Projections of Discovery Potentials of $0\nu\beta\beta$ Experiments

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OUTLINE:
Statistical concerns on $0\nu\beta\beta$ sensitivities
(ref : arXiv:2308.07049)

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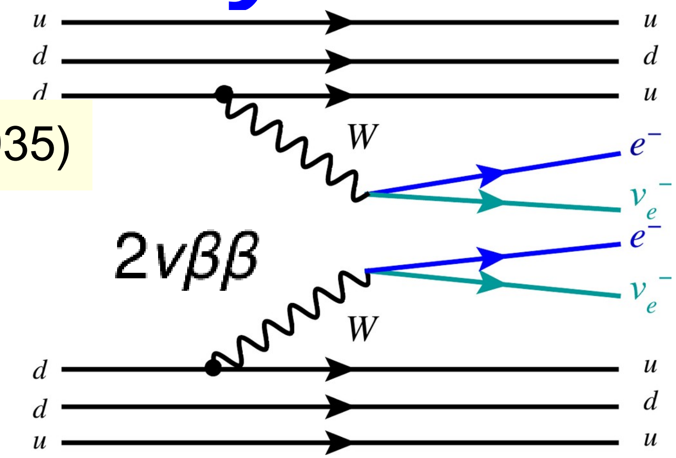
double beta decay

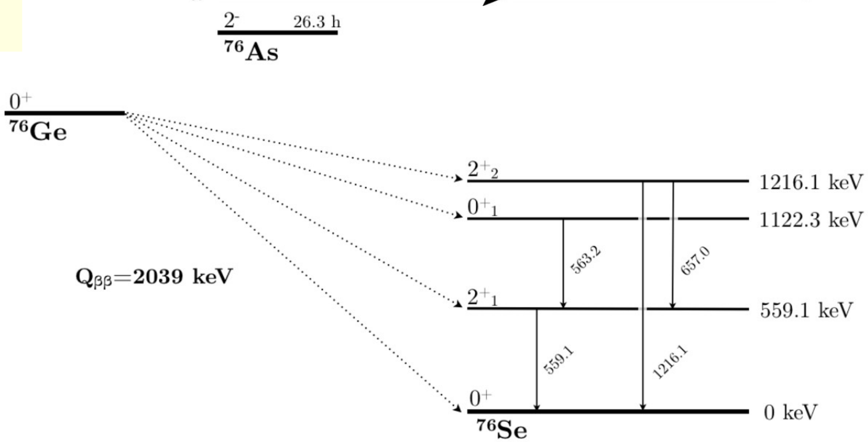
$2\nu\beta\beta: \quad {}^{N}_{Z}A_{\beta\beta} \rightarrow {}^{N-2}_{Z+2}A + 2e^{-} + 2\bar{\nu}_e$ (Goeppert-Mayer, 1935)

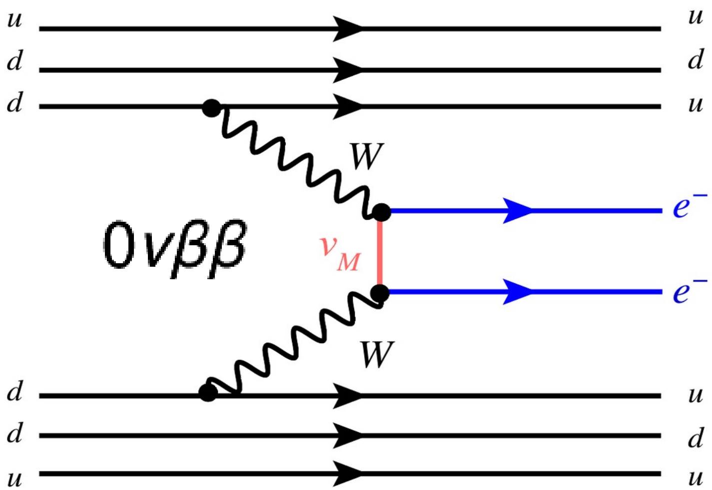
- a second-order weak process
- detectable if 1st order β-decay is forbidden
- in SM for even-N even-Z nuclei:

