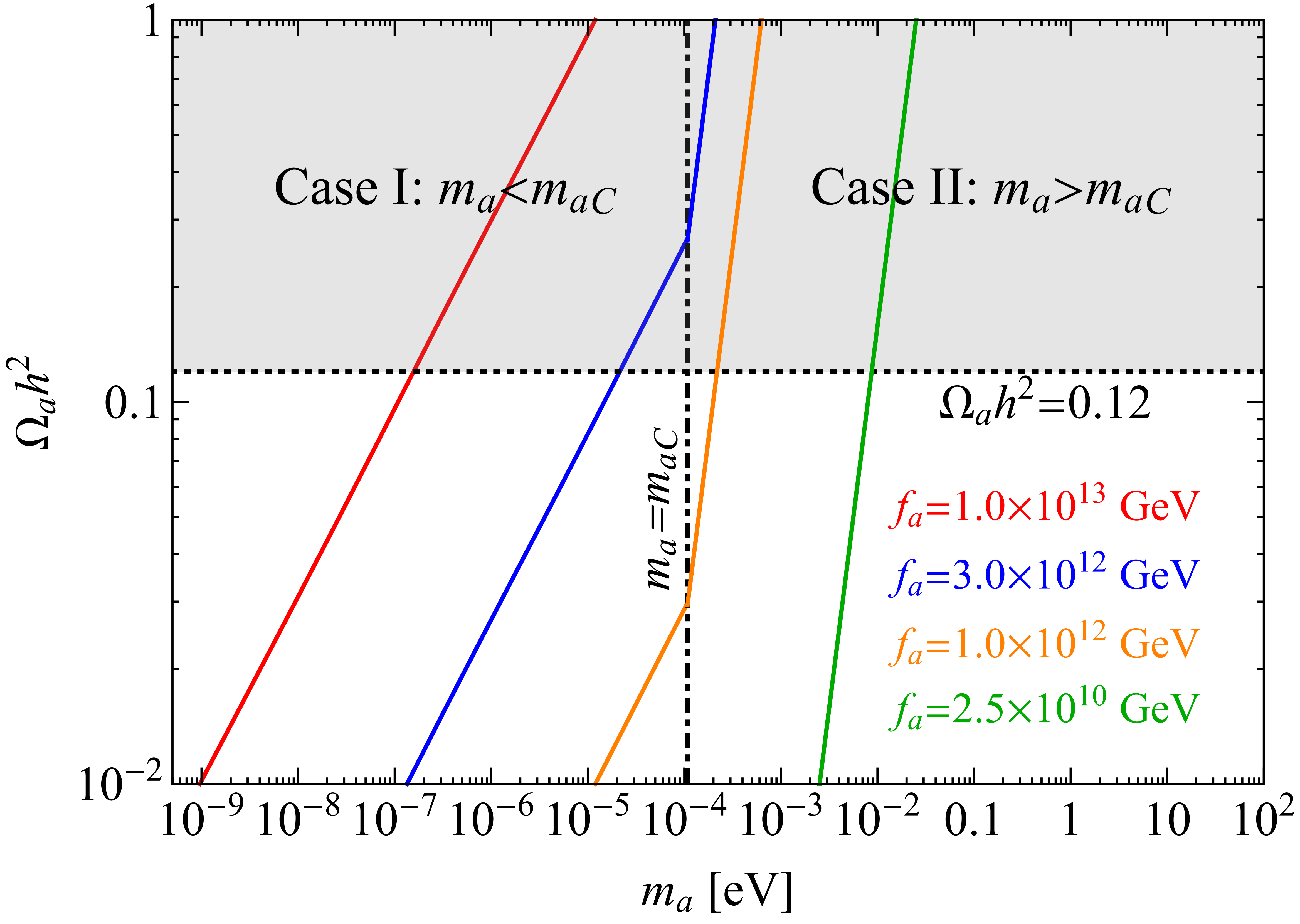
$$\theta(t) = -\pi\Big[-2m_a t_i J_{\frac{1}{4}}(m_a t) Y_{-\frac{3}{4}}(m_a t_i) + 2m_a t_i Y_{\frac{1}{4}}(m_a t) J_{-\frac{3}{4}}(m_a t_i)$$
$$-Y_{\frac{1}{4}}(m_a t) J_{\frac{1}{4}}(m_a t_i) - 2m_a t_i Y_{\frac{1}{4}}(m_a t)_{\frac{5}{4}}(m_a t_i) + J_{\frac{1}{4}}(m_a t) Y_{\frac{1}{4}}(m_a t_i)$$
$$+2m_a t_i J_{\frac{1}{4}}(m_a t) Y_{\frac{5}{4}}(m_a t_i)\Big] \Big/ \Big\{ 2\sqrt{3}\, t^{\frac{1}{4}} t_i^{\frac{3}{4}} \Big[J_{\frac{1}{4}}(m_a t_i) Y_{-\frac{3}{4}}(m_a t_i) - J_{-\frac{3}{4}}(m_a t_i) Y_{\frac{1}{4}}(m_a t_i)$$
$$+J_{\frac{5}{4}}(m_a t_i) Y_{\frac{1}{4}}(m_a t_i) - J_{\frac{1}{4}}(m_a t_i) Y_{\frac{5}{4}}(m_a t_i)\Big]$$

