# Gravitational waves from inflaton decay: an effective field theory approach

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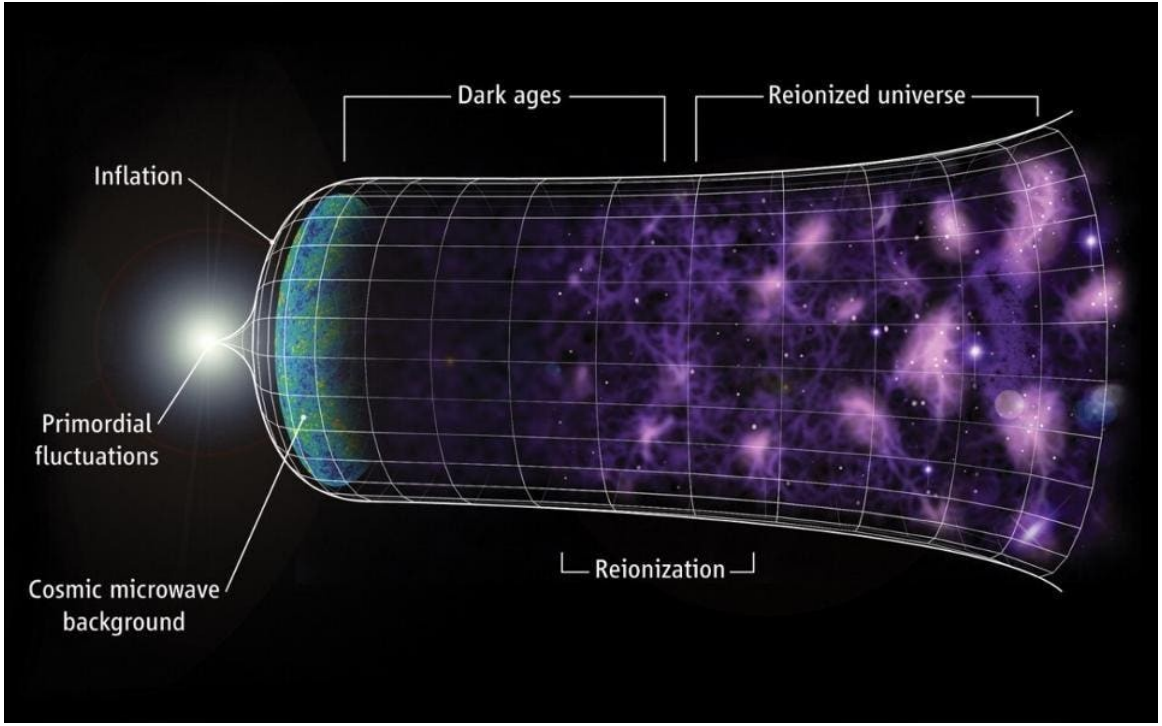
Based on A.Koshelev, A. Starobinsky, AT, Phys.Lett.B 838 (2023) 137686
AT, arXiv:2312.16691 (accepted to PLB)

# Early Universe inflation: Why do we need it?



- Initial conditions for Hot Big Bang
- The best explanation for homogeneity and isotropy of the present Universe
- Natural mechanism of generation of ‘seeds’ for CMB anisotropies and structures in the late Universe

$$a(t) = \mathrm{const} \cdot \mathrm{e}^{H_{vac}\, t}$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j$$

# Realization of inflation and reheating

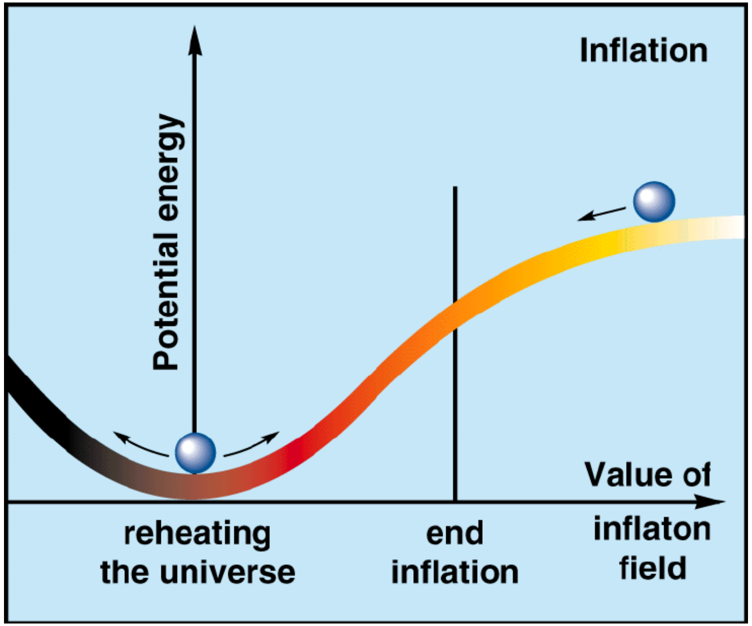
$$p = -\rho. \qquad a(t) = \mathrm{const} \cdot \mathrm{e}^{H_{vac}\, t}$$

$$S = \int d^4x \sqrt{-g} \left( \frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

Slowly rolling scalar field is a solution!

