Revisiting the fermion-field nontopological solitons and primordial black holes

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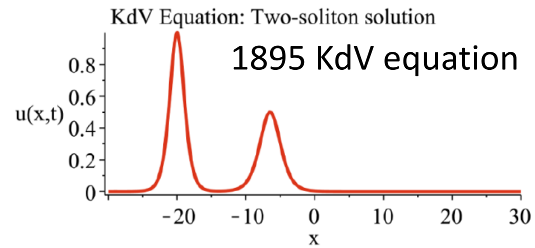
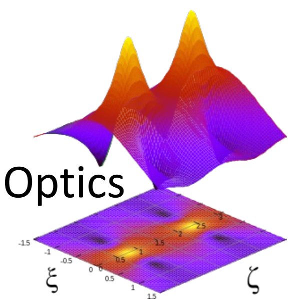
2024.5.10 @COUSP2024, Xichang

arXiv: 2405.01227 [hep-ph] & [astro-ph.CO]

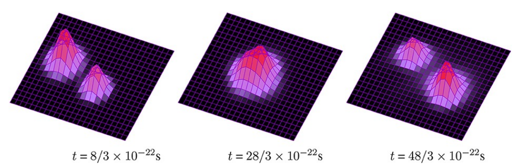
Solitons (孤子)

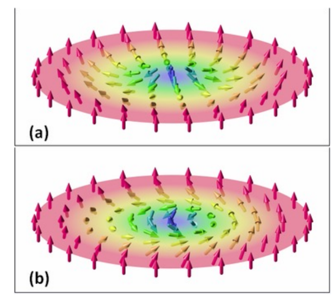
Extended and localized, stable or meta-stable “lumps”





Nuclear physics





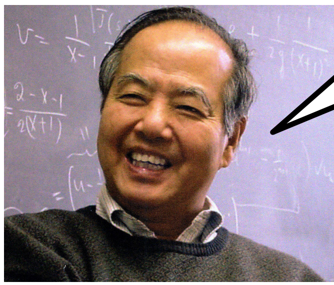
Condensed matter physics

Solitons in quantum field theory

- Topological solitons (cosmic strings, domain walls, etc)
- Nontopological solitons: carry conserved charge
  - ✓ Scalar solitons, e.g. Q-balls
  - ✓ Fermionic solitons (fermion + scalar) [This talk]

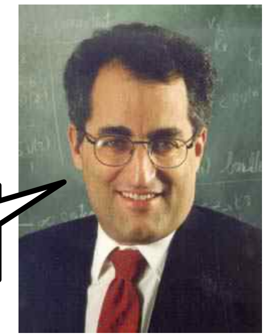
Fermionic solitons

Early work (1970s - 1980s) T. D. Lee *et al*, PRD 15 (1977) 1694, PRD 16 (1977) 1096, PRD 35 (1987) 3678; E. Witten, PRD 30 (1984) 272–285

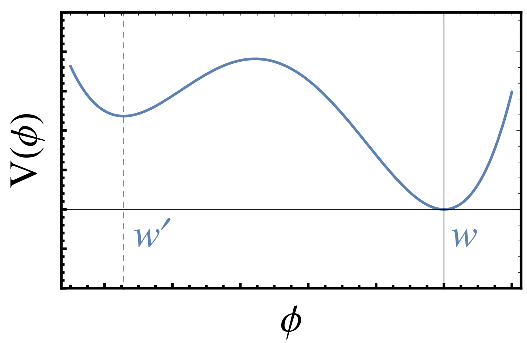


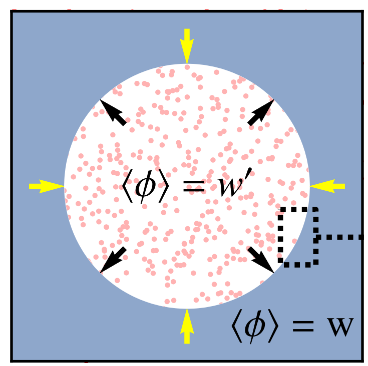
Fermion-field nontopological solitons

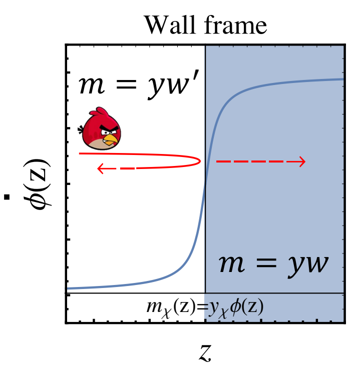
Quark nuggets



Basic setup: scalar ϕ and fermion χ with $\mathcal{L} \supset -y\phi\bar{\chi}\chi$







Balance between fermion degeneracy and vacuum pressures

Recent progress

Mass varies from GeV to galactic level

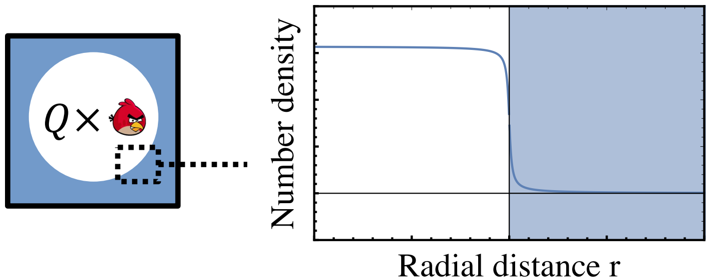
- Dark matter Bai *et al*, JHEP 06 (2018) 072, PRD 99 (2019) 055047; Hong, Jung, and **KPX**, PRD 102 (2020) 075028; Marfatia *et al*, JHEP 11 (2021) 068; Gross *et al*, JHEP 09 (2021) 033; etc
- Baryogenesis Zhitnitsky *et al*, PRD 71 (2005) 023519, PRD 94 (2016) 083502, MPLA 36 (2021) 2130017; Atreya *et al*, PRD 90 (2014) 045010; etc
- Primordial black holes Kawana and **KPX**, PLB 824 (2022) 136791; Huang and **KPX**, PRD 105 (2022) 115033; Marfatia *et al*, JHEP 08 (2022) 001, JHEP 04 (2023) 006; Lu *et al*, PRD 107 (2023) 103037; Kim *et al*, 2309.05703; etc