Majoron energy density

$$\rho_a(T_0) = \frac{1}{2} m_a^2 f_a^2 \langle \theta_{a,i}^2 \rangle \frac{g_{*s}(T_0)}{g_{*s}(T_{\rm osc})} \left(\frac{T_0}{T_{\rm osc}}\right)^3$$

Majoron DM—Relic Density





Majoron interactions from mixings

Interactions with scalars

Vertices	Coefficients
a^4	$\frac{1}{4}\lambda_1 V_{13}^4 + \frac{1}{4}\lambda_4 V_{13}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{23}^2 + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{33} + \frac{1}{4}\lambda_2 V_{23}^4 + \frac{1}{4}\lambda_3 V_{23}^4 + \frac{1}{4}\lambda_6 V_{33}^4$
$a^3 G$	$\lambda_1 V_{11} V_{13}^3 + \frac{1}{2}\lambda_4 V_{11} V_{13} V_{23}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{23}^2 + \lambda V_{11} V_{13} V_{23} V_{33} + \frac{1}{2}\lambda_4 V_{13}^2 V_{21} V_{23}$ $+\frac{1}{2}\lambda_5 V_{13}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{31} + \lambda_2 V_{21} V_{23}^3 + \lambda_3 V_{21} V_{23}^3 + \lambda_6 V_{31} V_{33}^3$
$a^2 h^2$	$\frac{1}{2}\lambda_1 U_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 U_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 U_{11}^2 V_{23}^2 - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{33} + \lambda U_{11} U_{21} V_{13} V_{33}$ $-\lambda U_{11} U_{31} V_{13} V_{23} + \frac{1}{4}\lambda_4 U_{21}^2 V_{13}^2 + \frac{1}{4}\lambda_5 U_{21}^2 V_{13}^2 + \frac{1}{2}\lambda_2 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda_3 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda U_{21} U_{31} V_{13}^2 + \frac{1}{2}\lambda_6 U_{31}^2 V_{33}^2$
$a^2 h$	$\lambda_1 U_{11} v_\phi V_{13}^2 + \frac{1}{2}\lambda_4 U_{11} v_\phi V_{23}^2 + \frac{1}{2}\lambda_5 U_{11} v_\phi V_{23}^2 - \lambda U_{11} v_\phi V_{23} V_{33} - \sqrt{2}\mu U_{11} V_{13} V_{23}$ $-\lambda U_{11} V_{13} V_{23} v_s + \lambda U_{11} V_{13} V_{33} v_\Delta + \lambda U_{21} v_\phi V_{13} V_{33} + \frac{1}{\sqrt{2}}\mu U_{21} V_{13}^2 + \frac{1}{2}\lambda_4 U_{21} V_{13}^2 v_\Delta + \frac{1}{2}\lambda_5 U_{21} V_{13}^2 v_\Delta$ $+\frac{1}{2}\lambda U_{21} V_{13}^2 v_s + \lambda_2 U_{21} V_{23}^2 v_\Delta + \lambda_3 U_{21} V_{23}^2 v_\Delta - \lambda U_{31} v_\phi V_{13} V_{23} + \frac{1}{2}\lambda U_{31} V_{13}^2 v_\Delta + \lambda_6 U_{31} V_{33}^2 v_s$
$a^2 G^2$	$\frac{3}{2}\lambda_1 V_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 V_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{11}^2 V_{23}^2 + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{33} + \lambda_4 V_{11} V_{13} V_{21} V_{23} + \lambda_5 V_{11} V_{13} V_{21} V_{23}$ $+\lambda V_{11} V_{13} V_{21} V_{33} + \lambda V_{11} V_{13} V_{23} V_{31} + \frac{1}{4}\lambda_4 V_{13}^2 V_{21}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{21}^2 + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{31} + \frac{3}{2}\lambda_2 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_3 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_6 V_{31}^2 V_{33}^2$
$a^2 G^+ G^-$	$\lambda_1 V_{13}^2 \cos^2\beta + \frac{1}{2}\lambda_4 V_{13}^2 \sin^2\beta + \frac{1}{4}\lambda_5 V_{13}^2 \sin^2\beta + \frac{1}{\sqrt{2}}\lambda_5 V_{13} V_{23} \sin\beta\cos\beta$ $+\sqrt{2}\lambda V_{13} V_{33} \sin\beta\cos\beta + \lambda_2 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_3 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_4 V_{23}^2 \cos^2\beta$
$a G^3$	$\lambda_1 V_{11}^3 V_{13} + \frac{1}{2}\lambda_4 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{11}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{31}$ $+\frac{1}{2}\lambda_4 V_{11} V_{13} V_{21}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{21}^2 + \lambda V_{11} V_{13} V_{21} V_{31} + \lambda_2 V_{21}^3 V_{23} + \lambda_3 V_{21}^3 V_{23} + \lambda_6 V_{31}^3 V_{33}$
$a h^2 G$	$\lambda_1 U_{11}^2 V_{11} V_{13} + \frac{1}{2}\lambda_4 U_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 U_{11}^2 V_{21} V_{23} - \frac{1}{2}\lambda U_{11}^2 V_{21} V_{33} - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{31}$ $+\lambda U_{11} U_{21} V_{11} V_{33} + \lambda U_{11} U_{21} V_{13} V_{31} - \lambda U_{11} U_{31} V_{11} V_{23} - \lambda U_{11} U_{31} V_{13} V_{21} + \frac{1}{2}\lambda_4 U_{21}^2 V_{11} V_{13}$ $+\frac{1}{2}\lambda_5 U_{21}^2 V_{11} V_{13} + \lambda_2 U_{21}^2 V_{21} V_{23} + \lambda_3 U_{21}^2 V_{21} V_{23} + \lambda U_{21} U_{31} V_{11} V_{13} + \lambda_6 U_{31}^2 V_{31} V_{33}$
$a\bar{\nu}\nu$	$V_{23} m_\nu / v_\Delta$