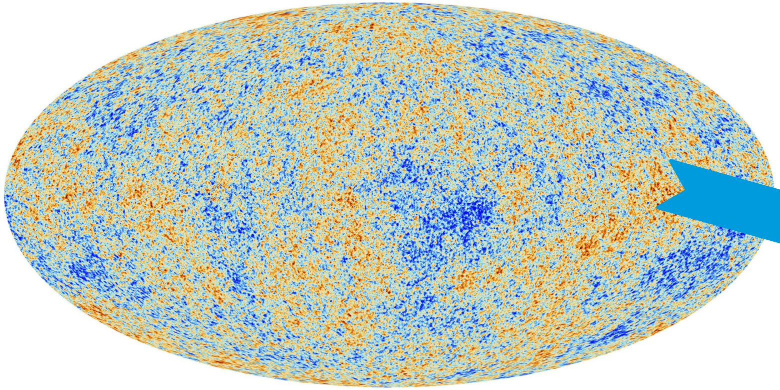


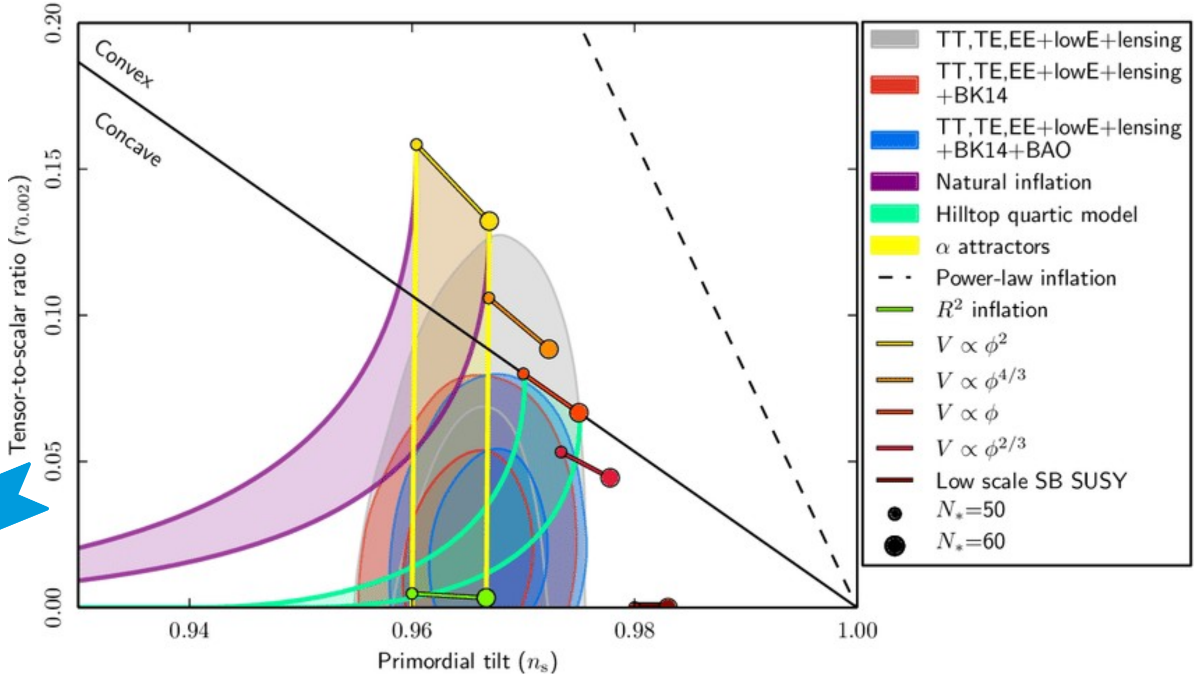
Oscillations after inflation decay to the SM particles $\Longrightarrow$ reheating of the Universe

Reheating temperature is unknown: from 1 GeV to $10^{16}$ GeV

# Planck Constraints on the potential

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H^2}{2\pi\dot{\phi}_c}\right)^2_{\eta_k}$$





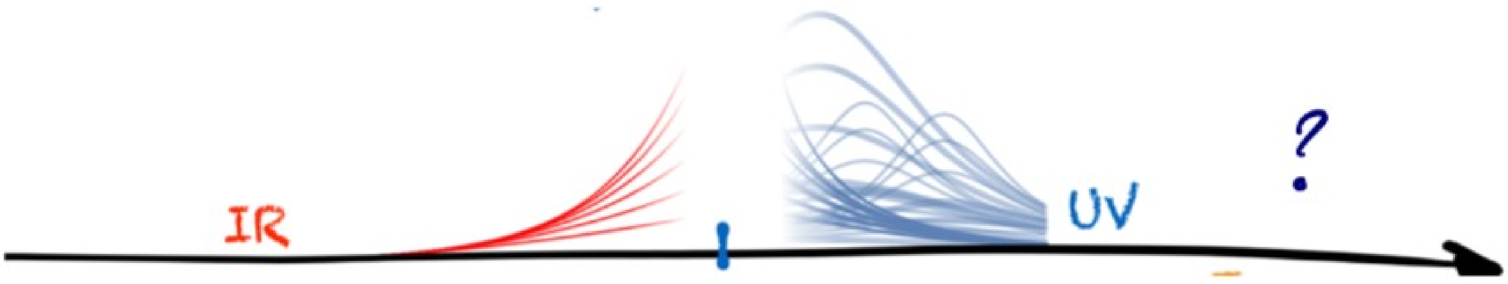
Serious QFT challenge – there is no renormalizable model left!

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = \frac{1}{\pi}\frac{M_{Pl}^2 V'^2}{V} = 16\epsilon$$

$$n_s(k) - 1 = \frac{M_{Pl}^2}{4\pi}\left(\frac{V''}{V} - \frac{3}{2}\left(\frac{V'}{V}\right)^2\right)$$

# How to deal with non-renormalizable theories?



- We write all couplings in the Lagrangian which are compatible with the symmetries of low energy theory
- The Wilson coefficients are arbitrary and should be got from experiment
- This approach is working for energies below cutoff scale (minimal suppression scale of higher derivative operators)

# EFT of inflaton and gravity

Expansion around the flat space:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