Two representative features of the calculation framework

1. Polynomial potentials as primary setup

$$U(\phi)=\frac{1}{2}a\phi^2+\frac{1}{3!}b\phi^3+\frac{1}{4!}c\phi^4$$

2. Uniform spatial distribution of fermions



Leads to analytical formulae

Improving the calculation

Apply to general potentials New

- ✓ Classically conformal theories[Iso *et al*, PLB 676 (2009) 81–87]

$$V(\phi)=\frac{3g_{B-L}^4}{2\pi^2}\phi^4\left(\log\frac{\phi}{w}-\frac{1}{4}\right)$$

- ✓ Finite-temperature field theories[Dolan *et al*, PRD 9 (1974) 3320–3341]

$$V(\phi,T)\supset\pm\frac{T^4}{2\pi^2}\int_0^\infty x^2\mathrm{d}x\log\left(1\mp e^{-\sqrt{x^2+\phi^2/T^2}}\right)$$

Include the influences between $\langle\bar{\chi}\chi\rangle$ and ϕ New

<u>Mean field theory</u>

Classical field $\phi(\mathbf{x})$

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)+\bar{\chi}i\gamma^\mu\partial_\mu\chi-y\phi\bar{\chi}\chi$$

Ensemble average of $f(\mathbf{x},\mathbf{p})=\frac{1}{e^{(\epsilon-\mu)/T}+1}$

Bare mass term $-M_f\bar{\chi}\chi$ eliminated by $\phi\to\phi-M_f/y$

- ✓ Non-uniform χ distributions

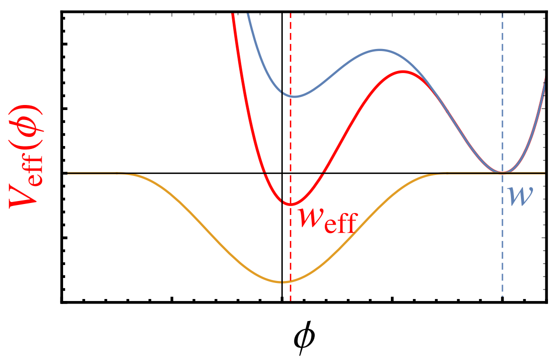
Improvement of methodology brings **new insights** to worldview!

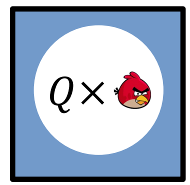
The effective potential

The equation of motion

$$\nabla^2\phi = \frac{\partial V}{\partial\phi} + y\langle\bar{\chi}\chi\rangle \equiv \frac{\partial V_{\rm eff}}{\partial\phi}$$

Effective potential

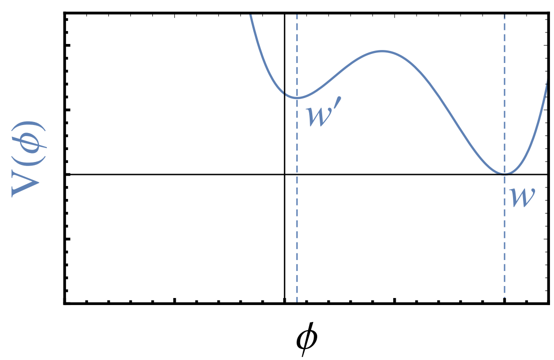




Promoting the aggregation of fermions (μ-dependent)

=

Bare scalar potential



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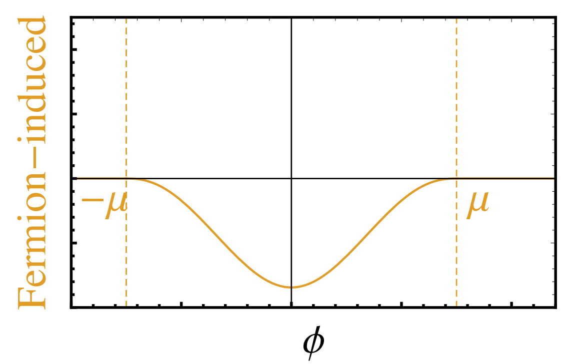
Fermion contribution



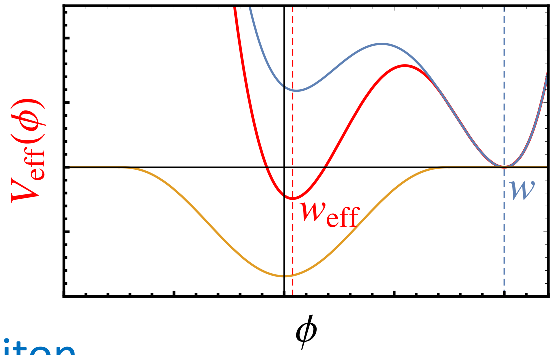
Solving the EoM

Spherical symmetric solution

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{\partial V_{\mathrm{eff}}}{\partial\phi}$$

Boundary: $\left.\frac{\mathrm{d}\phi}{\mathrm{d}r}\right|_{r=0}=0$ and $\lim_{r\to\infty}\phi=w$