Interactions with fermions

$$\overline{\nu_L^C} i a \lambda_{a\bar{\nu}\nu} \nu_L + \mathrm{h.c.}$$

$$\rightarrow \lambda_{a\bar{\nu}\nu} : V_{23} m_\nu / v_\Delta ,$$

$$Y_E \overline{\ell_L} H E_R + \mathrm{h.c.} \rightarrow$$

$$\lambda_{aee} \bar{e} i \gamma_5 e$$

$$\rightarrow \lambda_{aee} : V_{13} \frac{m_e}{v_h} ,$$

Majoron interactions from anomaly

Schemas

$$-\mathscr{L}_{\rm int} \supset \frac{\lambda}{\sqrt{2}} f_a e^{i\frac{a}{f_a}} \Phi^T i\tau_2 \Delta^\dagger \Phi + {\rm h.c..}$$

$$\Delta \to \Delta e^{-i\frac{a}{f_a}}$$

$$-\mathscr{L}_{\rm Yukawa} = y_{\alpha\beta} \overline{\ell_L^{\alpha c}} i\tau_2 \Delta' e^{i\frac{a}{f_a}} \ell_L^\beta + \text{h.c.}$$

$$-\mathscr{L}_{\rm Yukawa} = y^E_{\alpha\beta} \overline{\ell_L^{\alpha'}} H e^{\frac{ia}{2f_A}} E_R^\beta + \text{h.c.}$$

$$\left.\begin{array}{l} \ell_L \to e^{\frac{-ia}{2f}} \ell_L \\ E_R \to e^{\frac{-ia}{2f}} E_R \end{array}\right\} \longrightarrow$$

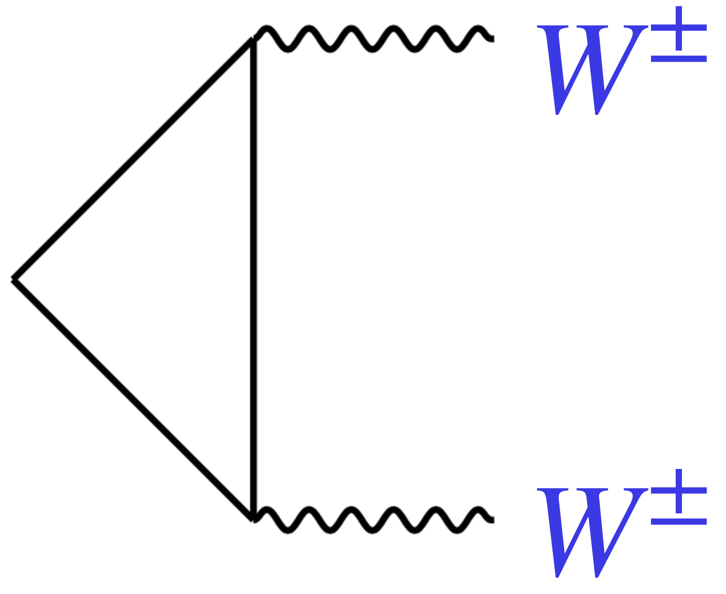
$$\mathscr{L} \to \mathscr{L} - \frac{a}{2f} \partial_\mu \left(\overline{\ell_L} \gamma^\mu \ell_L + \overline{E_R} \gamma^\mu E_R \right)$$