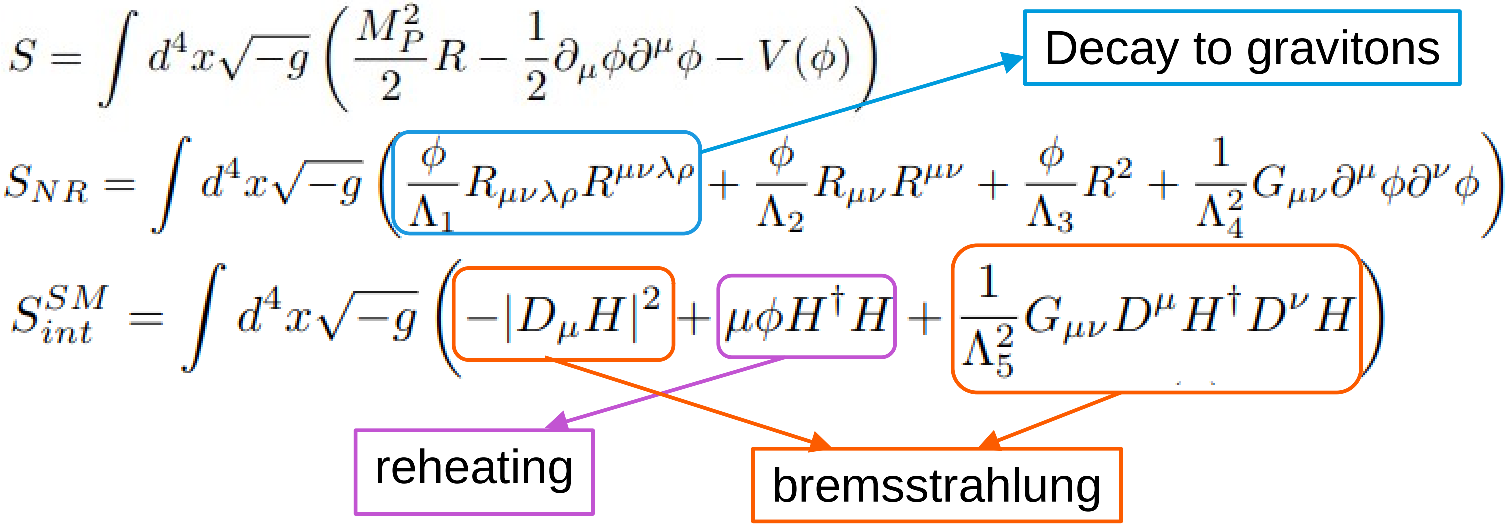
$$S_{NR} = \int d^4x\sqrt{-g}\left(\frac{\phi}{\Lambda_1}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \frac{\phi}{\Lambda_2}R_{\mu\nu}R^{\mu\nu} + \frac{\phi}{\Lambda_3}R^2 + \frac{1}{\Lambda_4^2}G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\right)$$

$$S_{int}^{SM} = \int d^4x\sqrt{-g}\left(-|D_\mu H|^2 + \mu\phi H^\dagger H + \frac{1}{\Lambda_5^2}G_{\mu\nu}D^\mu H^\dagger D^\nu H\right)$$

Leading contribution to graviton production after inflation?

# EFT of inflaton and gravity

## Expansion around the flat space:



$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

Decay to gravitons

$$\Gamma = \frac{m^7}{32\pi M_p^4\Lambda_1^2}$$

$$S_{NR} = \int d^4x\sqrt{-g}\left(\boxed{\frac{\phi}{\Lambda_1}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}} + \frac{\phi}{\Lambda_2}R_{\mu\nu}R^{\mu\nu} + \frac{\phi}{\Lambda_3}R^2 + \frac{1}{\Lambda_4^2}G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\right)$$

$$S_{int}^{SM} = \int d^4x\sqrt{-g}\left(\boxed{-|D_\mu H|^2} + \boxed{\mu\phi H^\dagger H} + \boxed{\frac{1}{\Lambda_5^2}G_{\mu\nu}D^\mu H^\dagger D^\nu H}\right)$$

reheating

bremsstrahlung

Other operators are suppressed by higher powers of Λs

Results are valid for ANY UV completion for quantum gravity

# Example: non-local UV completion to gravity

$$S = -\frac{M_P^2}{2}\int\sqrt{-g}d^4x\left(R-\frac{R^2}{6M^2}\right)$$

A. A. Starobinsky, Phys. Lett. B **91** (1980)

What could be the UV completion?

$+\beta W_{\mu\nu\lambda\rho}W^{\mu\nu\lambda\rho}$ – renormalizable Stelle gravity $\Rightarrow ghost$

K. S. Stelle, Phys. Rev. D **16** (1977), 953-969

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R+\frac{\lambda}{2}\left(RF(\Box)R+WF_W(\Box)W\right)\right)$$

E. T. Tomboulis, [arXiv:hep-th/9702146 [hep-th]].

Ghost-free if

$$F(\Box) = \frac{M_P^2}{6M^2\Box}\left[(\Box-M^2)e^{\sigma(\Box)}+M^2\right]$$

$$F_W(\Box) = M_P^2\frac{e^{\sigma(\Box)}-1}{2\Box}$$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

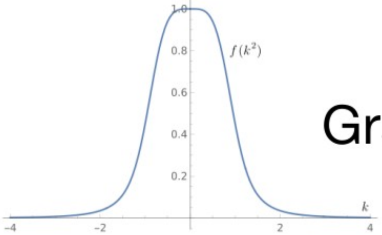
How does it work?

Examlple:

$$L = \frac{1}{2}\phi f(\Box)\phi - V(\phi)\,, \quad f(\Box) = (\Box - m^2)\,e^{\sigma(\Box)}$$

$\sigma(\Box)$ is an entire function, for example $\sigma(\Box) = \Box/\Lambda^2$

UV-finite theory for ANY $V(\phi)$!