Center of the soliton   Exterior of the soliton



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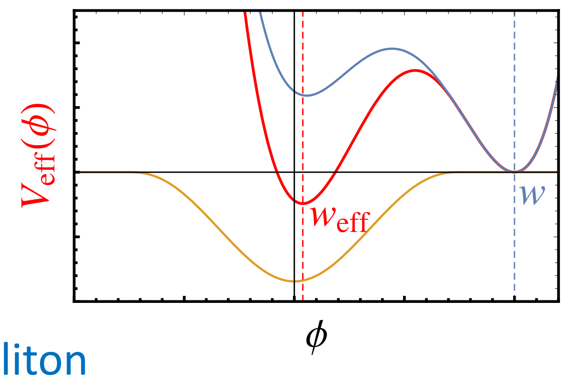
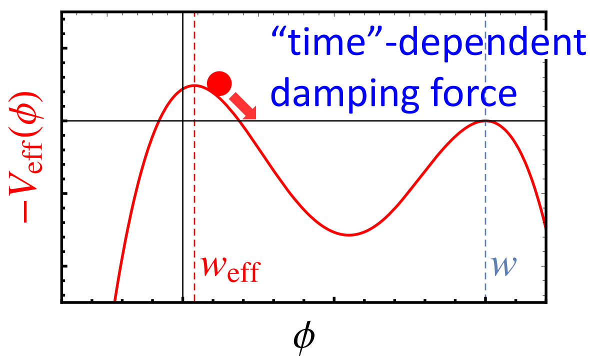
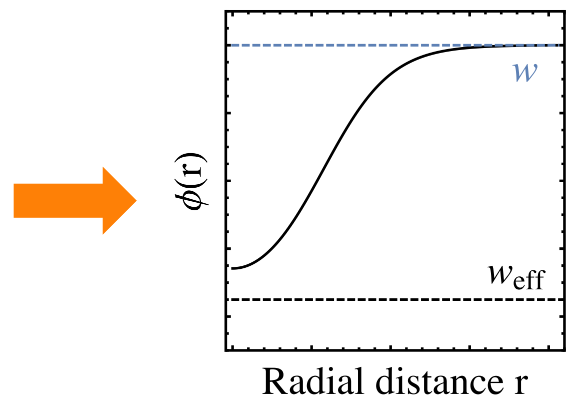


(Just a trick) r as "time" and ϕ as "position"



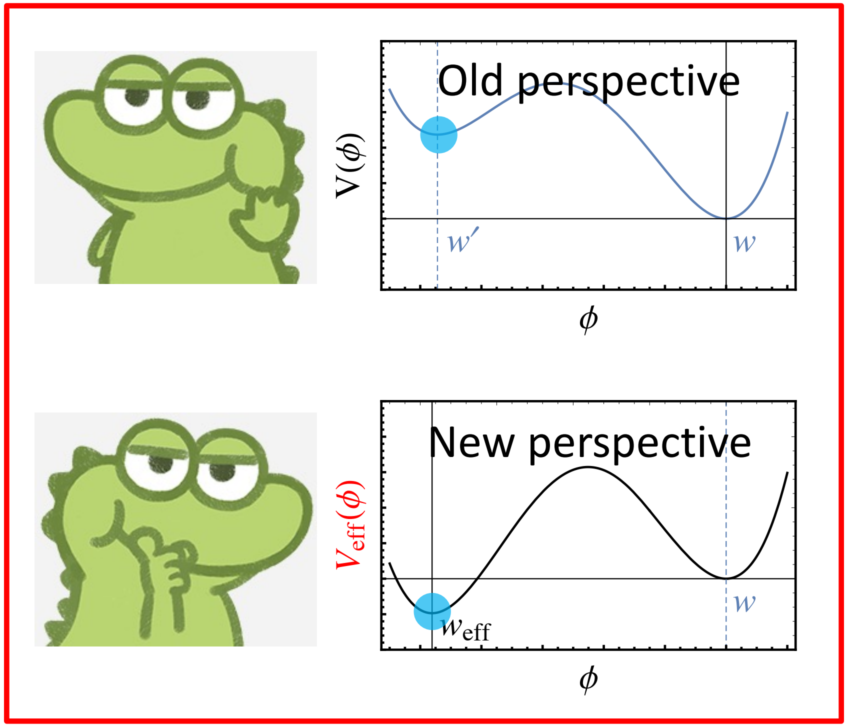


Coleman's overshoot-undershoot argument[PRD 15 (1977) 2929–2936]

When the left-hilltop is higher, the solution must exist!

The soliton profile

New insight 1: Solitons actually live in the true vacuum of $V_{\mathrm{eff}}(\phi)$, rather than the false vacuum of $V(\phi)$

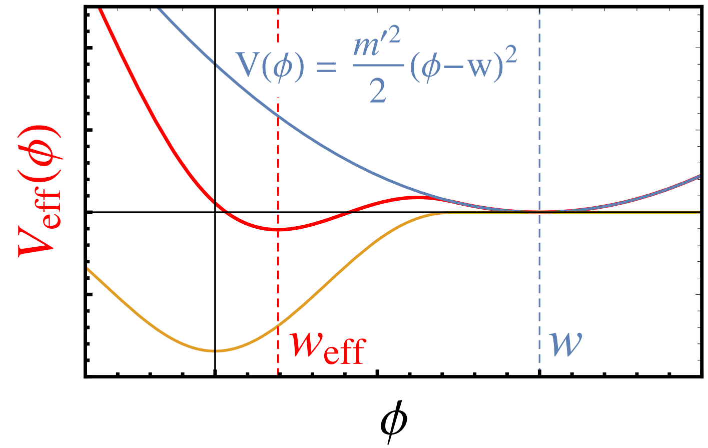


Note $w_{\mathrm{eff}} \neq w'$ (although could be close to)

Extending the concept of solitons

A multi-vacuum $V(\phi)$ is NOT necessary!