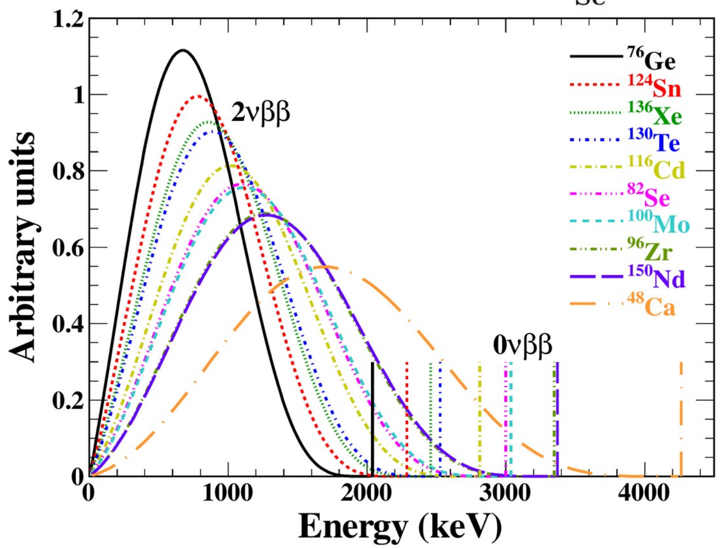


$0\nu\beta\beta: \quad {}^{N}_{Z}A_{\beta\beta} \rightarrow {}^{N-2}_{Z+2}A + 2e^{-}$ (Furry, 1939)

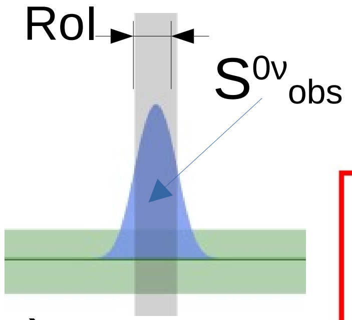
- forbidden in Standard Model
- never been observed, rare.
- only way to detect Majorana neutrino
- $\langle m_{\beta\beta} \rangle \neq 0$, and lepton number violation
- signature: mono-energetic peak at $Q_{\beta\beta}$







half-life of 0νββ



measure half-life of 0νββ :

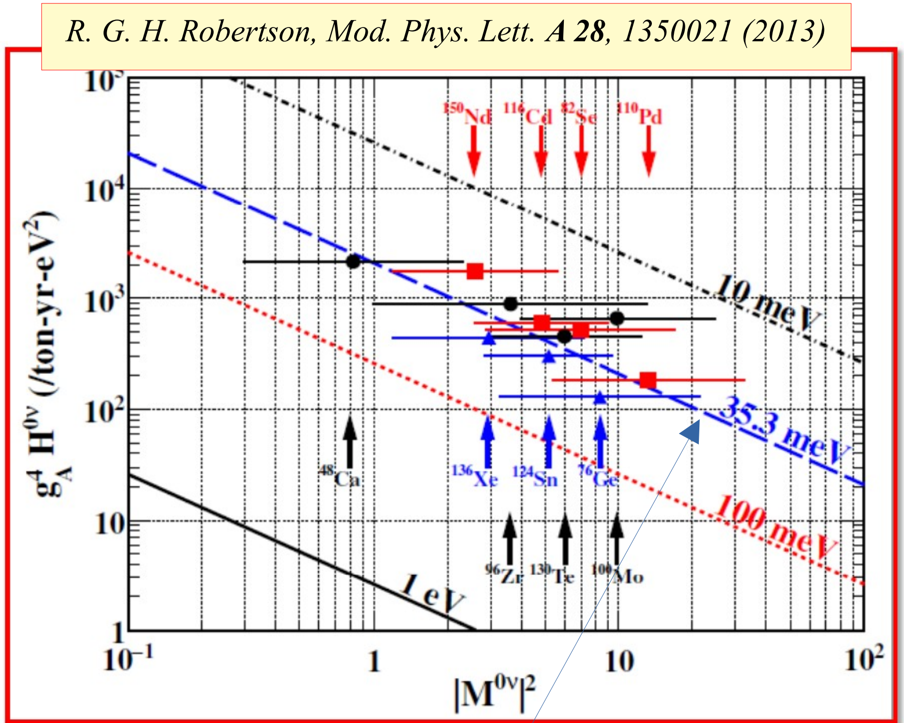
$$T^{0\nu}_{1/2} = ln2\cdot\left[\frac{N_A}{N+Z}\right]\cdot\Sigma\cdot\left[\frac{\varepsilon_{RoI}}{\boxed{S^{0\nu}_{obs}}}\right]$$

N_A : Avogadro Number

∑ : exposure (livetime×detector-mass)