Graviton propagator

# Graviton production in non-local model

We prove that graviton production is determined only by the term $W\Box W$, irregardless of the presence of higher derivatives

$$L = A\,W_{\alpha\beta\gamma\delta}\,\Box W^{\alpha\beta\gamma\delta} + B\,R\,W_{\alpha\beta\gamma\delta}W^{\alpha\beta\gamma\delta}$$

$$\Gamma = \frac{6}{\pi}\frac{M^{11}}{M_P^6}(A+2B)^2 = \frac{3}{2\pi}\alpha_1^2\frac{M^3}{M_P^2}\left(\frac{M}{\Lambda}\right)^8, \ \alpha_1 \sim 1,$$

$\Lambda$ – scale of non-locality

Connecting to the observables:

$$\Delta N_{eff} = 2.85\frac{\rho_{GW}}{\rho_{SM}} = 2.85\frac{\Gamma_{GW}}{\Gamma_H} = 821\alpha_1^2\frac{M^8}{\Lambda^8}$$

**Planck:** $\Delta N_{eff} < 0.2$

Bound on the non-locality scale: $\Lambda \gtrsim 3M$

# Graviton production in non-local model

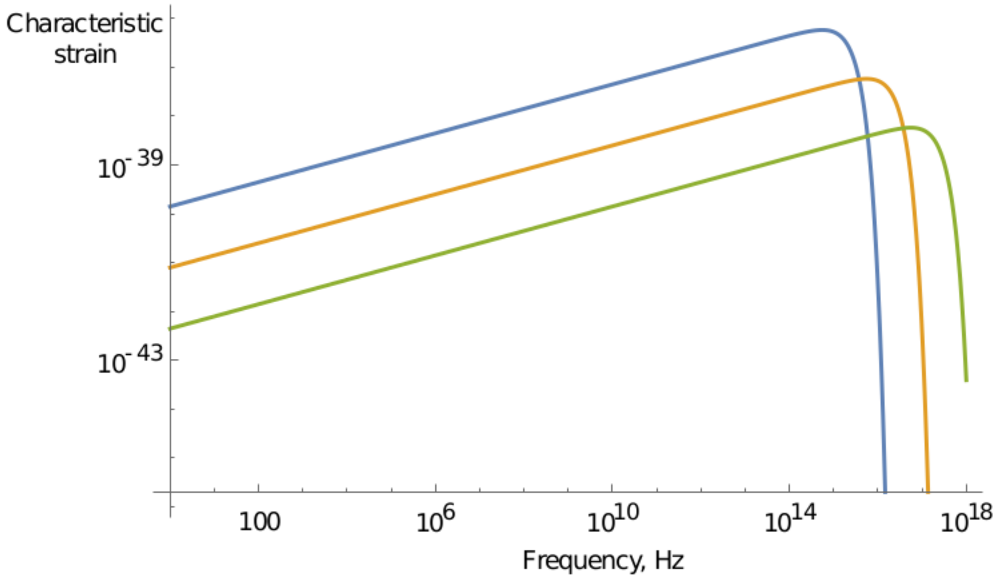


Figure 1: The blue curve shows the gravitational wave signal for $T_{reh} = 10^{10}$ GeV. The orange curve is for $T_{reh} = 10^{9}$ GeV and the green curve is plotted for $T_{reh} = 10^{8}$ GeV. In all cases we assume $\Gamma_{GW}/\Gamma_{SM} = 10^{-3}$. The low-frequency slopes of the plots correspond to the universal $h_c \propto f^{1/4}$ behaviour following from (21). The lowest characteristic strain available for future gravitational wave detectors is $10^{-24}$ for the frequencies $1 - 10^{6}$ Hz. One can see that the predicted signal is well below that level even for more intensive reheating and GW production.

# Inflaton decay to gravitons: selected results

- Planck-suppressed operators **do matter** for low $T_{reh}$ !

$$T_{reh} \lesssim 0.15\, g_{reh}^{1/4} \frac{m^{7/2}}{M_P^{3/2}\Lambda_1} \left(\frac{\Delta N_{eff}}{0.2}\right)^{-1/2}$$

Overproduction of dark radiation

$$m = 10^{13} \text{ GeV} \quad T_{reh}^{min} = 1 \text{ GeV}$$

$$m = 10^{16} \text{ GeV} \quad T_{reh}^{min} = 10^{10} \text{ GeV}$$

$$\Delta N_{eff} = 2.85 \frac{\rho_{GW}}{\rho_{SM}} = 2.85 \frac{\Gamma_{GW}}{\Gamma_H}$$

$$\Delta N_{eff} \lesssim 0.2$$

$$T_{reh} = 0.3\, g_{reh}^{1/4} \sqrt{\Gamma_{SM} M_P}.$$

$$\Gamma_{GW} = \frac{m^7}{64\pi M_p^4 \Lambda_1^2}$$

$$\Gamma_{SM} = \frac{\mu^2}{8\pi M}.$$

$$S_{NR} = \int d^4x \sqrt{-g} \left(\frac{\phi}{\Lambda_1} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}\right)$$