One possibility:
$V(\phi) \sim (\phi - w)^2$ single-vacuum
But $\langle\bar{\chi}\chi\rangle$ deforms the potential
$\Downarrow$
Multi-vacuum $V_{\text{eff}}(\phi)$ get!

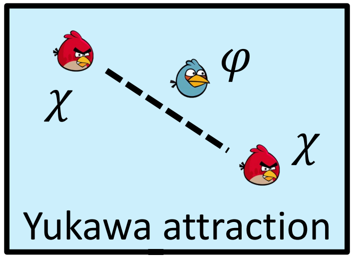


Define $\varphi = \phi - w$,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m'^2}{2}\varphi^2 + \bar{\chi}\left(i\gamma^\mu\partial_\mu - yw\right)\chi - y\varphi\bar{\chi}\chi$$

The conventional *fermion bound states*

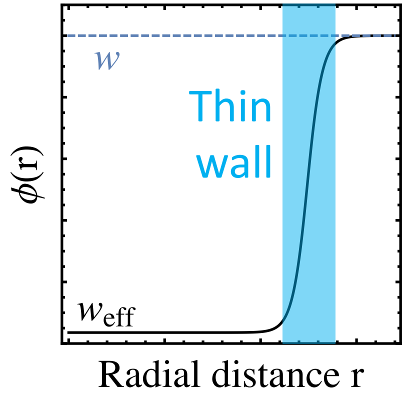
Stephenson *et al*, IJMPA 13 (1998) 2765–2790;
Wise *et al*, PRD 90 (2014) 055030, JHEP 02 (2015) 023;
Gresham *et al*, PRD 96 (2017) 096012, PRD 98 (2018) 096001;
Smirnov *et al*, JHEP 08 (2022) 170, [2201.00939]; etc



New insight 2: Bound states are a subset of solitons

Two analytical limits

Saturation limit: $Q \to \infty$, $\phi(r) \to$ uniform, $R \propto Q^{1/3}$



Implicitly assumed in most studies

Example: relativistic constituent

$$E = Q(12\pi^2 V_0)^{1/4},\ R = Q^{1/3}\left[\frac{3}{16}\left(\frac{3}{2\pi}\right)^{2/3}\frac{1}{V_0}\right]^{1/4}$$

Consistent with [Hong, Jung and **KPX**, PRD 102 (2020) 075028]

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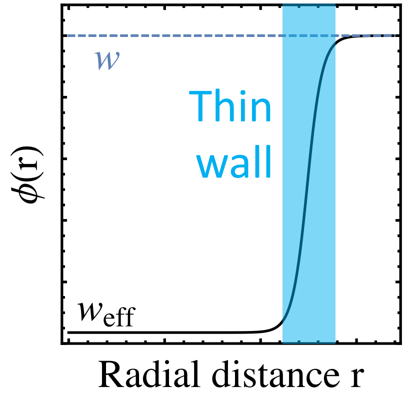

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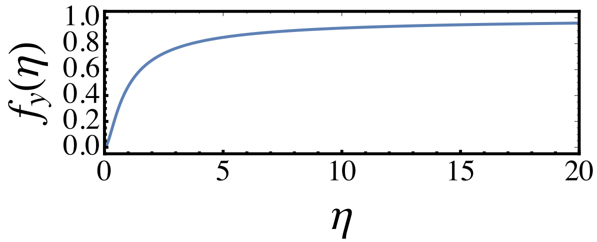
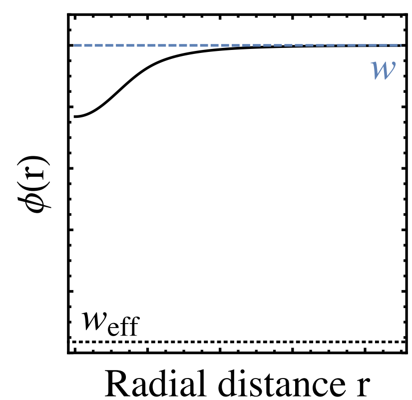
$$\langle\bar{\chi}\chi\rangle = g_{\rm dof}\int\frac{d^3p}{(2\pi)^3}\left(\frac{m}{\epsilon}\right)\frac{1}{e^{(\epsilon-\mu)/T}+1}$$

Yukawa attraction limit: Q small & fermions non-relativistic

- $E_{\rm Yuk} \approx -\frac{3y^2Q^2}{20\pi R}f_y\left(\frac{1}{m'R}\right)$





Consistent with [Kawana and **KPX**, PLB 824 (2022) 136791]

Interplay between charge and radius

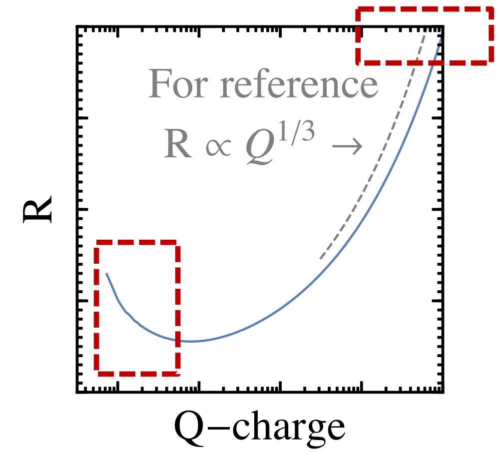
New insight 3:

- When charge Q ↑, the radius R ↘↗
- Eventually $R \sim Q^{1/3}$

When Q is small, saturation limit analytical formulae do not apply!