ε_{RoI} : fraction of signal (0νββ) region in data analysis

$S^{0\nu}_{obs}$: measured signal counts



relation with 〈 $m_{\beta\beta}$ 〉 :

axial vector coupling constant

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 |M^{0\nu}|^2 \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$$

phase space factor

nuclear matrix element

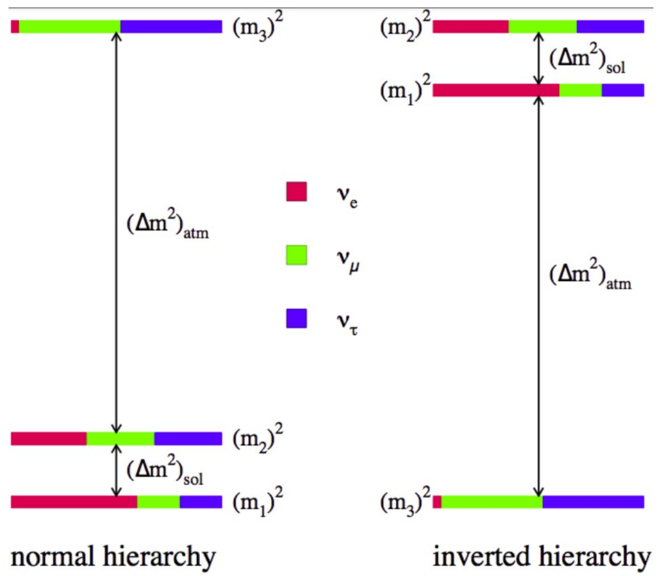
in this report:
$G^{0\nu} g^4_A |M^{0\nu}|^2$ is fixed
to this line
($H^{0\nu} = \ln 2[N_A/(A\cdot m_e^2]G^{0\nu}$)

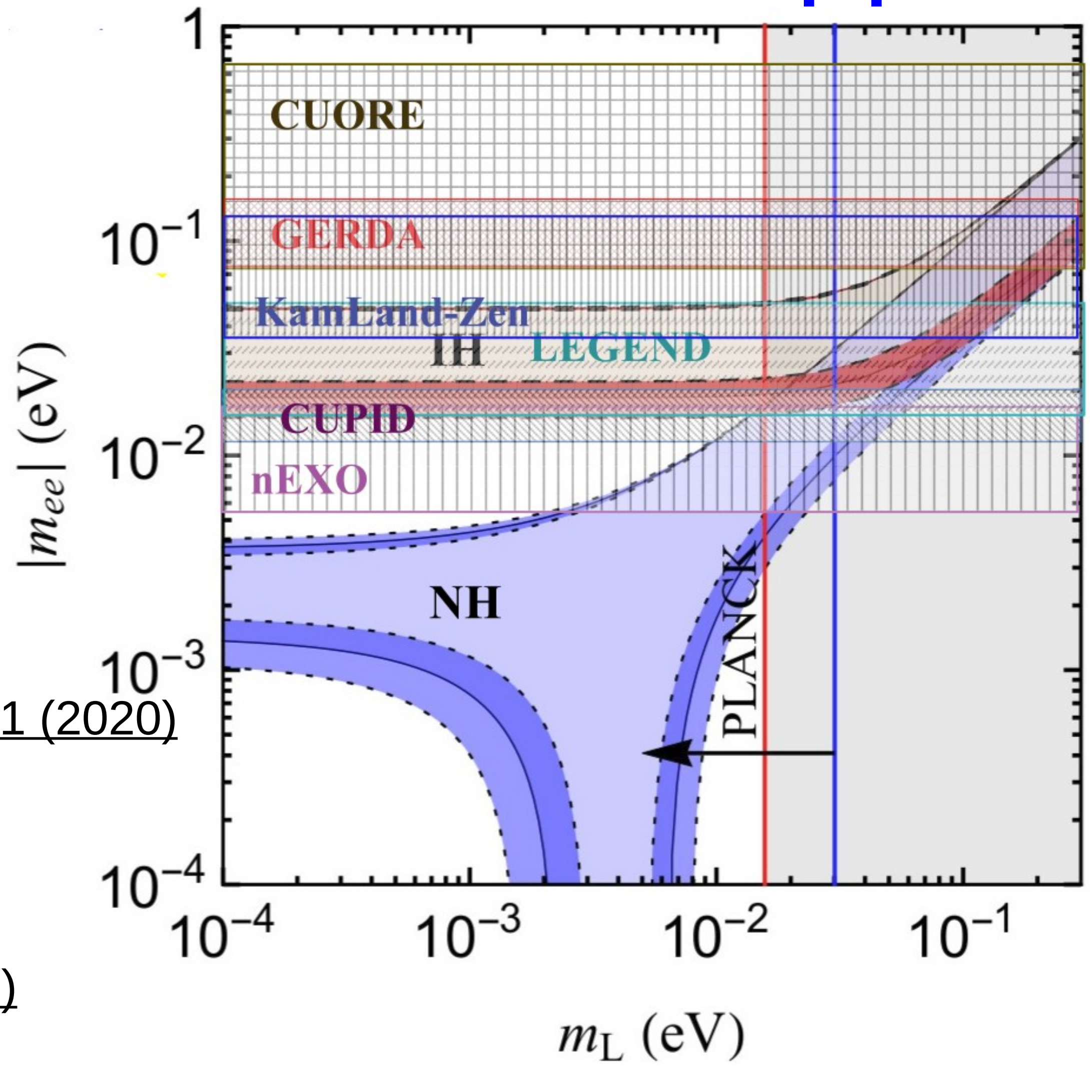
effective Majorana mass:

$$\langle m_{\beta\beta}\rangle = |U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\alpha} + U_{e3}^2 m_3 e^{i\beta}|$$

|Uei2| : neutrino mixing matris (PMNS)

current/projected sensitivities on 0νββ



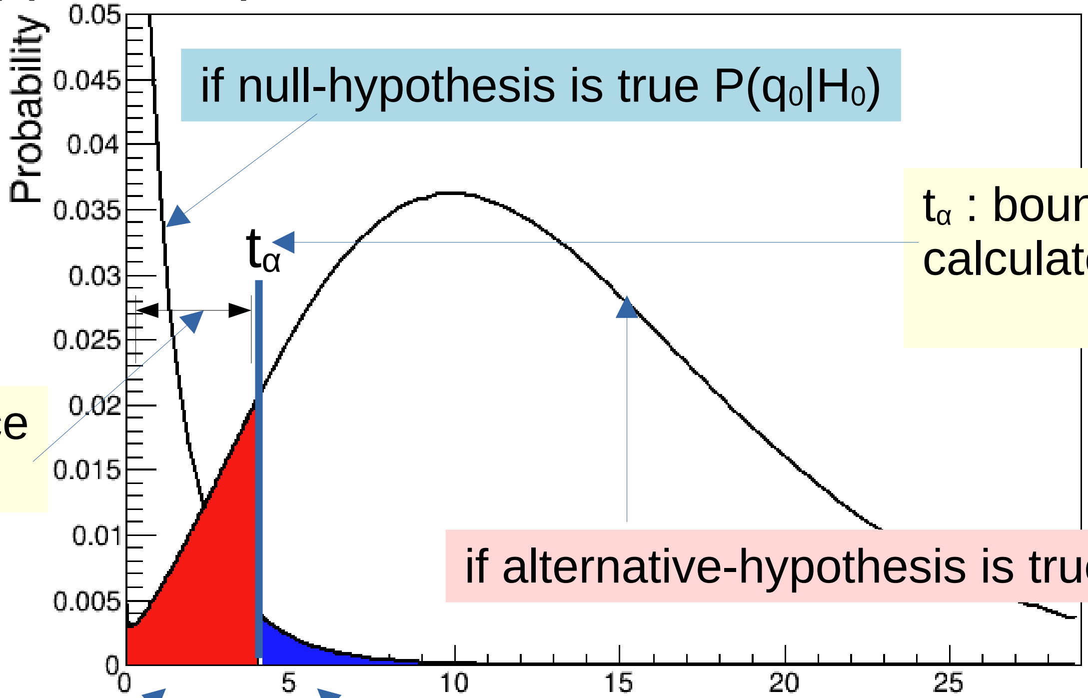


Achieved:
Cryogenic Underground Observatory for
Rare Events (CUORE) 75-350 meV; PRL 124, 122501 (2020)
GERmanium Detector Array (GERDA)
79-180 meV; PRL 125, 252502 (2020)
Kamioka Liquid scintillator Anti-Neutrino Detector
- Zero neutrino double-beta decay experiment
(KamLand-Zen)36-156 meV; PRL 130, 051801 (2023)
Projected:
Large Enriched Germanium Experiment for Neutrinoless ββ Decay (LEGEND)
15-50 meV; AIP Conference Proceedings 1894, 020027 (2017)
CUORE Upgrade with Particle Identification (CUPID)
12-20 meV; arXiv:1907.09376v1
next Enriched Xenon Observatory (nEXO)
5.7-17.7 meV; PRC 97, 065503 (2018)

hypothesis test

what is $T^{0\nu}_{1/2}$ (or $S^{0\nu}_{obs}$) to define positive claim of 0νββ?