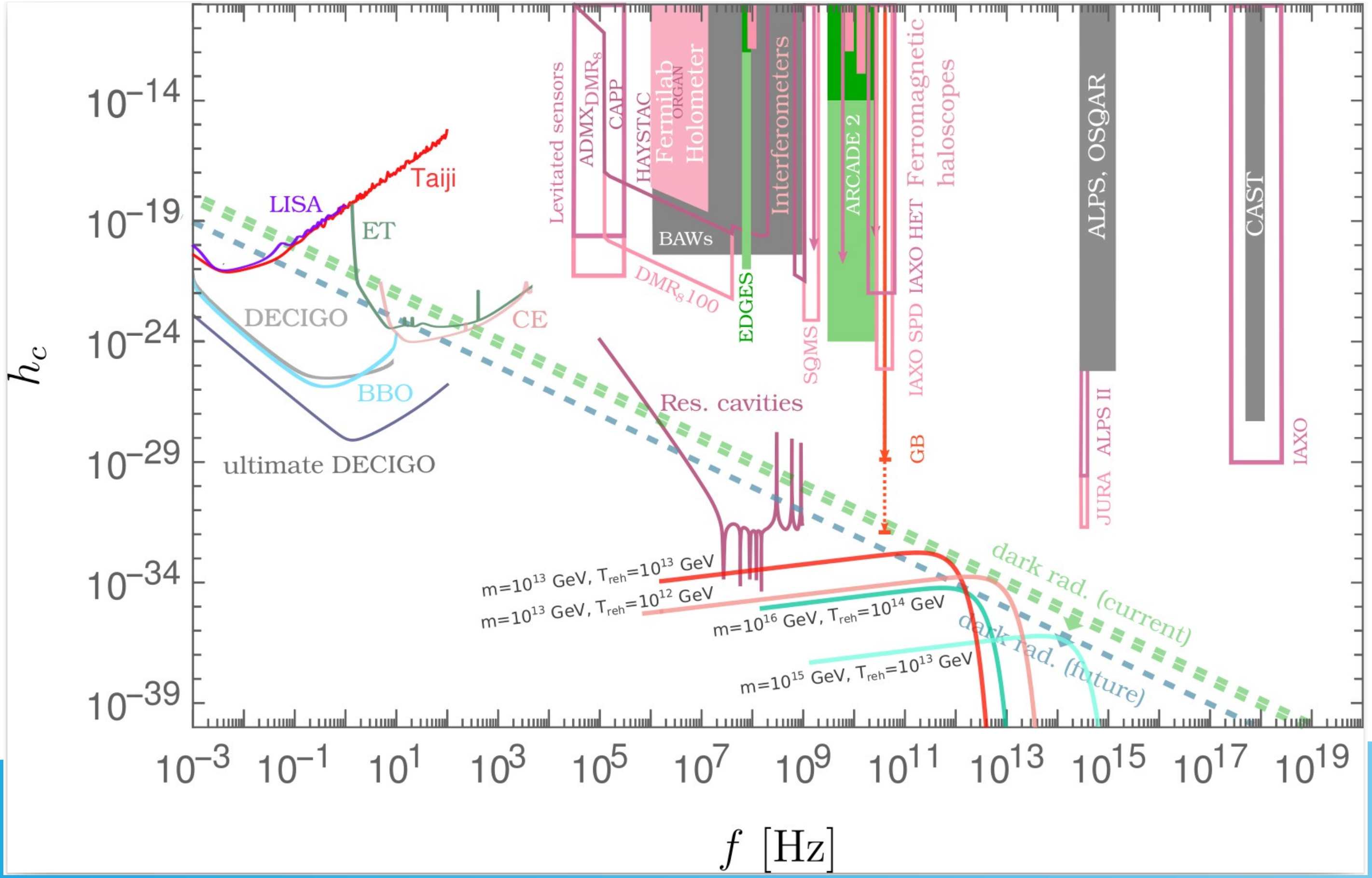
# Gravitational waves from inflaton decay

$$\frac{d\Omega_{GW}}{d\log E} = \frac{16E^4}{M^4}\frac{\rho_{reh}}{\rho_0}\frac{\Gamma_{GW}}{H_{reh}}\frac{1}{\gamma(E)}e^{-\gamma(E)}$$

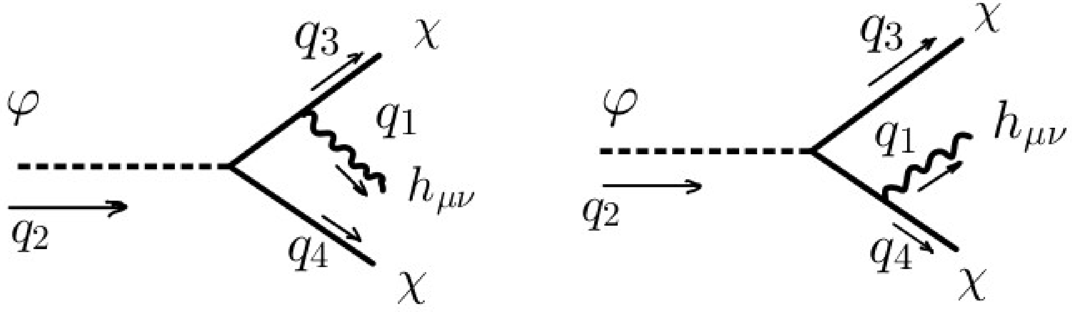
$$\gamma(E) = \left(\left(\frac{g_{reh}}{g_0}\right)^{1/3}\frac{T_{reh}}{T_0}\frac{2E}{M}\right)^{3/2}$$

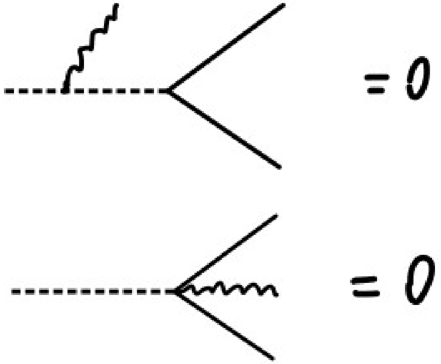
A. Koshelev, A. Starobinsky, AT, PLB, arXiv:2211.02070

$$h_c(f) = \sqrt{\frac{3H_0^2}{\pi f^2}\frac{d\Omega_{GW}}{df}}$$



# Graviton bremsstrahlung during reheating





$$G(k)=\frac{\partial\Gamma}{\partial k}=A\frac{(m-2k)^2}{m\,k},\quad A=\frac{1}{64\pi^3}\frac{\mu^2}{3M_p^2}$$

