What do current experiments tell us about supersymmetry?

Junjie Cao

junjiec@alumni.itp.ac.cn

May 8, 2024

1. Fine tuning problem of QCD axion DM
2. Cruicial experimental restrictions
3. Why does Bino-dominated DM become less attractive?
4. Why does Z_3-NMSSM not naturally explain DM experiments?
5. Advantages of Singlino-dominated DM in GNMSSM
6. Conclusion

Fine tuning problem of QCD axion DM

Assuming: $\phi = \frac{\rho + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$.

Broken PQ symmetry: $V_0(\phi) = \lambda \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2$.

Instanton effects: $V_{\rm QCD}(a) = \left(0.4 \frac{f_\pi m_\pi}{f_a} \right)^2 f_a^2 \left[1 - \cos\left(\frac{a}{f_a} + \theta \right) \right]$

PQ mechanism: $\mathcal{L} = \frac{g_s^2}{32\pi^2} \left(\theta + \frac{a}{f_a} \right) G^{\mu\nu,a} \tilde{G}^a_{\mu\nu}$.

Gravity effects: $V_g(\phi) = \frac{|g| e^{i\delta}}{M_{Pl}^{2m+n-4}} |\phi|^{2m} \phi^n + h.c.$

$$V_g(a) = \left(|g| M_{Pl}^2 \left(\frac{f_a}{\sqrt{2} M_{Pl}} \right)^{2m+n-2} \right)^2 f_a^2 \left[1 - \cos\left(\frac{na}{f_a} + \delta \right) \right].$$

Axion potential: $V(a) = V_{\rm QCD}(a) + V_g(a)$.

Solving strong CP problem: $|g| \lesssim 10^{-55}$

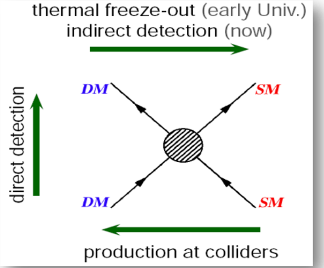
for dimension-5 symmetry breaking operator.

Misalignment mechanism: $\Omega_a h^2 \approx 0.18 \times \left(\frac{f_a}{10^{12}{\rm GeV}} \right)^{1.19} \left(\frac{3\theta_i^2}{\pi^2} \right)$.

Experimental Restrictions: DM DD experiments

Fermi-LAT, AMS...

XENON
PandaX
LUX...



LEP, Tevatron, LHC...

No new physics signal !

arXiv:2207.03764

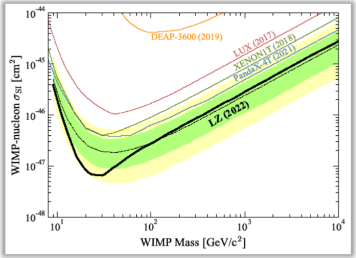


FIG. 5

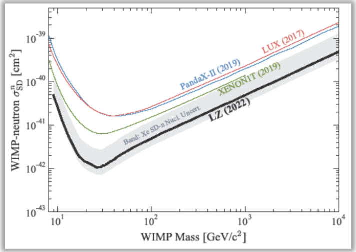


FIG. 7

Simple WIMP DM Theory:

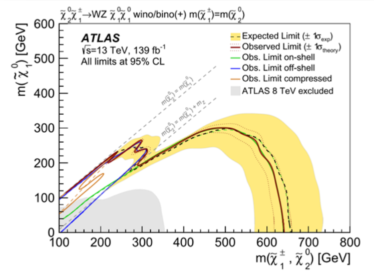
DM mass: $m_{\rm DM} \sim 100$ GeV

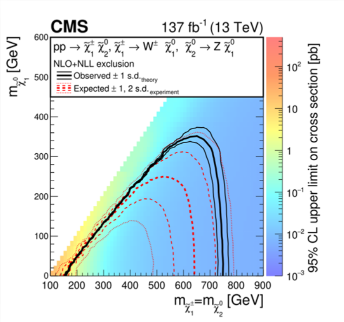
Relic density: $\langle \sigma v \rangle \simeq 10^{-26} {\rm cm}^3 s^{-1}$

SI scattering: $\sigma^{\rm SI}_{\tilde{\chi}-N} \sim 10^{-45} {\rm cm}^2$,

SD scattering: $\sigma^{\rm SD}_{\tilde{\chi}-N} \sim 10^{-39} {\rm cm}^2$.

Experimental Restrictions: LHC's search for SUSY





Latest LHC searches for Tri- and Bi-lepton signals.