→ frequentist approach, profile likelihood ratio as test-statistic.



if null-hypothesis is true $P(q_0|H_0)$

t_α : boundary of acceptance region calculated from user defined α

acceptance region

if alternative-hypothesis is true $P(q_0|H_1)$

test statistic $q_0 = -2\,ln[\frac{L(S=0,\hat{\hat{B}},\hat{\hat{\nu}})}{L(\hat{S},\hat{B},\hat{\nu})}]$

$$\beta \equiv \int_0^{t_\alpha} P(q_0|H_1)\; dq_0$$

$$\alpha \equiv \int_{t_\alpha}^{\infty} P(q_0|H_0)\; dq_0$$

type-II error
search for hypothesis
of certain β

p-value
type-I error
user defined

Neyman-Pearson Lemma:
for every choice of α,
likelihood ratio minimize β

in this report we adopted α=0.00135 (3σ) for positive claim and β=0.5.
→ alternative-hypothesis that has 50% of chance to see >3σ effect
(equivalent to 50% of the best-fit S with 3σ-error-bar not include 0)

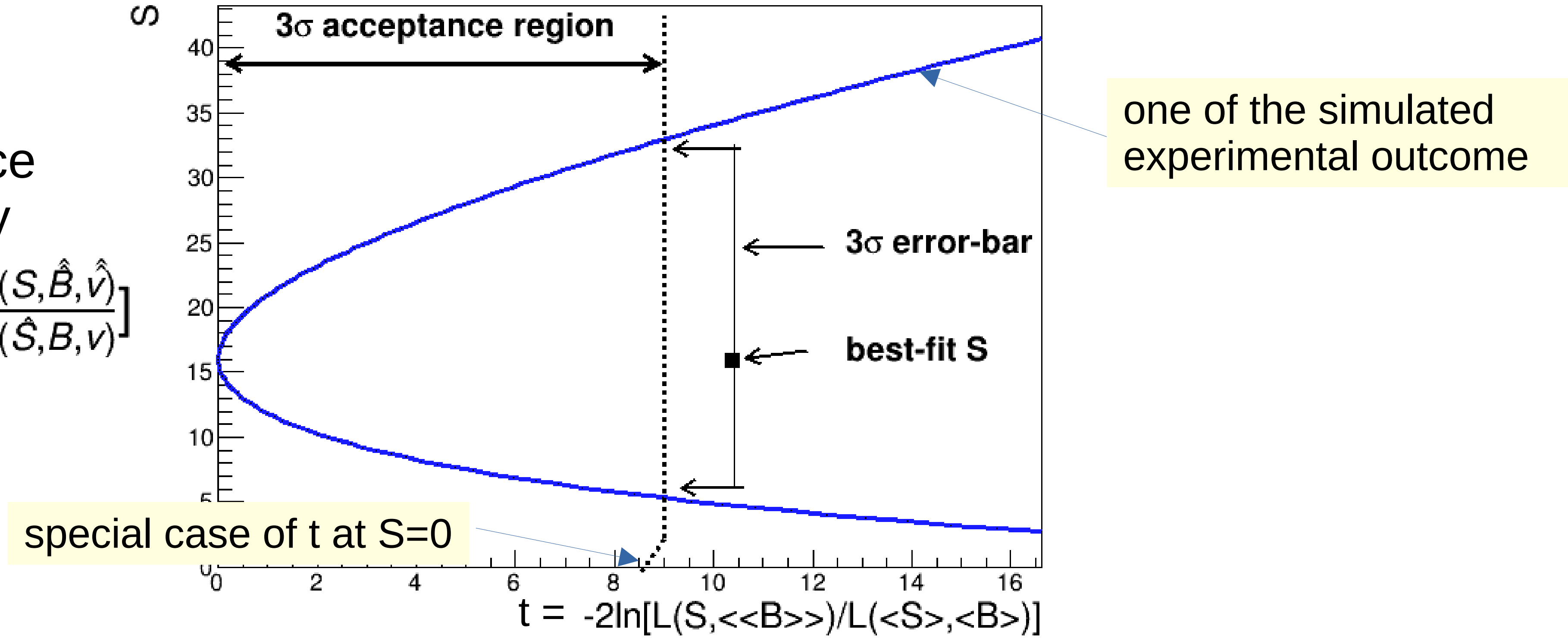
$P^{3\sigma}_{50}$ criteria

Neyman construstion

in frequentist statistic, C.I. is defined by cooverage:

acceptance defined by

$$t = -2\,ln[\frac{L(S,\hat{\hat{B}},\hat{\hat{\nu}})}{L(\hat{S},B,\nu)}]$$



the $P^{3\sigma}_{50}$ criteria:

$$\alpha \equiv \int_{t_\alpha}^{\infty} P(q_0|H_0)\; dq_0 \quad = 0.00135\ (1-P_{3\sigma})$$