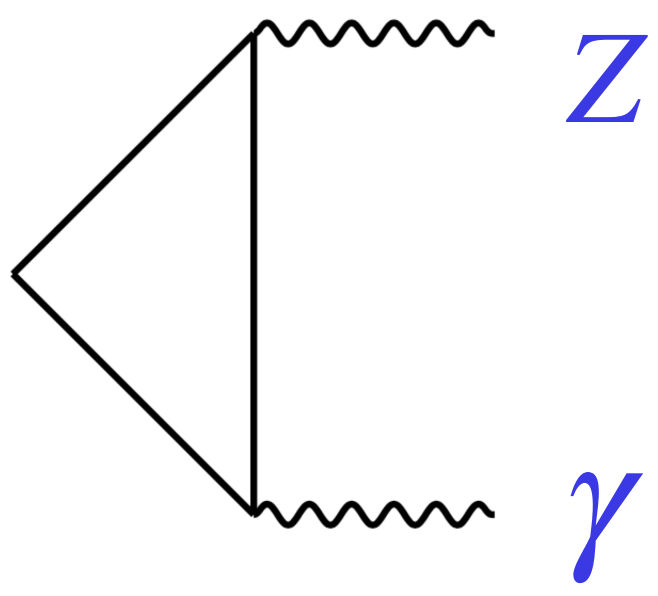
$$= \mathscr{L} - \frac{a}{2f} \partial_\mu J^L_\mu$$

$$= \mathscr{L} + \frac{a}{2f} \frac{N_f}{32\pi^2} \left(g^2 W^a_{\mu\nu} \widetilde{W}^{\mu\nu,a} - g'^2 B_{\mu\nu} \widetilde{B}^{\mu\nu} \right)$$

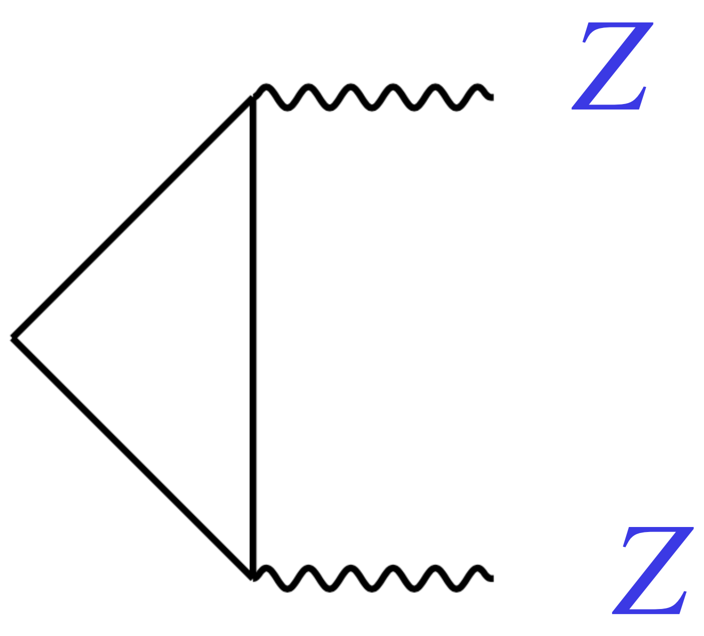
Majoron interactions from anomaly

Interactions in mass eigenstates



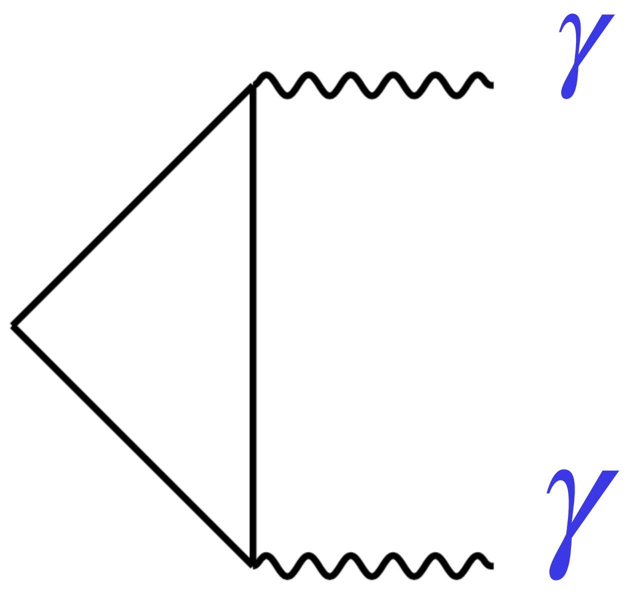


$$\alpha = \frac{g^2}{4\pi}$$



$$\frac{\alpha N_f}{16\pi f_a} a W_{\mu\nu} \widetilde{W}^{\mu\nu}$$

$$\frac{\alpha \tan\theta_W}{32\pi f_a} a \left(Z_{\mu\nu} \widetilde{F}^{\mu\nu} + F_{\mu\nu} \widetilde{Z}^{\mu\nu} \right)$$



$$\frac{\alpha}{8\pi \cos^2\theta_w f_a} \left(\frac{1}{2} - \sin^2\theta_w \right) a Z_{\mu\nu} \widetilde{Z}^{\mu\nu}$$