(<u>Not totally new</u>: known in *fermion bound state* studies, now obtained in the general fermionic soliton study)

Most studies assume



Interplay between charge and radius

New insight 3:

- When charge $Q \uparrow$, the radius $R \searrow\nearrow$
- Eventually $R \sim Q^{1/3}$

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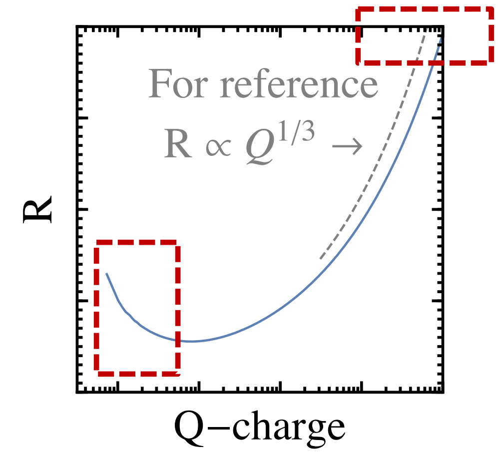
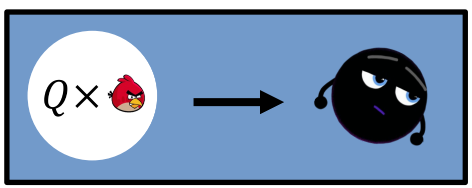




New insight 4:

Bad news: unlikely collapse via Yukawa

- $R_{\rm Sch}/R \lesssim (w/M_{\rm Pl})^2$, no collapse unless $w \sim M_{\rm Pl}$

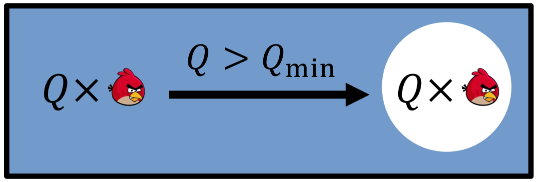
Good news: saturation $R_{\rm Sch}/R \propto Q^{2/3}$

- collapse happens if Q is large enough

Production mechanism

Can exist ≠ must exist!
The actual production of soliton is another nontrivial topic.

Direct fusion from free fermions



Trapping fermions via walls (can simultaneously realize baryogenesis)

- First-order phase transitions Bai *et al*, JHEP 06 (2018) 072, PRD 99 (2019) 055047; Hong, Jung and **KPX**, PRD 102 (2020) 075028, etc
- Domain walls Zhitnitsky *et al*, PRD 71 (2005) 023519, PRD 94 (2016) 083502, etc

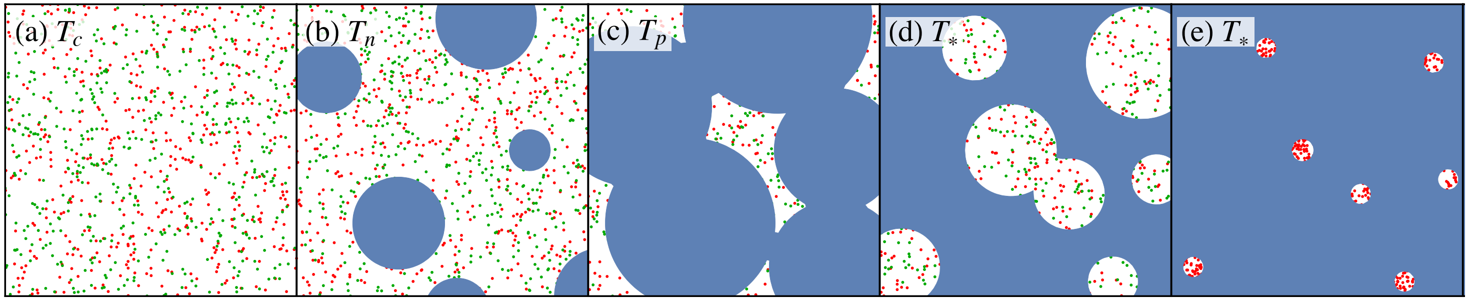


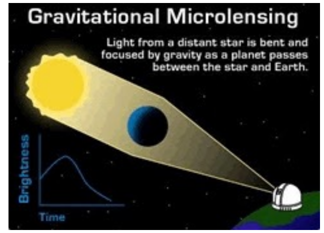
Figure from: Kawana and **KPX**, PLB 824 (2022) 136791

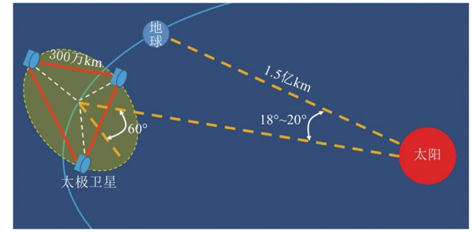
Fragmentation of the scalar field

Experimental signals

<u>Gravitational effects</u>:

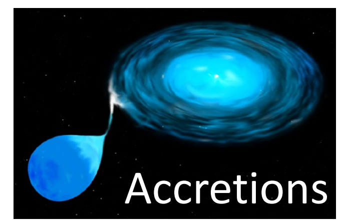
- Lensing
- FOPT or domain wall GWs





<u>Astrophysical particle signals</u>:

- Emitting ϕ quanta, decaying to SM particles via $\phi^2|H|^2$
- χ interacts with SM particles



<u>Direct detection</u>: Energy density Mass Detector size Operating time

$$N_{\rm dd} \approx 6\times\left(\frac{\rho}{\rho_{\rm dm}}\right)\left(\frac{10^{-4}\ {\rm g}}{E}\right)\left(\frac{L}{10\ {\rm m}}\right)^2\left(\frac{\Delta t}{1\ {\rm yr}}\right)$$

Scalar Q-balls already constrained by Xenon1T

Fermionic solitons can also be probed!