Not sensitive to inflaton-graviton coupling

$$G(k)=\frac{\partial\Gamma}{\partial k}=A\frac{(m-2k)^2}{m\,k}+B_{UV}(k)\qquad A=\frac{1}{64\pi^3}\frac{\mu^2}{2M_p^2}\left(\frac{m^2}{\Lambda_5^2}+1\right)^2$$

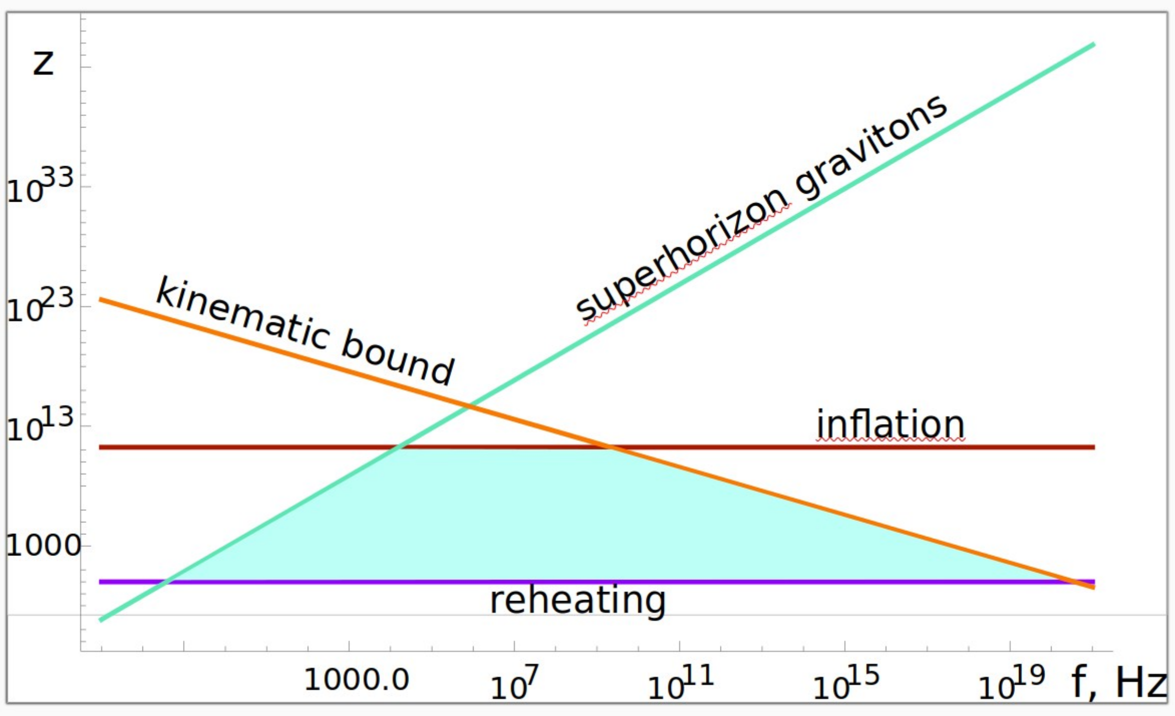
$$B_{UV}(k)=\frac{1}{64\pi^3}\frac{\mu^2}{2M_p^2}\frac{2(m-2k)^2}{15\Lambda_5^2}\left(\frac{m\,(7k-10m)}{\Lambda_5^2}-10\right)$$

$$\frac{d\rho_{GW}}{dk}=\int\frac{kdN}{a_0^3}=\int dt\frac{kn_\phi(t)a(t)^3}{a_0^3}G(k\frac{a_0}{a(t)})$$

$$n_\phi=\frac{\rho_{reh}}{M}\left(\frac{a_{reh}}{a}\right)^3e^{-\Gamma_{tot}t}$$

# Limits on GW frequencies

$$\frac{d\Omega_{GW}}{d\log k} = \frac{k^2}{M\,H_{reh}}\frac{a_{reh}^2}{a_0^2}\frac{\rho_{reh}}{\rho_0}\int_{z_{min}}^{z_{max}} dz\, G(kz\frac{a_0}{a_{reh}})z^{-3/2}e^{-2z^{-3/2}/3}$$