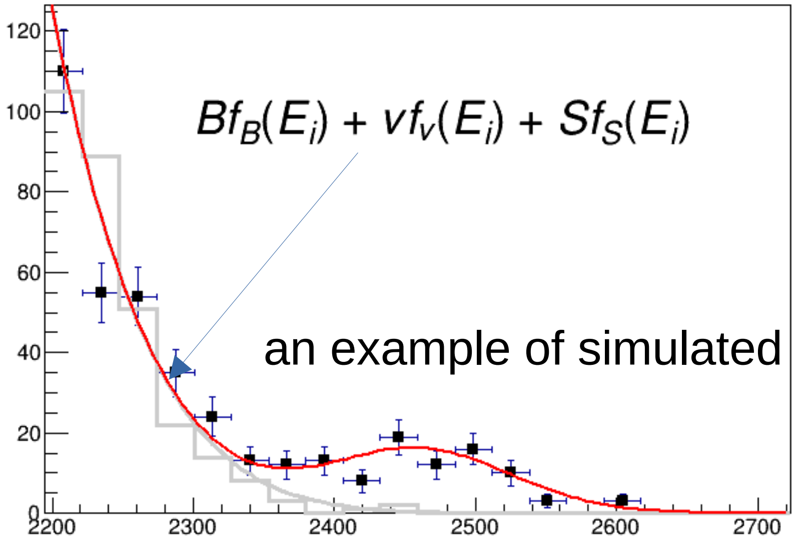
$$\beta \equiv \int_{0}^{t_\alpha} P(q_0|H_1)\; dq_0 \quad = 0.5$$

is equivalent to 50% of the outcomes (of alternative hypothesis) have 0 outside 3σ-error-bar.

likelihood function

$$L(S, B, \nu | N, E_i) = \frac{e^{-(B+\nu+S)}(B+\nu+S)^N}{N!} \prod_{i=0}^{N} \left[\frac{Bf_B(E_i)+\nu f_\nu(E_i)+Sf_S(E_i)}{B+\nu+S}\right] \cdot F_{sys.}(\ldots)$$

simulated data, MC for N and E_i



S : expected 0νββ count
B : expected flat-bkg count
ν : expected 2νββ count

f_B, f_ν, f_S : normalized spectrum

$(\hat{S}, \hat{B}, \hat{\nu})$: maximize L

$(S=0, \hat{\hat{B}}, \hat{\hat{\nu}})$: maximize L at null-hypothesis

test-statistic: likelihood ratio $q_0 = -2 ln\left[\frac{L(S=0,\hat{\hat{B}},\hat{\hat{\nu}})}{L(\hat{S},\hat{B},\hat{\nu})}\right]$

Feldman & Cousin, PRD 1998

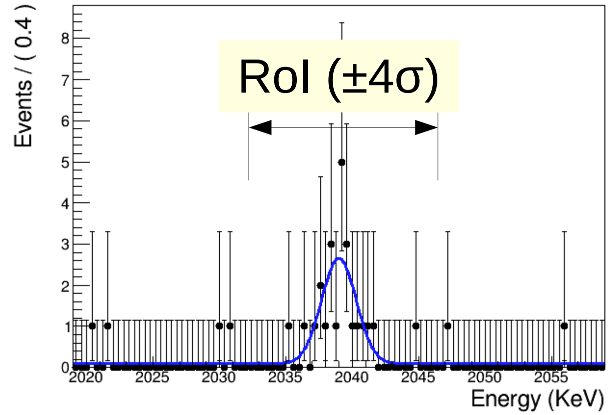
$P(q_0|H_0)$ and $P(q_0|H_1)$

H_0 : data generated with S=0
H_1: data generated with S=some number (>0)