1. Simplified model for a specified process.
2. Invalid for a specific theory: complex decay chain, multiple production processes, and various signals to be analyzed.
3. Elaborated Monte Carlo simulations are necessary.

Experimental Restrictions: FT of the MSSM

Example: Fine-tuning of MSSM

$$W_{\rm MSSM} = y_u \hat{Q}\cdot\hat{H}_u\hat{U} + y_d\hat{H}_d\cdot\hat{Q}\hat{D} + \mu\hat{H}_u\cdot\hat{H}_d + \cdots$$

μ **parameter:** Natural values are $\mu = 0$ or $\mu = \Lambda_{\rm GUT}$.

Z-boson mass: $\mu \lesssim 1$ TeV, LHC: $\mu \gtrsim 180$ GeV.

Giudice-Maserio Mechanism:

Generate SUSY-conserving term by gravity-mediated SUSY-breaking.

SM: $m_h^2 = m_h^2|_{tree} + \delta m_h^2 = m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}\Lambda^2;$

MSSM:

$$\begin{aligned} m_h^2 &= m_h^2|_{tree} + \delta m_h^2 \\ &\simeq m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}} \\ &\simeq m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}\frac{2g_s^2}{3\pi^2}m_{\tilde{g}}^2\left(\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}\right)^2; \end{aligned}$$

since $\delta m_{\tilde{t}_i}^2 \simeq \frac{g_s^2}{3\pi^2}m_{\tilde{g}}^2\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}$. $M_3|_{\rm Weak\ scale} = 2.91 M_3|_{\rm GUT}$.

$m_h^2 \simeq m_h^2|_{tree} - 13.6 \times m_{\tilde{g}}^2$ **if** $\Lambda = \Lambda_{\rm GUT}$ **and** $m_{\tilde{t}} = 1$ **TeV!**

Experimental Restrictions: FT of the MSSM

Considering SUSY breaking at GUT scale and RGE running effects,

$$
\begin{aligned}
m_Z^2 \equiv & \{2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)/(\tan^2\beta - 1) - 2\mu^2 \}|_{\textbf{Weak scale}} \\
= & \{(3.84 M_3^2 + 0.32 M_3 M_2 + 0.047 M_1 M_3 - 0.42 M_2^2 + 0.01 M_2 M_1 \\
& -0.012 M_1^2 - 0.65 M_3 A_t - 0.15 M_2 A_t - 0.025 M_1 A_t + 0.22 A_t^2 \\
& +0.004 M_3 A_b - 1.27 m_{H_u}^2 - 0.053 m_{H_d}^2 + 0.73 m_{Q_3}^2 + 0.57 m_{U_3}^2 \\
& +0.049 m_{D_3}^2 - 0.052 m_{L_3}^2 + 0.053 m_{E_3}^2 + 0.051 m_{Q_2}^2 - 0.110 m_{U_2}^2 \\
& +0.051 m_{D_2}^2 - 0.052 m_{L_2}^2 + 0.053 m_{E_2}^2 + 0.051 m_{Q_1}^2 - 0.110 m_{U_1}^2 \\
& +0.051 m_{D_2}^2 - 0.052 m_{L_1}^2 + 0.053 m_{E_1}^2) - 2.18\mu^2 \}|_{\textbf{GUT}}, \\
& \text{for } \tan\beta = 10.
\end{aligned}
$$

$$
m_Z^2 = \underline{(0.45 m_{\tilde{g}}^2 + 0.82 m_{\tilde{t}_L}^2 + 0.74 m_{\tilde{t}_R}^2 - 1.27 m_{H_u}^2|_{\mathrm{GUT}} + \cdots)} - \mathbf{2m_{\tilde{H}}^2}.
$$

The first term: very large considering LHC ..., suppression mechan.

- No log enhancements by symmetry, e.g., R-symmetry;
- Initial condition, e.g., all coefficients proportional to .. ;
- Relaxing the LHC restrictions, e.g., cascade decay, lower prod..

The second term: related with DM physics, **focus of this talk.**

Why Does Bino-dominated DM Become Less Attract.?