Kinematic bound – comoving momentum is less than m/2

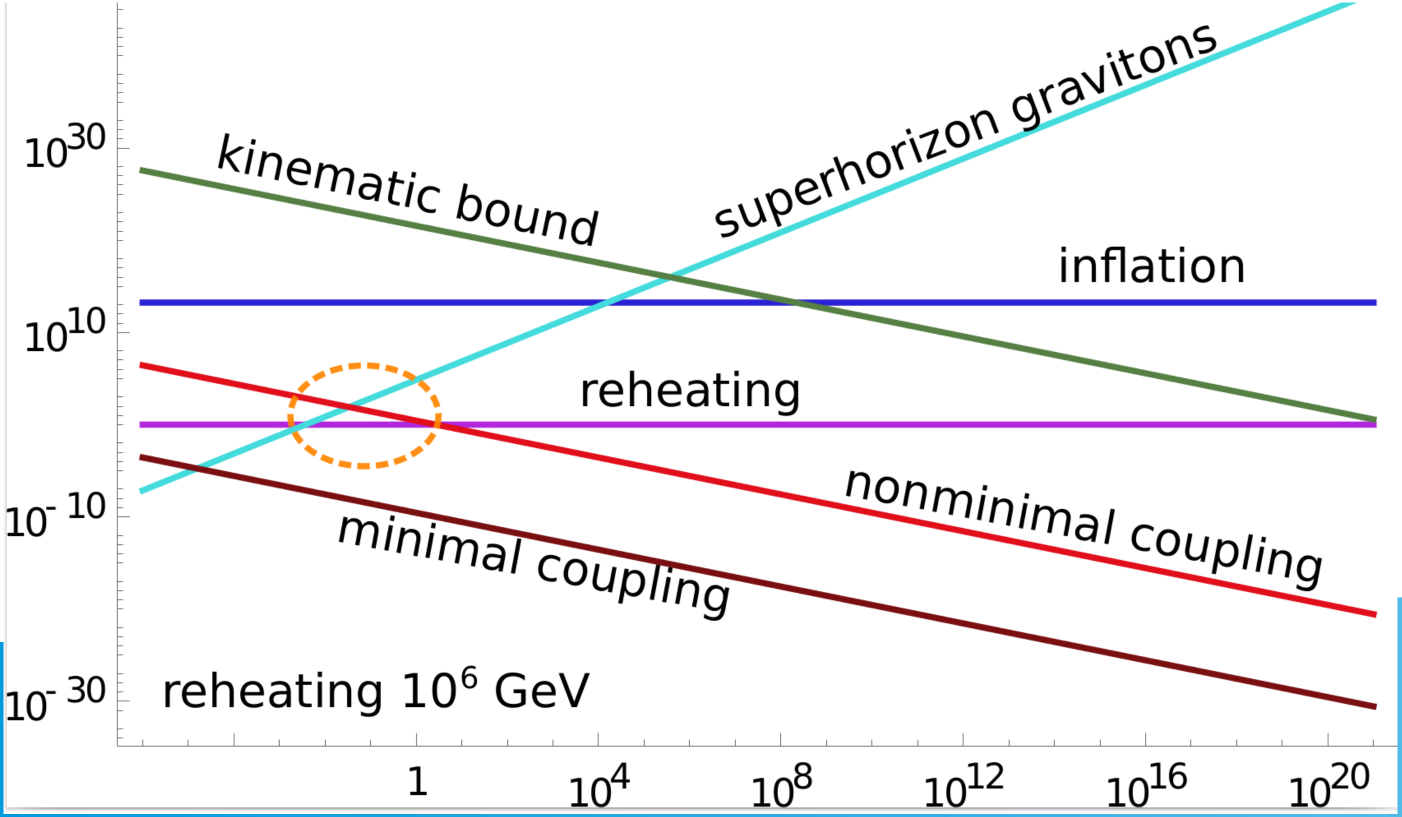
- Gravitons were emitted between inflation and reheating
- Causality requirement - no superhorizon gravitons!

**Also found in**
**- G.Choi, Wenqi Ke,Keith A. Olive, Phys.Rev.D 109 (2024) 8, 083516**
**-Mikko Laine, AT, work in preparation**

# More limitations: IR singularity

$$G(k)=\frac{\partial\Gamma}{\partial k}=A\frac{(m-2k)^2}{m\,k}$$

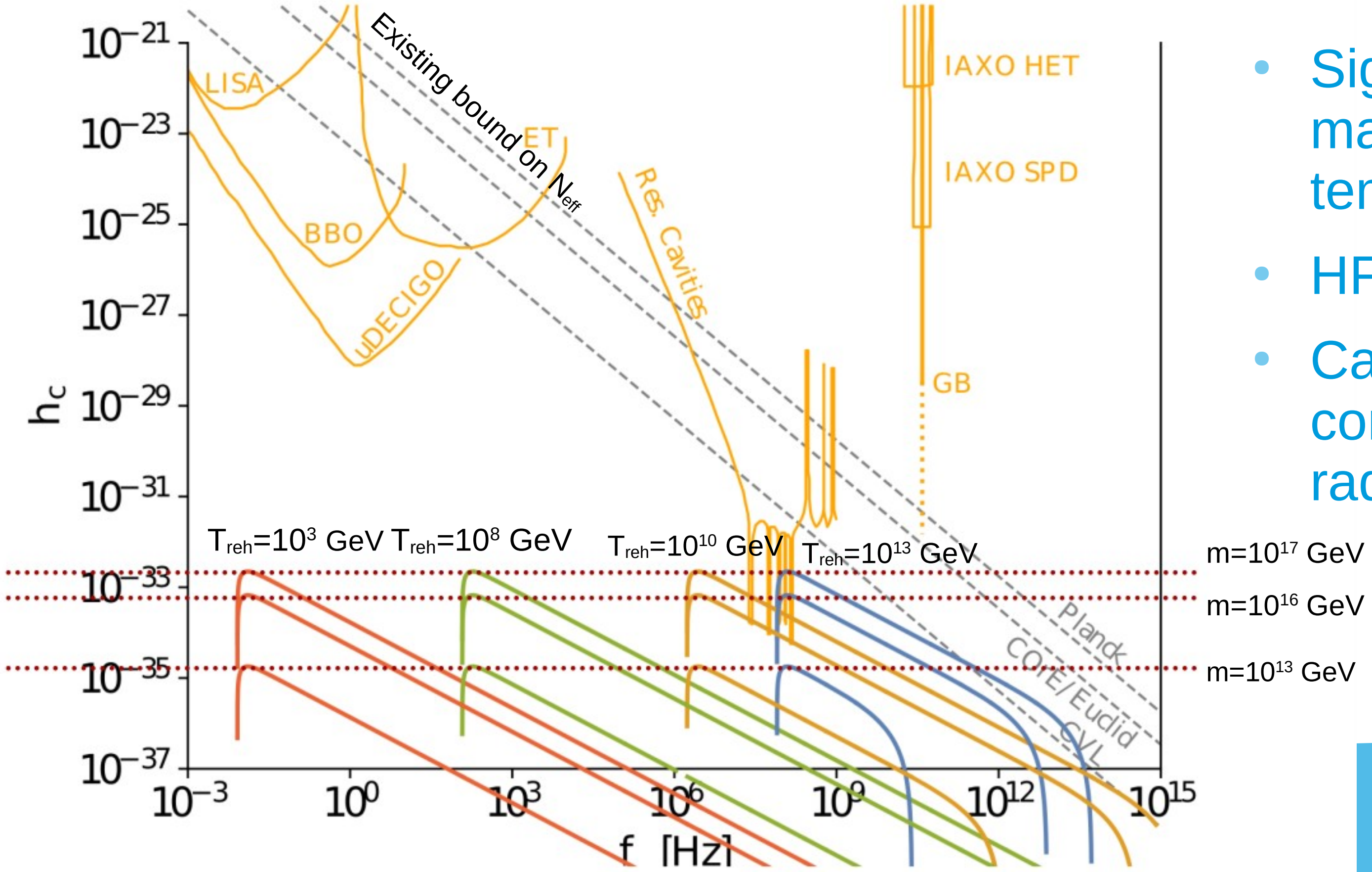
**The computation is incorrect when** $G(k)\sim 1$