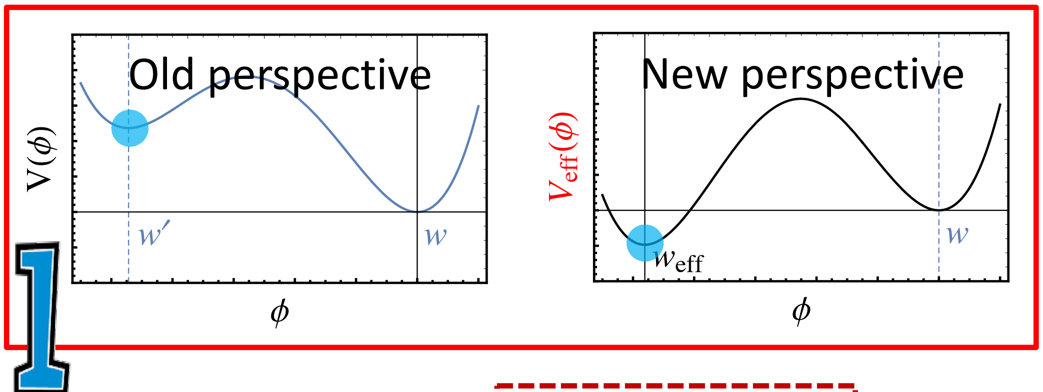
Bai *et al*, JHEP 09 (2019) 011; Huang *et al*, 2404.16509

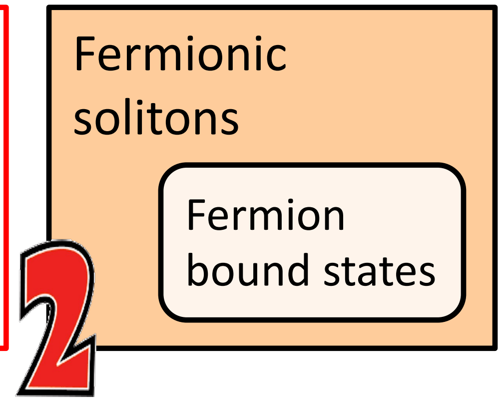
Closing remarks

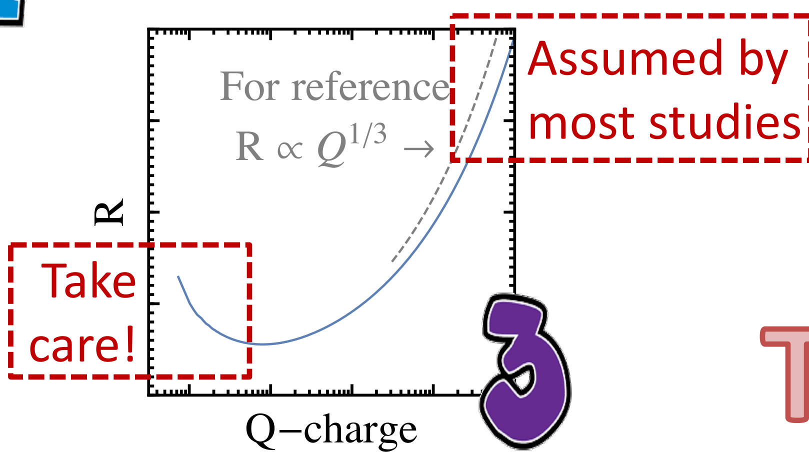
A framework to calculate the fermionic soliton profile, including

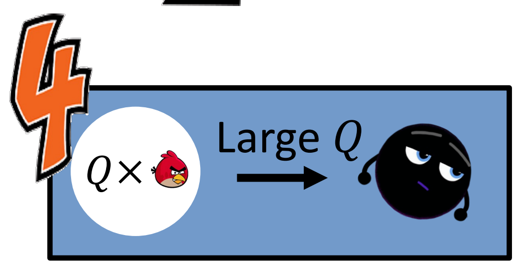
1. General non-polynomial potential $V(\phi)$
2. Influences between $\langle\bar{\chi}\chi\rangle$ and ϕ

Improvement of methodology brings **new insights** to worldview









Thank you!

Backup: setup of the framework

Mean field theory

Classical field $\phi(\mathbf{x})$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \bar{\chi}i\gamma^\mu\partial_\mu\chi - y\phi\bar{\chi}\chi$$

Ensemble average of $f(\mathbf{x},\mathbf{p}) = \frac{1}{e^{(\epsilon-\epsilon_F)/T}+1}$

Single fermion energy $\epsilon = \sqrt{\mathbf{p}^2 + m^2}$, with $m(\mathbf{x}) = y\cdot\phi(\mathbf{x})$
Bare mass term $-M_f\bar{\chi}\chi$ eliminated by $\phi \to \phi - M_f/y$

System described by $\phi(\mathbf{x})$ and $\epsilon_F(\mathbf{x})$

Charge (particle number) $\mathcal{Q}[\phi,\epsilon_F] = g_{\rm dof}\int \mathrm{d}^3x\int\frac{\mathrm{d}^3p}{(2\pi)^3}f$

Energy (mass) $\mathcal{E}[\phi,\epsilon_F] = \int \mathrm{d}^3x\left[\frac{1}{2}(\nabla\phi)^2 + V(\phi) + g_{\rm dof}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\epsilon f\right]$

Entropy $\mathcal{S}[\phi,\epsilon_F] = g_{\rm dof}\int \mathrm{d}^3x\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left[\left(\frac{\epsilon-\epsilon_F}{T}\right)f + \log\left(1+e^{-(\epsilon-\epsilon_F)/T}\right)\right]$