- MSSM: Full expression complicated; $\mu/m_{\tilde{\chi}_1^0}$ is Higgsino/DM mass.

$$\sigma^{SI}_{\tilde{\chi}_1^0-N} \simeq 5\times 10^{-45}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{h}}}{0.1}\right)^2 \left(\frac{\mathrm{m_h}}{125\mathrm{GeV}}\right)^2$$

$$\sigma^{SD}_{\tilde{\chi}_1^0-N} \simeq 10^{-39}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{Z}}}{0.1}\right)^2$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h} \simeq e\tan\theta_W \frac{m_Z}{\mu\left(1-m^2_{\tilde{\chi}_1^0}/\mu^2\right)}\left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right)$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z} \simeq \frac{e\tan\theta_W\cos 2\beta}{2}\frac{m_Z^2}{\mu^2-m^2_{\tilde{\chi}_1^0}}$$

- **Conservative bounds on Higgsino mass:**

  LZ Experiment: $\mu \gtrsim 380$ GeV, **LZ + LHC +** a_μ**:** $\mu \gtrsim 500$ GeV.

- **Higgsino mass is related with electroweak symmetry breaking!**

$$m_Z^2 = 2(m^2_{H_d} - m^2_{H_u}\tan^2\beta)/(\tan^2\beta - 1) - \mathbf{2\mu^2}.$$

A tuning of 1% in EWSB. **Significantly worsen in Giudice-Masiero Mechan**

Why Does Bino-dominated DM Become Less Attract.?

Solutions: Go beyond minimal realizations of WIMP miracle.

DM EFTs	Examples	DM Abundance	$\tilde{\chi} - N$ Scattering	Remarks
SM+DM	SM+S_{real}	Weak/contact interactions	$\sigma_{\rm SI} \gtrsim 10^{-45}{\rm cm}^2$ and/or $\sigma_{\rm SD} \gtrsim 10^{-39}{\rm cm}^2$	Experimentally excluded.
			Suppressed by cancellation	Symmetry!
		Feeble interaction: h/Z funnels	Suppressed	Increasingly Fine-tuned: $\Delta > 150$.
SM+DM+X	MSSM with Light Gauginos	Coannihilation/Mediator	Suppressed	Fine-tuning: $\Delta > 30$; Tight LHC constraints.
SM+DM+XY	GNMSSM ISS-NMSSM	May form **secluded DM sector**	Suppressed	No tuning; **three portals to SM**.

Why is the dark matter still called WIMP?
Weak interactions in the DM sector to predict proper Ωh^2, feeble connections between SM and DM sectors to suppress ...

At least two directions to build models:

- Naturally solve μ-problem: **MSSM** → **Z_3-NMSSM** → **General NMSSM**.
- Generate neutrino mass: **Type-I NMSSM** → **ISS-NMSSM** → **B-L NMSSM**.

Why Does Z_3-NMSSM not naturally Explain DM Exp.?

- Field content and gauge group

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{\mathbf{s}}$	$\mathbf{S}$	$\tilde{S}$	**1**	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential — an ad hoc Z_3 discrete symmetry

$$W_{\mathrm{NMSSM}} = W_{\mathrm{Yukawa}} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3$$

Try to economically solve μ-problem and ....

- DM may be Bino- or Singlino-dominated. For Bino-dom. case:

LZ Experiment: $\mu \gtrsim 380$ GeV, **Higgs Data:** $\lambda\mu \lesssim 100$ GeV.

DM physics is the same as that of MSSM since $\lambda \lesssim 0.3$.

Singlino-dominated DM:

- Neutralino mass matrix — diagonalized by a rotation matrix N

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_1 v_u}{\sqrt{2}} & 0 \\ & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda v_u \\ & & & 0 & -\lambda v_d \\ & & & & \frac{2\kappa}{\lambda}\mu \end{pmatrix}$$

- DM mass and its couplings are approximated by: $\mu \equiv \mu_{\rm eff} \equiv \frac{\lambda}{\sqrt{2}} v_s$

$$m_{\tilde{\chi}_1^0} \approx \frac{2\kappa}{\lambda}\mu + \frac{\lambda^2 v^2}{\mu^2}(\mu \sin 2\beta - \frac{2\kappa}{\lambda}\mu) \simeq \frac{2\kappa}{\lambda}\mu, \qquad N_{15} \simeq 1,$$

$$\frac{N_{13}}{N_{15}} = \frac{\lambda v}{\sqrt{2}\mu} \frac{(m_{\tilde{\chi}_1^0}/\mu)\sin\beta - \cos\beta}{1 - \left(m_{\tilde{\chi}_1^0}/\mu\right)^2}, \qquad \frac{N_{14}}{N_{15}} = \frac{\lambda v}{\sqrt{2}\mu} \frac{(m_{\tilde{\chi}_1^0}/\mu)\cos\beta - \sin\beta}{1 - \left(m_{\tilde{\chi}_1^0}/\mu\right)^2},$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_i} \simeq \frac{\sqrt{2}\mu}{v} \left(\frac{\lambda v}{\mu}\right)^2 \frac{V_{h_i}^{\rm SM}(m_{\tilde{\chi}_1^0}/\mu - \sin 2\beta)}{1 - (m_{\tilde{\chi}_1^0}/\mu)^2} + \ldots,$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z} \simeq \frac{m_Z}{\sqrt{2}v} \left(\frac{\lambda v}{\mu}\right)^2 \frac{\cos 2\beta}{1 - (m_{\tilde{\chi}_1^0}/\mu)^2},$$