**- Resummation of the result including soft graviton emission may be required**

**- The GW spectrum cannot be pushed to high values in a consistent weakly coupled EFT**

# Gravitational waves from bremsstrahlung: $\Lambda_5 = M_P$



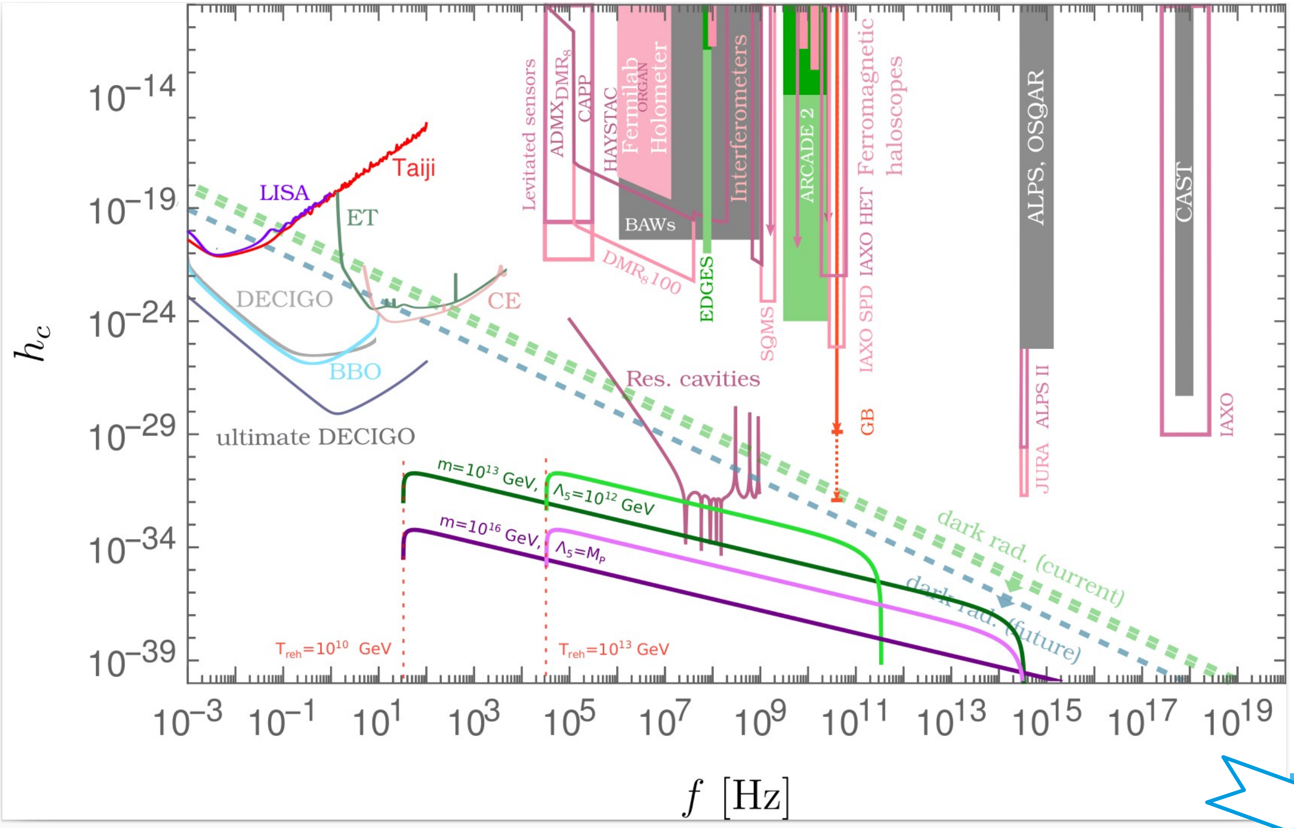
Detection prospects from Barman, Bernal, Xu, Zapata, 2301.11325

- Signals for different inflaton masses and reheating temperatures
- HF GW domain
- Can be also probed as contribution to the dark radiation

$$h_c(f) = \sqrt{\frac{3H_0^2}{\pi f^2}\frac{d\Omega_{GW}}{df}}$$

Results coincide with 2301.11325,
except the IR cutoff

# What if the quantum gravity scale is lower?



- GW signals for inflaton mass m=$10^{13}$ GeV
- The shape does not change, the amplitude is becoming higher
- The unitarity breaking scale is $\Lambda_{UV}=(\Lambda_5\ M_P)^{1/2}$ >m
- From $\Lambda_{UV}=10^{15}$ GeV – tension with $N_{eff}$ bound

Reheating-dependend bounds on quantum gravity scale!

# Conclusions

- High frequency gravitational waves can be sensitive to the quantum gravity effects
- Perturbative decay of inflation to gravitons can be non-negligible for low reheating temperatures → high frequency GWs
- Graviton bremsstrahlung during reheating can provide a sizable HF GW signal → constraints on EFT
- Reheating-dependent constraints on quantum gravity scale from gravitational waves !

Thank you!