$$0 \times a F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

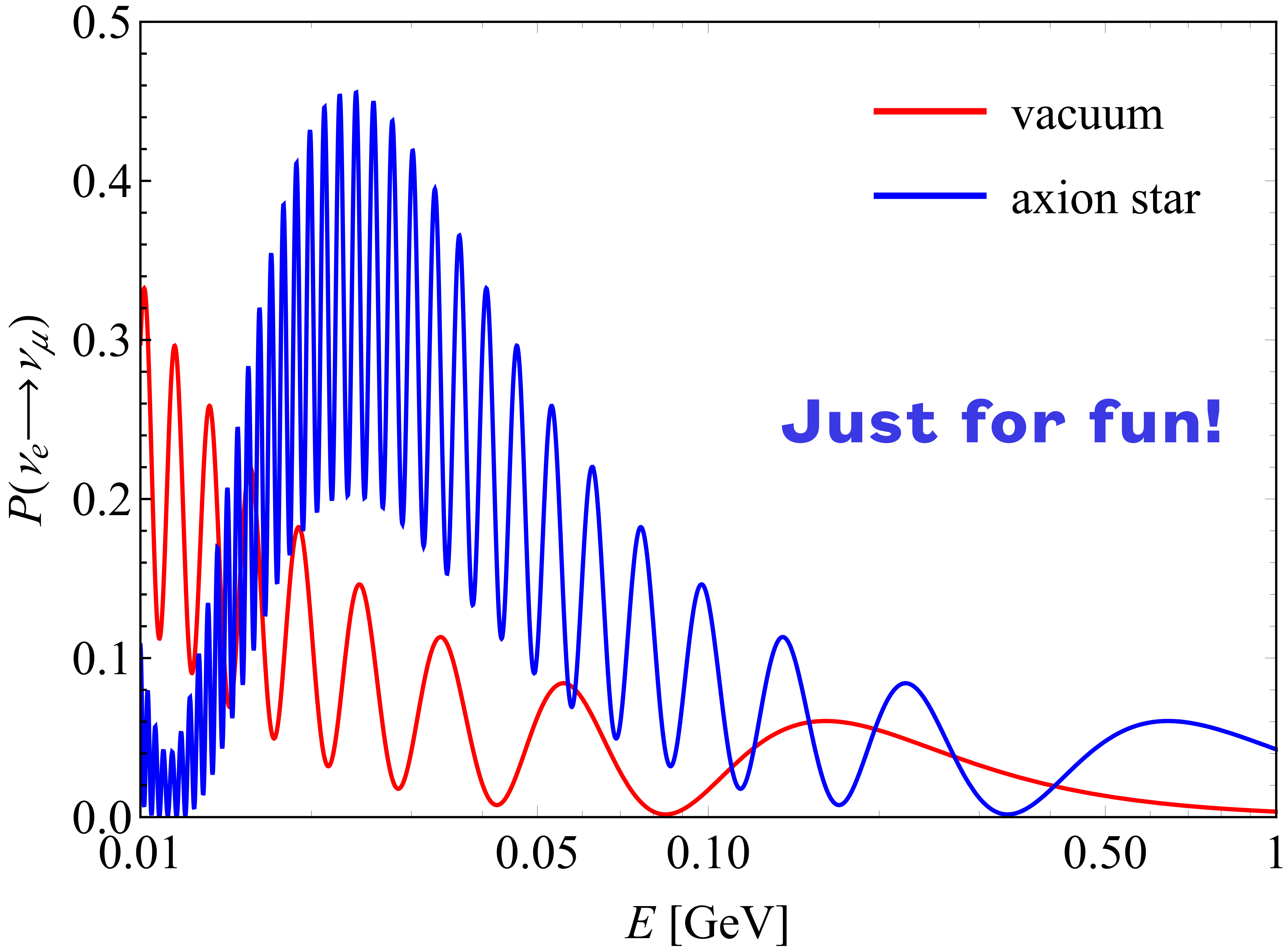
Neutrino oscillation in Majoron star

Effective potential

$$V_{\rm eff} = i\sqrt{2\rho_a} V_{23} m_a^{-1} v_\Delta^{-1} \cos(m_a t) \overline{\nu_L^C} m_\nu \nu_L + {\rm h.c.}$$

Amplitude:

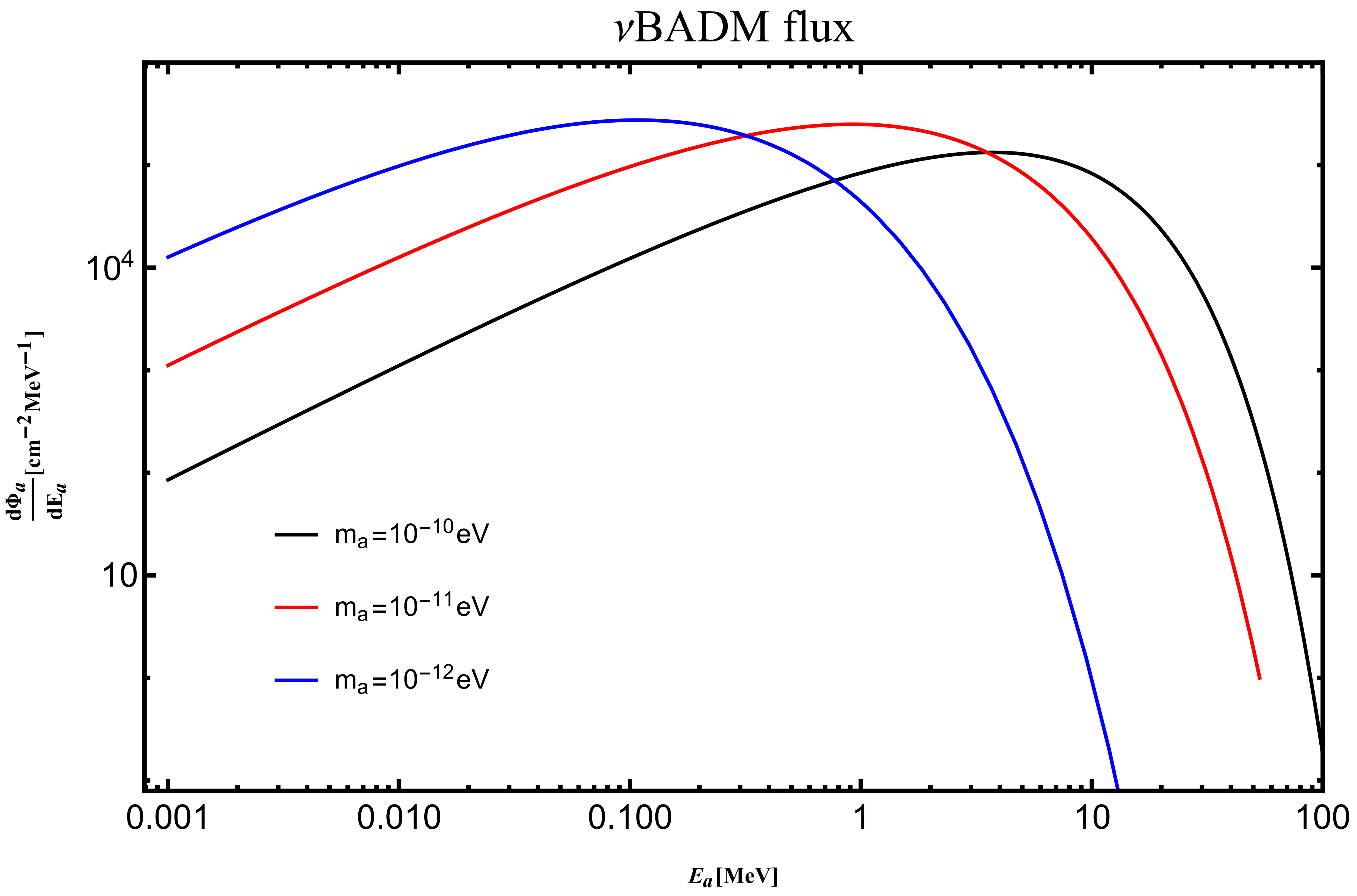
$$A_{\alpha\to\beta} = \sum_i \widehat{U}_{\beta i} \widehat{U}^*_{\alpha i} \exp\left[-i \frac{m_i^2 x}{2E} \left(1 + \frac{\rho_a V_{23}^2}{m_a^2 v_\Delta^2} + \frac{\rho_a V_{23}^2 \cos 2m_a x}{2x m_a^3 v_\Delta^2} \right) \right]$$