Z_3-NMSSM: DM Properties

Singlino-dominated DM:

- DM-Nucleon Scattering in the alignment limit:

$$\sigma^{\rm SI}_{\tilde\chi^0_1-N} \simeq 5\times 10^{-45}{\rm cm}^2 \times \left(\frac{\mathcal{A}}{0.1}\right)^2, \quad \sigma^{\rm SD}_{\tilde\chi^0_1-N} \simeq 10^{-39}\ {\rm cm}^2 \left(\frac{C_{\tilde\chi^0_1\tilde\chi^0_1 Z}}{0.1}\right)^2,$$

$$\mathcal{A} \simeq \left(\frac{125{\rm GeV}}{m_h}\right)^2 V_h^{\rm SM} C_{\tilde\chi^0_1\tilde\chi^0_1 h} + \left(\frac{125{\rm GeV}}{m_{h_s}}\right)^2 V_{h_s}^{\rm SM} C_{\tilde\chi^0_1\tilde\chi^0_1 h_s}$$

$$\simeq \sqrt{2}\left(\frac{125{\rm GeV}}{m_h}\right)^2 \lambda \frac{\lambda v}{\mu} \frac{(m_{\tilde\chi^0_1}/\mu - \sin 2\beta)}{\mathbf{1} - (\mathbf{m}_{\tilde\chi^0_1}/\mu)^{\mathbf{2}}},$$

$$C_{\tilde\chi^0_1\tilde\chi^0_1 Z} \simeq \frac{m_Z}{\sqrt{2}v}\left(\frac{\lambda v}{\mu}\right)^2 \frac{\cos 2\beta}{\mathbf{1} - (\mathbf{m}_{\tilde\chi^0_1}/\mu)^{\mathbf{2}}}.$$

- DM properties are described by **four** independent parameters:

$$\tan\beta, \quad \lambda, \quad \mu, \quad m_{\tilde\chi^0_1} \ \text{or}\ \kappa, \quad \text{and}\ 2|\kappa|/\lambda < 1.$$

Experiment: $\lambda \lesssim 0.1$, **DM-$\tilde H$ coannihilation to obtain proper abudance.**

Bayesian evidence is heavily suppressed → A fine-tuning theory!

Advantages of Singlino-dominated DM in GNMSSM

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	(U(1) $\otimes$ SU(2) $\otimes$ SU(3))
$\hat{q}$	$\tilde{q}$	q	3	$\left(\frac{1}{6}, \mathbf{2}, \mathbf{3}\right)$
$\hat{l}$	$\tilde{l}$	l	3	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$\left(-\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$\left(\frac{1}{2}, \mathbf{2}, \mathbf{1}\right)$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$\left(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$\left(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}}\right)$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential — no ad hoc symmetry!

$$W_{\rm GNMSSM} = W_{\rm Y} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3 + \mu \hat{H}_u \cdot \hat{H}_d + \frac{1}{2}\mu' \hat{S}^2 + \xi \hat{S}$$

1. Solving domain wall and tadpole problems in Z_3-NMSSM.
2. Z_3-violating terms originate from unified theories with a Z_4^n or Z_8^n sym..
3. The $\xi \hat{S}$ term can be eliminated by field redefinitions.