Free energy $\mathcal{F}[\phi,\epsilon_F] = \mathcal{E}[\phi,\epsilon_F] - T\mathcal{S}[\phi,\epsilon_F]$

Backup: getting EoM of soliton

Given constraint $\mathcal{Q}[\phi, \epsilon_F] = Q$, minimizing the free energy

Lagrange multiplier

$$\Omega[\phi, \epsilon_F] = \mathcal{F}[\phi, \epsilon_F] + (Q - \mathcal{Q}[\phi, \epsilon_F]) \cdot \mu$$

$$\boxed{\frac{\delta \Omega}{\delta \epsilon_F} = 0, \qquad \frac{\delta \Omega}{\delta \phi} = 0, \qquad \frac{\partial \Omega}{\partial \mu} = 0}$$

$$g_{\rm dof} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} (\epsilon_F - \mu) \left(\frac{\partial f}{\partial \epsilon_F} \right) = 0$$

$$\epsilon_F(\mathbf{x}) \equiv \mu$$

Chemical potential

The μ-Q mapping

Equation of motion

$$\nabla^2 \phi = \frac{\partial V}{\partial \phi} + y \langle \bar{\chi} \chi \rangle \equiv \frac{\partial V_{\rm eff}}{\partial \phi}$$

($T = 0$ for simplicity)

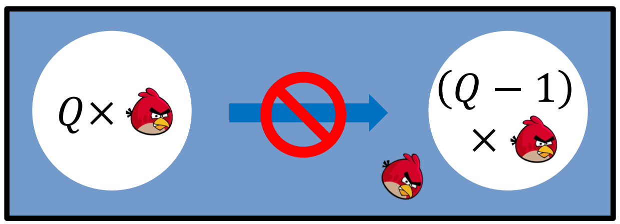
$$V_{\rm eff}(\phi) = V(\phi) + \frac{g_{\rm dof}}{16\pi^2} \left[\frac{\mu}{3} \sqrt{\mu^2 - m^2} (5m^2 - 2\mu^2) + m^4 \log \left(\frac{|m|}{\mu + \sqrt{\mu^2 - m^2}} \right) \right]$$

Backup: the chemical potential μ

Dressed mass of a χ particle inside the soliton

$$\frac{\delta E}{\delta Q} = \mu$$

Stability condition against evaporation to free particles:



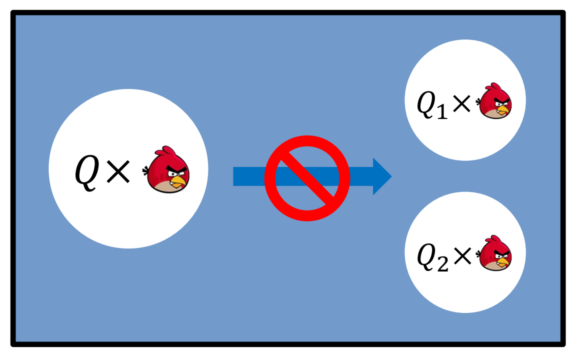
$$\mu < M = yw$$

Free fermion mass
(outside the soliton)

Stability condition against fission

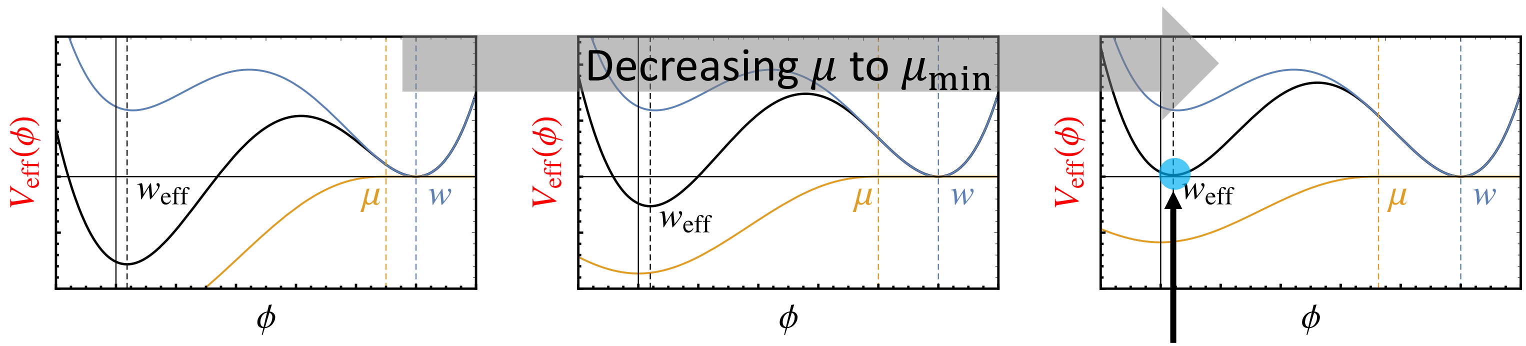
$$\frac{\delta \mu}{\delta Q} = \frac{\delta^2 E}{\delta Q^2} < 0$$

μ decreases with Q.