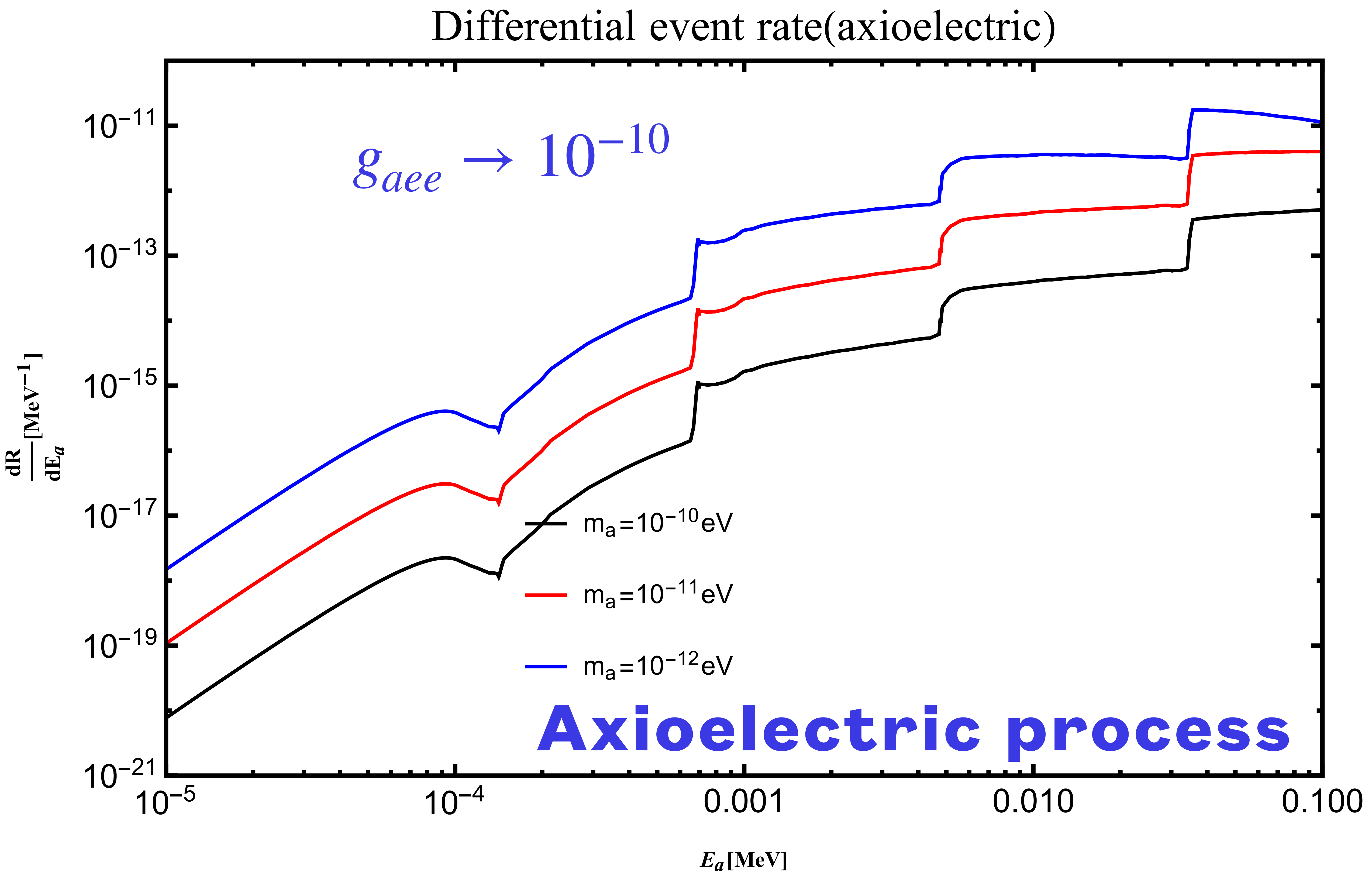
Just for fun!

Direct detections of Majoron DM

Boosted Majoron by supernova ν



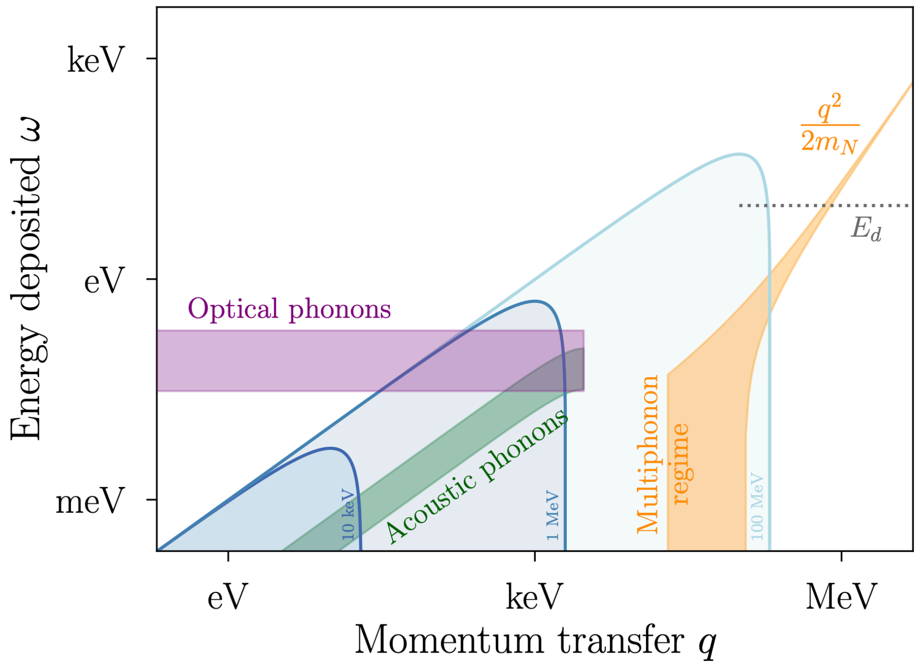
Differential event rate



Direct detections of Majoron DM

Direct detections in condensed matter systems

DM mass	DM energy or momentum	CM scale
50 MeV	$p_\chi \sim 50$ keV	zero-point ion momentum in lattice
20 MeV	$E_\chi \sim 10$ eV	atomic ionization energy
2 MeV	$E_\chi \sim 1$ eV	semiconductor band gap
100 keV	$E_\chi \sim 50$ meV	optical phonon energy