GNMSSM: DM Mass and Couplings

Singlino-dominated DM:

- Neutralino mass matrix: $\mu_{eff} \equiv \frac{\lambda}{\sqrt{2}} v_s$, $\mu_{tot} \equiv \mu + \mu_{eff}$.

$$m_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u & 0 \\ -\frac{1}{2} g_1 v_d & \frac{1}{2} g_2 v_d & 0 & -\mu_{\mathbf{tot}} & -\frac{1}{\sqrt{2}} v_u \lambda \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & -\mu_{\mathbf{tot}} & 0 & -\frac{1}{\sqrt{2}} v_d \lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}} v_u \lambda & -\frac{1}{\sqrt{2}} v_d \lambda & \mathbf{m_N} \end{pmatrix}$$

Mass and couplings of the singlino-dominated DM are given by:

$$m_{\tilde{\chi}_1^0} \simeq m_N + \frac{1}{2} \frac{\lambda^2 v^2 (m_{\tilde{\chi}_1^0} - \mu_{tot} \sin 2\beta)}{m_{\tilde{\chi}_1^0}^2 - \mu_{tot}^2} \simeq m_N, \quad \mathbf{m_N} \equiv \sqrt{\mathbf{2}} \kappa \mathbf{v_s} + \mu',$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_i} = C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h_i}^{\mathrm{Z_3-NMSSM}}|_{\mu \to \mu_{tot}}, \qquad C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z} = C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z}^{\mathrm{Z_3-NMSSM}}|_{\mu \to \mu_{tot}}.$$

- DM properties are described by **five** independent parameters:

$$\tan\beta, \ \lambda, \ \kappa, \ \mu_{\mathrm{tot}}, \ and \ m_{\tilde{\chi}_1^0}. \qquad \mu_{\mathbf{tot}} : \textbf{Higgsino mass}.$$

Note:
Different from Z_3-NMSSM, $m_{\tilde{\chi}_1^0}$, λ, and κ are not correlated!

- In the limit $\lambda \to 0$, matrix decomposition: $5 \times 5 = 4 \oplus 1$, **decoupled**!

GNMSSM: Higgs Sector

Soft-breaking terms:

$$-\mathcal{L}_{soft} = \left[\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3} A_\kappa \kappa S^3 + m_3^2 H_u \cdot H_d + \frac{1}{2} {m'_S}^2 S^2 + h.c.\right] + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2.$$

CP-odd Higgs mass matrix in bases $(A_{\rm NSM}, {\rm Im}(S))$:

$$\mathcal{M}^2_{P,11} = \frac{2\left[\mu_{eff}(\lambda A_\lambda + \kappa\mu_{eff} + \lambda\mu') + \lambda m_3^2\right]}{\lambda \sin 2\beta} \equiv \mathbf{m_A^2},$$

$$\mathcal{M}^2_{P,22} = \frac{(\lambda A_\lambda + 4\kappa\mu_{eff} + \lambda\mu')\sin 2\beta}{4\mu_{eff}} \lambda v^2 - \frac{\kappa\mu_{eff}}{\lambda}(3A_\kappa + \mu') - \frac{\mu}{2\mu_{eff}}\lambda^2 v^2 - 2{m'_S}^2$$

$$\mathcal{M}^2_{P,12} = \frac{v}{\sqrt{2}}(\lambda A_\lambda - 2\kappa\mu_{eff} - \lambda\mu') \equiv \frac{\boldsymbol{\lambda} \mathbf{v}}{\sqrt{\mathbf{2}}}(\mathbf{A}_\lambda - \mathbf{m_N}).$$

- In the limit $\lambda \to 0$, matrix decomposition: $2 \times 2 = 1 \oplus 1$, **singlet decoupled**!
- m_A: heavy doublet mass scale, $m_B \equiv \sqrt{\mathcal{M}^2_{P,22}}$: CP-odd singlet Higgs mass.

$$m_3^2 = \frac{\lambda \mathbf{m_A^2} sin2\beta - 2\kappa\mu_{\rm eff}^2 - 2\lambda\mu_{\rm eff}\mu' - 2\lambda\mu_{\rm eff} A_\lambda}{2\lambda}$$

$$m'^2_S = -\frac{1}{2}\left[\mathbf{m_B^2} + \frac{\mu}{2\mu_{\rm eff}}\lambda^2 v^2 + \frac{\kappa\mu_{\rm eff}}{\lambda}(3A_\kappa + \mu') - \frac{(\lambda A_\lambda + 4\kappa\mu_{\rm eff} + \lambda\mu') sin2\beta}{4\mu_{\rm eff}}\lambda v^2\right]$$