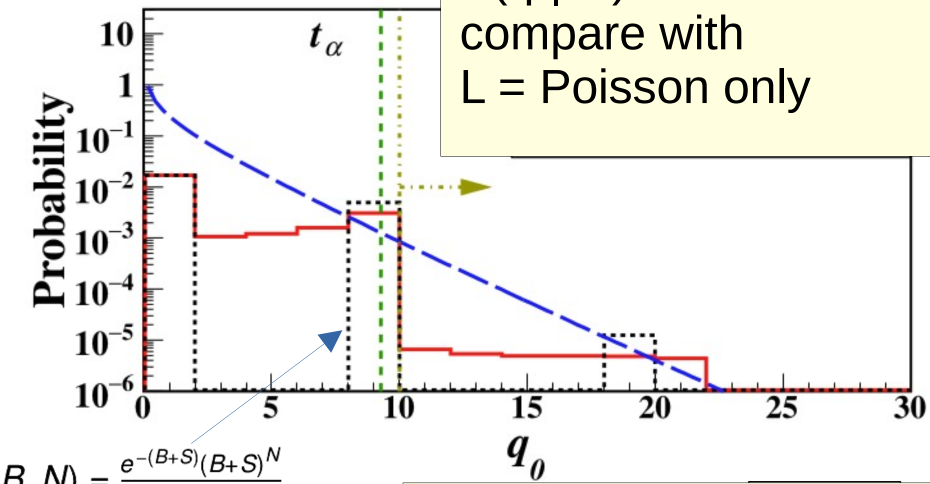
flat background only

$$L(S|B,N,E_i) = \frac{e^{-(B+S)}(B+S)^N}{N!}\prod_{i=0}^{N}\left[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}\right]$$

S: expected 0νββ count in RoI
B: expected background count in RoI
Choice of RoI : cover whole Gaussian

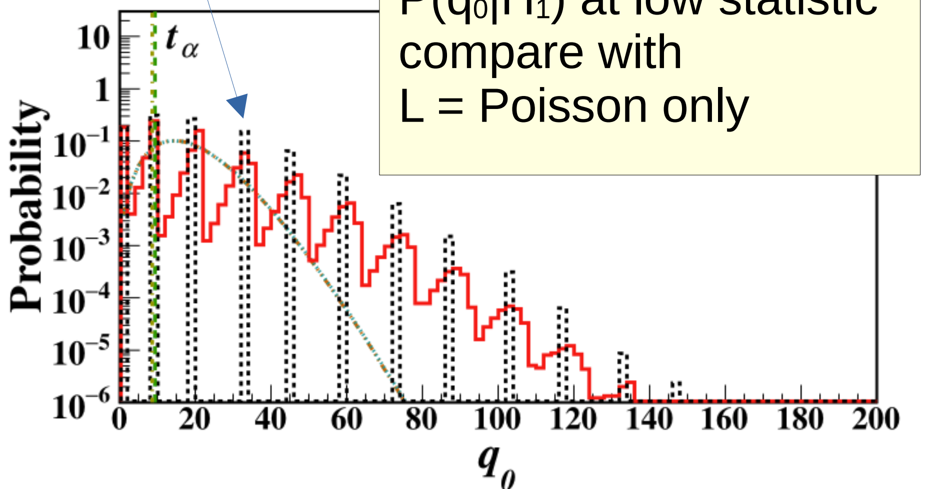


P(q_0|H_0) at low statistic
compare with
L = Poisson only



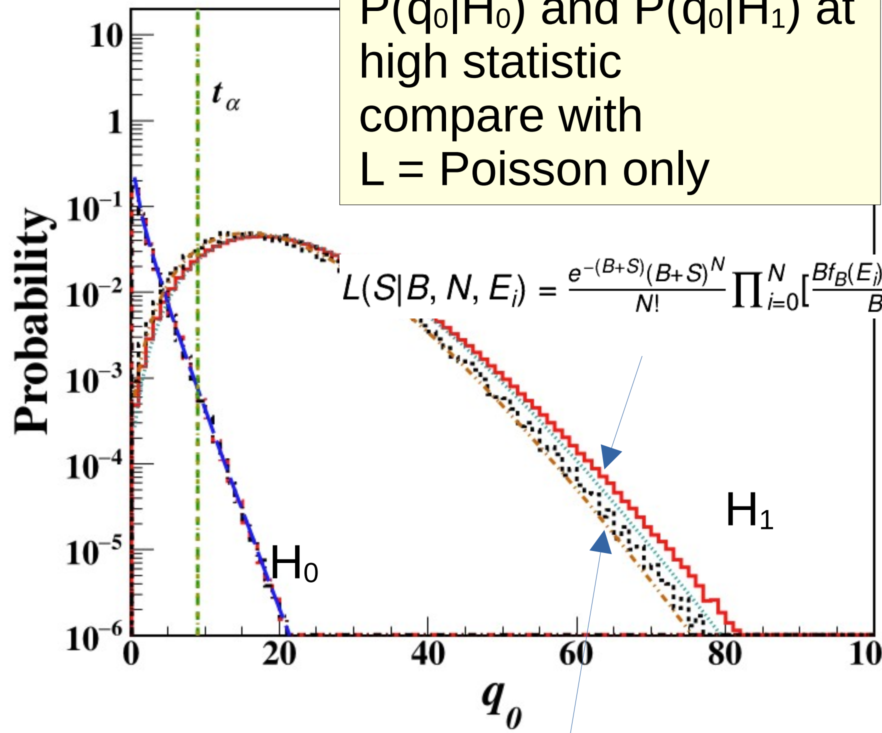
$$L(S|B,N) = \frac{e^{-(B+S)}(B+S)^N}{N!}$$