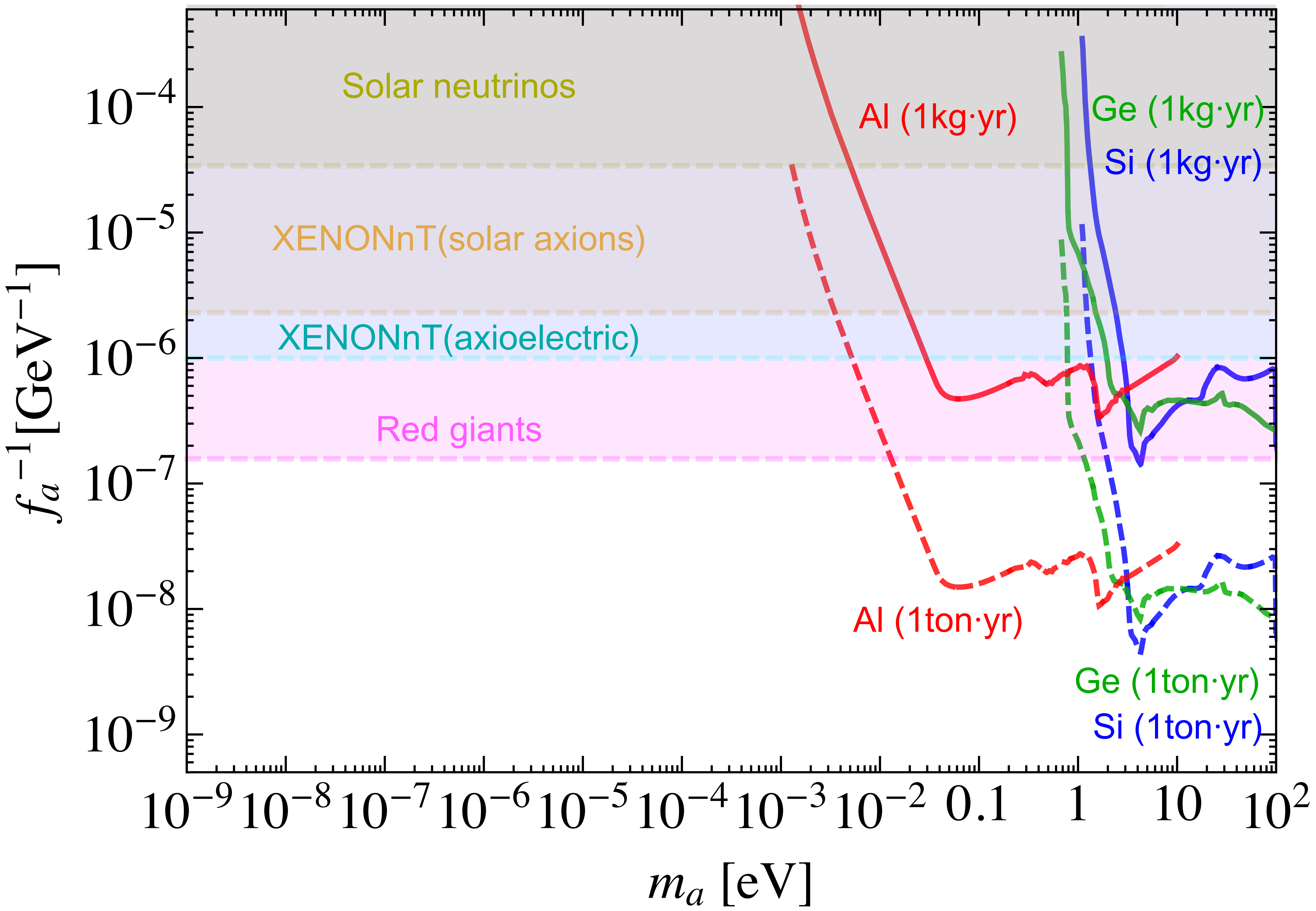


$$R \sim \frac{1}{\rho}\frac{\rho_a}{m_a}\frac{3m_a^2}{4m_e^2}\frac{g_{aee}^2}{e^2}\langle n_e\sigma_{abs}v_{rel}\rangle_\gamma$$

$$\langle n_e\sigma_{abs}v_{rel}\rangle_\gamma = -\frac{\mathrm{Im}\Pi(\omega)}{\omega}$$

Absorption rate for photon in material

Combined Constraints



Summary

Issue-I:

We have proposed a novel mass generation mechanism for Axion-like particles: explicit global symmetry breaking $\rightarrow$ALP mass

Issue-II

We have presented two very simple and natural models that can address the neutrino mass, the dark matter problem and the BAU simultaneously!

Thank you for your attention!