Upper limit $\mu < \mu_{\max} = M$ means $Q > Q_{\min}$ - Minimal charge

Backup: saturation limit



The "particle" stays near $w_{\rm eff}$ for a long "time" before rolling

$\phi \to$ uniform distribution

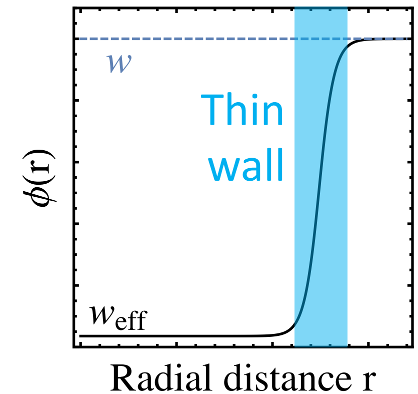
$Q \to \infty$ and $R \propto Q^{1/3}$

$$E \approx E_{\rm kin} + E_{\rm vac} + E_{\rm surf}$$

$E_{\rm kin}$: $\sim \frac{Q^{4/3}}{R}$ (relativistic), $\sim \frac{Q^{5/3}}{R^2}$ (non-relativistic)

$E_{\rm vac}$: $\frac{4\pi R^3}{3} V(w_{\rm eff})$

$E_{\rm surf}$: $4\pi R^2 \sigma$



Example: relativistic constituent

$$E = Q(12\pi^2 V_0)^{1/4},\ R = Q^{1/3}\left[\frac{3}{16}\left(\frac{3}{2\pi}\right)^{2/3}\frac{1}{V_0}\right]^{1/4}$$

Consistent with [Hong, Jung and **KPX**, PRD 102 (2020) 075028]

Implicitly assumed in most studies

Backup: soliton profiles from FOPT scenarios

Estimates: charge Q, mass E, radius R, dark matter fraction $f_{\rm dm}$

$Q \sim Y \cdot s \cdot R_*^3$ (preexisting) or$Q \sim \sqrt{N_\chi}$ (thermal fluctuation)

	Preexisting χ-asymmetry	Thermal fluctuations
$\langle Q \rangle$	$10^{47} \times v_w^3 \left(\frac{Y}{10^{-10}}\right) \left(\frac{\rm GeV}{T_*}\right)^3 \left(\frac{H_*}{\beta}\right)^3$	$10^{27} \times v_w^{3/2} \left(\frac{\rm GeV}{T_*}\right)^{3/2} \left(\frac{H_*}{\beta}\right)^{3/2}$
$\langle E \rangle$	10^{24} g $\times v_w^3 \left(\frac{Y}{10^{-10}}\right) \left(\frac{\rm GeV}{T_*}\right)^2 \left(\frac{H_*}{\beta}\right)^3 \alpha^{1/4}$	10^4 g $\times v_w^{3/2} \left(\frac{\rm GeV}{T_*}\right)^{1/2} \left(\frac{H_*}{\beta}\right)^{3/2} \alpha^{1/4}$
$\langle R \rangle$	10 cm $\times v_w \left(\frac{Y}{10^{-10}}\right)^{1/3} \left(\frac{\rm GeV}{T_*}\right)^2 \left(\frac{H_*}{\beta}\right) \alpha^{-1/4}$	10^{-6} cm $\times v_w^{1/2} \left(\frac{\rm GeV}{T_*}\right)^{3/2} \left(\frac{H_*}{\beta}\right)^{1/2} \alpha^{-1/4}$
$f_{\rm dm}$	$\left(\frac{Y}{10^{-10}}\right) \left(\frac{T_*}{\rm GeV}\right) \alpha^{1/4}$	$10^{-20} \times v_w^{-3/2} \left(\frac{T_*}{\rm GeV}\right)^5 \left(\frac{\beta}{H_*}\right)^{3/2} \alpha^{1/4}$

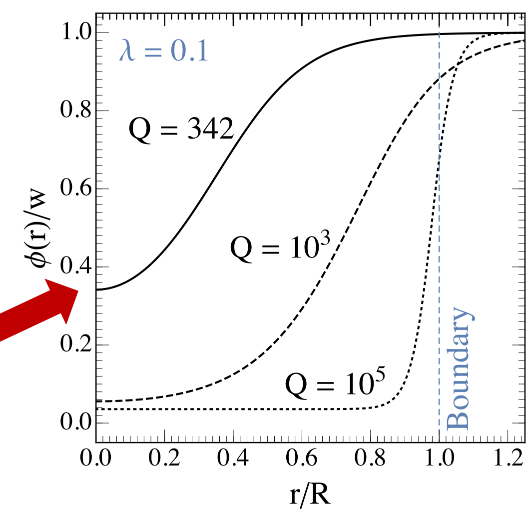
α: (latent heat)/(radiation energy)

β/H_*: (Hubble time)/(FOPT duration)

Temperature T_*

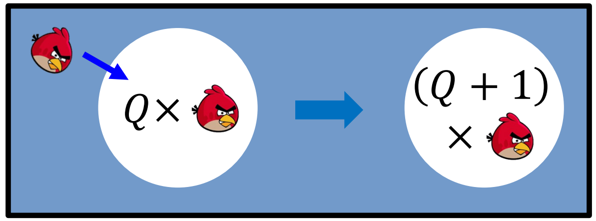
Bubble expansion velocity v_w

Analytical formulae may not apply to solitons formed at high-scales



Backup: accretion and evaporation

Solitosynthesis[Griest *et al*, PRD 40 (1989) 3231; Bai *et al*, JHEP 10 (2022) 181]



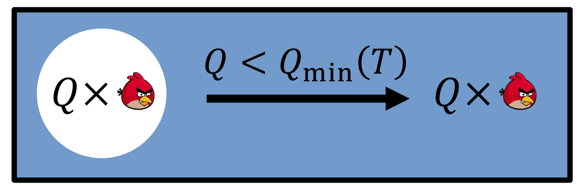
Active when free χ fermions in the plasma are abundant

Evaporation to free χ fermions

$V(\phi) \to V(\phi, T)$ in the early Universe.

$Q > Q_{\min}(T_*)$ at formation,

while $Q < Q_{\min}(T)$ at low temperatures



Evaporation to other particles via decaying χ

Decay via small breaking of $U(1)_Q$

Mainly through surface

For example $-y_\nu \bar{\ell}_L \widetilde{H} \chi_R$