GNMSSM: Higgs Sector

CP-even Higgs mass matrixin in bases $(H_{\rm NSM}, H_{\rm SM}, {\rm Re}[S])$**:**

$$\mathcal{M}^2_{S,11} = m_A^2 + \frac{1}{2}(2m_Z^2 - \lambda^2 v^2)\sin^2 2\beta,$$

$$\mathcal{M}^2_{S,12} = -\frac{1}{4}(2m_Z^2 - \lambda^2 v^2)\sin 4\beta,$$

$$\mathcal{M}^2_{S,13} = -\frac{1}{\sqrt{2}}(\lambda A_\lambda + 2\kappa\mu_{eff} + \lambda\mu')v\cos 2\beta \equiv -\frac{\lambda}{\sqrt{2}}(A_\lambda + m_N)v\cos 2\beta,$$

$$\mathcal{M}^2_{S,22} = m_Z^2\cos^2 2\beta + \frac{1}{2}\lambda^2 v^2\sin^2 2\beta,$$

$$\mathcal{M}^2_{S,23} = \frac{v}{\sqrt{2}}\left[2\lambda(\mu_{eff} + \mu) - (\lambda A_\lambda + 2\kappa\mu_{eff} + \lambda\mu')\sin 2\beta\right],$$

$$\equiv \frac{\lambda \mathbf{v}}{\sqrt{\mathbf{2}}}\left[\mathbf{2}\mu_{\mathbf{tot}} - (\mathbf{A}_\lambda + \mathbf{m_N})\sin\mathbf{2}\beta\right],$$

$$\mathcal{M}^2_{S,33} = \frac{\lambda(A_\lambda + \mu')\sin 2\beta}{4\mu_{eff}}\lambda v^2 + \frac{\mu_{eff}}{\lambda}(\kappa A_\kappa + \frac{4\kappa^2\mu_{eff}}{\lambda} + 3\kappa\mu') - \frac{\mu}{2\mu_{eff}}\lambda^2 v^2,$$

- In the limit $\lambda \to 0$, matrix decomposition: $3 \times 3 = 2 \oplus 1$, **singlet decoupled**!
- $m_C \equiv \sqrt{\mathcal{M}^2_{S,33}}$: CP-even singlet Higgs mass.

$$A_\kappa = \frac{\mathbf{m_C^2} + \frac{\mu}{2\mu_{\rm eff}}\lambda^2 v^2 - \frac{\lambda(A_\lambda+\mu')sin2\beta}{4\mu_{\rm eff}}\lambda v^2 - \frac{4\kappa^2}{\lambda^2}\mu_{\rm eff}^2 - \frac{3\kappa}{\lambda}\mu_{\rm eff}\mu'}{\frac{\mu_{\rm eff}}{\lambda}\kappa}$$

GNMSSM: Input Parameters

Input parameters in the original Lagrangian:

- Soft-breaking masses: $m^2_{H_u}$, $m^2_{H_d}$, and m^2_S;
- Yukawa couplings in Higgs sector: λ and κ;
- Soft-breaking trilinear coefficients A_λ and A_κ;
- Bilinear mass parameters μ and μ', and their soft-breaking parameters m^2_3 and ${m'_S}^2$.

Physical inputs: λ, κ, $\tan\beta$, v_s, $m_{H^\pm}$, m_{h_s}, m_{A_s}, $m_{\tilde{\chi}^0_1}$, and μ_{tot}.

- Vacuum expectation values: v_u, v_d, v_s;
- Yukawa couplings in Higgs sector: λ and κ;
- Electroweakino masses: $m_{\tilde{\chi}^0_1} \simeq m_N$, and Higgsino mass μ_{tot};
- Higgs boson masses: $m^2_{H^\pm} \simeq m^2_A$, $m_{A_s} \simeq m_B$, and $m_{h_s} \simeq m_C$;
- Soft-breaking trilinear coefficients A_λ, **which is an insensitive parameter for all observables.**

GNMSSM: Key Features

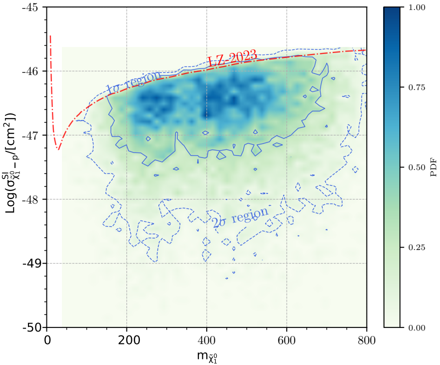
Important conclusions on the Singlino-dominated DM:

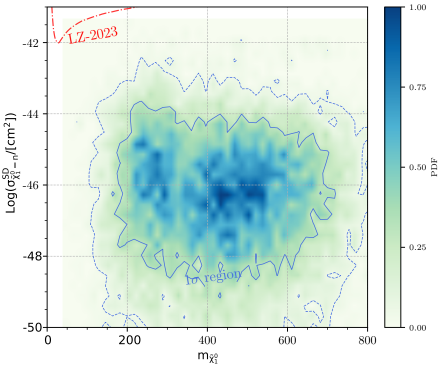
- **Bayesian analyses (assuming all inputs flat distributed):**
  DM is primarily preferred to be Singlino-dominated.
- **Singlet-dominated particles form a secluded DM sector:**
  Measured DM abundance is generated by
  - s**-wave process** $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s A_s$**, occuring by** s**-channel exchange of Higgs bosons and** t**-channel exchange of neutralinos**:

$$|C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_s}| = |C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A_s}| = \sqrt{2}\kappa \simeq 0.21 \times \left(\frac{m_{\tilde{\chi}_1^0}}{300\ \text{GeV}}\right)^{1/2};$$

  - p**-wave process** $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s h_s, A_s A_s$**, via adjusting** κ;
  - h_s/A_s**-funnels, via adjusting** m_{h_s}/m_{A_s}.
- **DM-nucleon scatterings suppressed by** λ^4**:**
  Current LZ experiment requires $\lambda \lesssim 0.1$**. Future DD expt. will further suppress** λ**, but not affect GNMSSM phenomenology.**

GNMSSM: Key Features





Assuming $\lambda = 1/(16\pi^2)$ and $\mu_{\rm tot} = 500$ GeV, we conclude

$$\sigma^{\rm SI}_{\tilde{\chi}_1^0-N} \sim (10^{-48} - 10^{-52})\ {\rm cm}^2 \text{ depending on } m_{h_s}.$$

GNMSSM: Other Distinct Features

Characteristics:

1. **Free from the domain wall and tadpole problems**;
2. **More stable vacuum than the MSSM**;

$$V_{\rm min}^{\rm des} = \cdots - \frac{\kappa^2}{\lambda^4}\mu_{eff}^4 - \frac{1}{3}\frac{\kappa A_\kappa}{\lambda^3}\mu_{eff}^3.$$

3. **Significant alleviation of the LHC constraints**.
   Heavy sparticles prefer to decay into NLSP or NNLSP first. Their decay chains are thus lengthened and their decay products become more complex.
4. **Every EW parameter takes natural values**.
   Considering LZ + LHC + a_μ,
   Z_3-NMSSM: $m_{\tilde{\chi}_1^0} \gtrsim 260\text{GeV}, \ \mu \gtrsim 550\text{GeV}, \ v_s \gtrsim 2$ TeV;
   GNMSSM: $m_{\tilde{\chi}_1^0} \gtrsim 100\text{GeV}, \ \mu_{tot} \gtrsim 200\text{GeV}, \ v_s < 1$ TeV.
5. **Bayesian evidence is much larger than that of Z_3-NMSSM**.

Technical Support

Results are based on global fits of supersymmetric theories.

1. SARAH suite for calculation.
   - Model building: SARAH-4.14.3;
   - Spectrum generator: SPheno-4.0.4;
   - DM physics calculator: MicrOMEGAs-5.0.4;
   - Higgs physics calculator: HiggsSingal-2.6.2, HiggsBounds-5.10.2;
   - Flavor physics calculator: FlavorKit;
   - MC simulation: MadGraph_aMC@NLO, PYTHIA8, and Delphes;
   - LHC SUSY search: SModelS-2.1.1, CheckMATE-2.0.29.
2. Scan strategy: parallel MultiNest algorithm.
   High performance:
   Simultaneous computation of more than 10^6 programes.
3. Members of the developers for the package CheckMATE.
   Reproduce more than 70 experimental analyses.
4. Specially designed clusters → **Different from GUM**
   IB Network, Data transition speed: 200G/s.

Technical Support

Table 1: Experimental analyses of the electroweakino production processes.

Scenario	Final State	Name
$\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to WZ\tilde{\chi}_1^0\tilde{\chi}_1^0$	$n\ell(n\geq 2)+nj(n\geq 0)+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	CMS-SUS-20-001($137fb^{-1}$) ATLAS-2106-01676($139fb^{-1}$) CMS-SUS-17-004($35.9fb^{-1}$) CMS-SUS-16-039($35.9fb^{-1}$) ATLAS-1803-02762($36.1fb^{-1}$) ATLAS-1806-02293($36.1fb^{-1}$)
$\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to \ell\tilde{\nu}\ell\tilde{\ell}$	$n\ell(n=3)+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	CMS-SUS-16-039($35.9fb^{-1}$) ATLAS-1803-02762($36.1fb^{-1}$)
$\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to \tilde{\tau}\nu\ell\tilde{\ell}$	$2\ell+1\tau+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	CMS-SUS-16-039($35.9fb^{-1}$)
$\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to \tilde{\tau}\nu\tilde{\tau}\tau$	$3\tau+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	CMS-SUS-16-039($35.9fb^{-1}$)
$\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to Wh\tilde{\chi}_1^0\tilde{\chi}_1^0$	$n\ell(n\geq 1)+nb(n\geq 0)+nj(n\geq 0)+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	ATLAS-1909-09226($139fb^{-1}$) CMS-SUS-17-004($35.9fb^{-1}$) CMS-SUS-16-039($35.9fb^{-1}$) ATLAS-1812-09432($36.1fb^{-1}$) CMS-SUS-16-034($35.9fb^{-1}$) CMS-SUS-16-045($35.9fb^{-1}$)
$\tilde{\chi}_1^\mp\tilde{\chi}_1^\pm \to WW\tilde{\chi}_1^0\tilde{\chi}_1^0$	$2\ell+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	ATLAS-1908-08215($139fb^{-1}$) CMS-SUS-17-010($35.9fb^{-1}$)
$\tilde{\chi}_1^\mp\tilde{\chi}_1^\pm \to 2\tilde{\ell}\nu(\tilde{\nu}\ell)$	$2\ell+\mathrm{E}_\mathrm{T}^{\mathrm{miss}}$	ATLAS-1908-08215($139fb^{-1}$) CMS-SUS-17-010($35.9fb^{-1}$)

GNMSSM: Possible problems and solutions

Problems:

- What's the origin of the S field?
- Why is λ small? Are there other reasons than the Higgs data?
- Which value A_λ is preferred?
- What's the origin of neutrino mass?

Possible solutions: Seesaw Extended MRSSM, B-L NMSSM, $\cdots$
R-symmetry: the largest subgroup of automorphism group of supersymmetry algebra which commutes with Lorenz group.

R-symmetry + Seesaw mechanism!

Secluded DM sector: $\hat{S}$ and $\hat{\nu}_R$ form ..., Higgs or Neutrino Portal.
$\tilde{S}\tilde{S} \to \nu_R \bar{\nu}_R$ or $\tilde{\nu}_R \tilde{\nu}_R \to SS$.

Crucial characteristic of R-symmetric SUSY: Super-safe

SM: $m_h^2 = m_h^2|_{tree} + \delta m_h^2 = m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}\Lambda^2;$

MSSM:
$$m_h^2 = m_h^2|_{tree} + \delta m_h^2$$
$$\simeq m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}$$
$$\simeq m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}\frac{2g_s^2}{3\pi^2}m_{\tilde{g}}^2\left(\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}\right)^2;$$

since $\delta m_{\tilde{t}_i}^2 \simeq \frac{g_s^2}{3\pi^2}m_{\tilde{g}}^2\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}.$

MRSSM: Soft-breaking terms → Supersoft opterators

$$m_h^2 = m_h^2|_{tree} + \delta m_h^2$$
$$\simeq m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)\log\frac{M_3^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}};$$

$\delta m_{\tilde{t}_i}^2 = \frac{g_s^2}{3\pi^2}m_{\tilde{g}}^2\log\frac{\tilde{m}^2}{m_{\tilde{g}}^2}$, $\tilde{m}$: Sgluon mass.

$m_Z^2 = a_1 M_3^2 + a_2 m_{Q_3}^2 + a_3 m_{U_3}^2 - 2\mu^2 + \cdots$

a_i: one-loop or two-loop suppressed, no log enhancement.

μ: irrelevant to DM physics, maybe light without conflicting with the LHC restrictions.

Conclusion

1. **The so-called WIMP crisis just means that the simplest realizations of the WIMP miracle are facing challenges** $\longrightarrow$ More elaborate theories are encouraged.
2. **Occam razor was incorrectly applied to the NMSSM**. Specifically, the Z_3-NMSSM is too restricted to exhibit all the essential characteristics of the NMSSM.
3. **It is time to explore the phenomenon of GNMSSM**, which is one of the simplest supersymmetric theories to naturally coincide with current experiments.
   - **Singlino DM is primarily preferred by Bayesian statistics!**
   - **The GNMSSM has many distinct theoretical advantages!**
4. Seemingly independent problems may have common physical origins! DM correlates with SUSY search!
   Go forward to explore them with fancy ideas and more sophisticated techniques.

thanks!