P(q_0|H_1) at low statistic
compare with
L = Poisson only



P(q_0|H_0) and P(q_0|H_1) at
high statistic
compare with
L = Poisson only

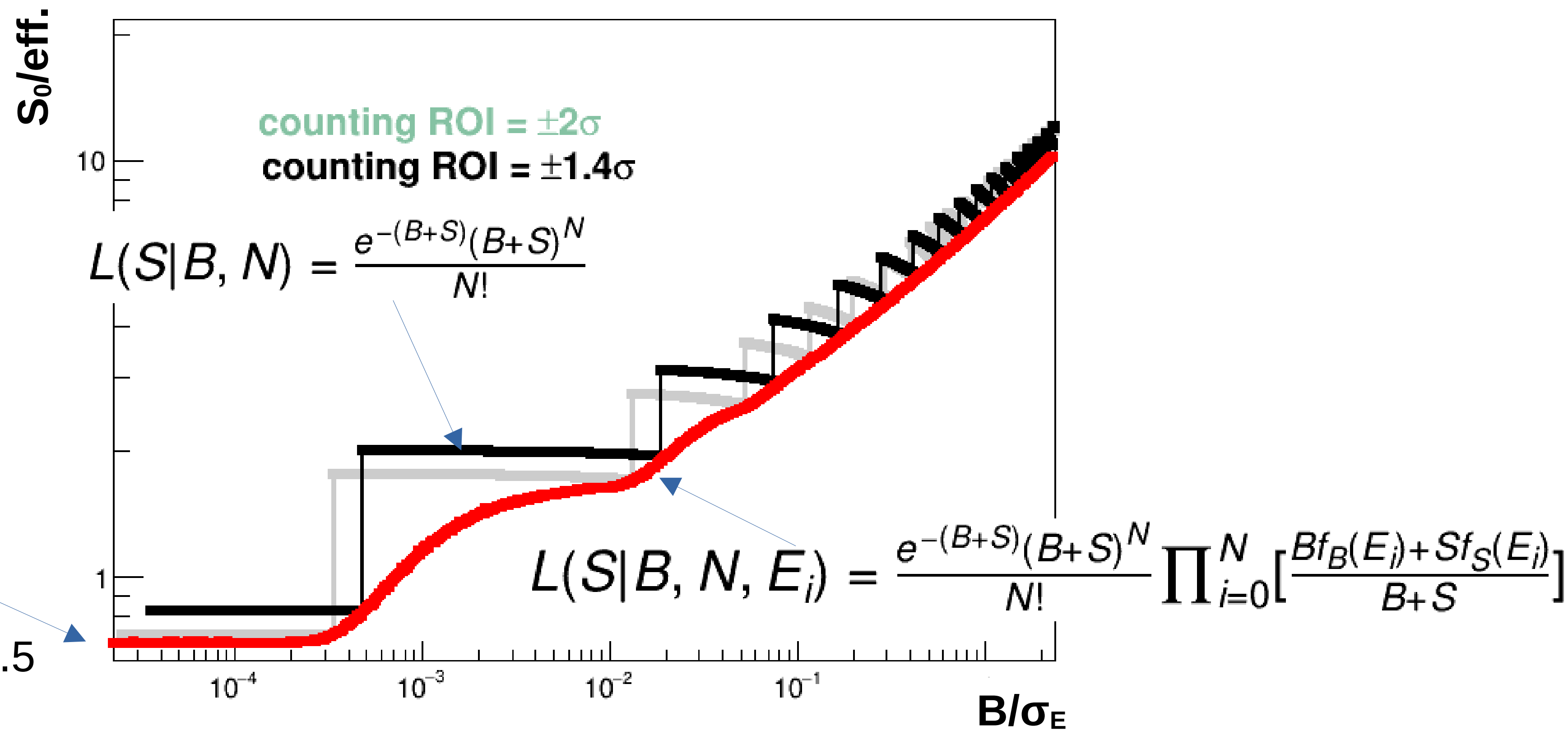
$$L(S|B,N,E_i) = \frac{e^{-(B+S)}(B+S)^N}{N!}\prod_{i=0}^{N}\left[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}\right]$$



$$L(S|B,N) = \frac{e^{-(B+S)}(B+S)^N}{N!}$$

0νββ sensitivities

eff. : area of Gaussian in RoI



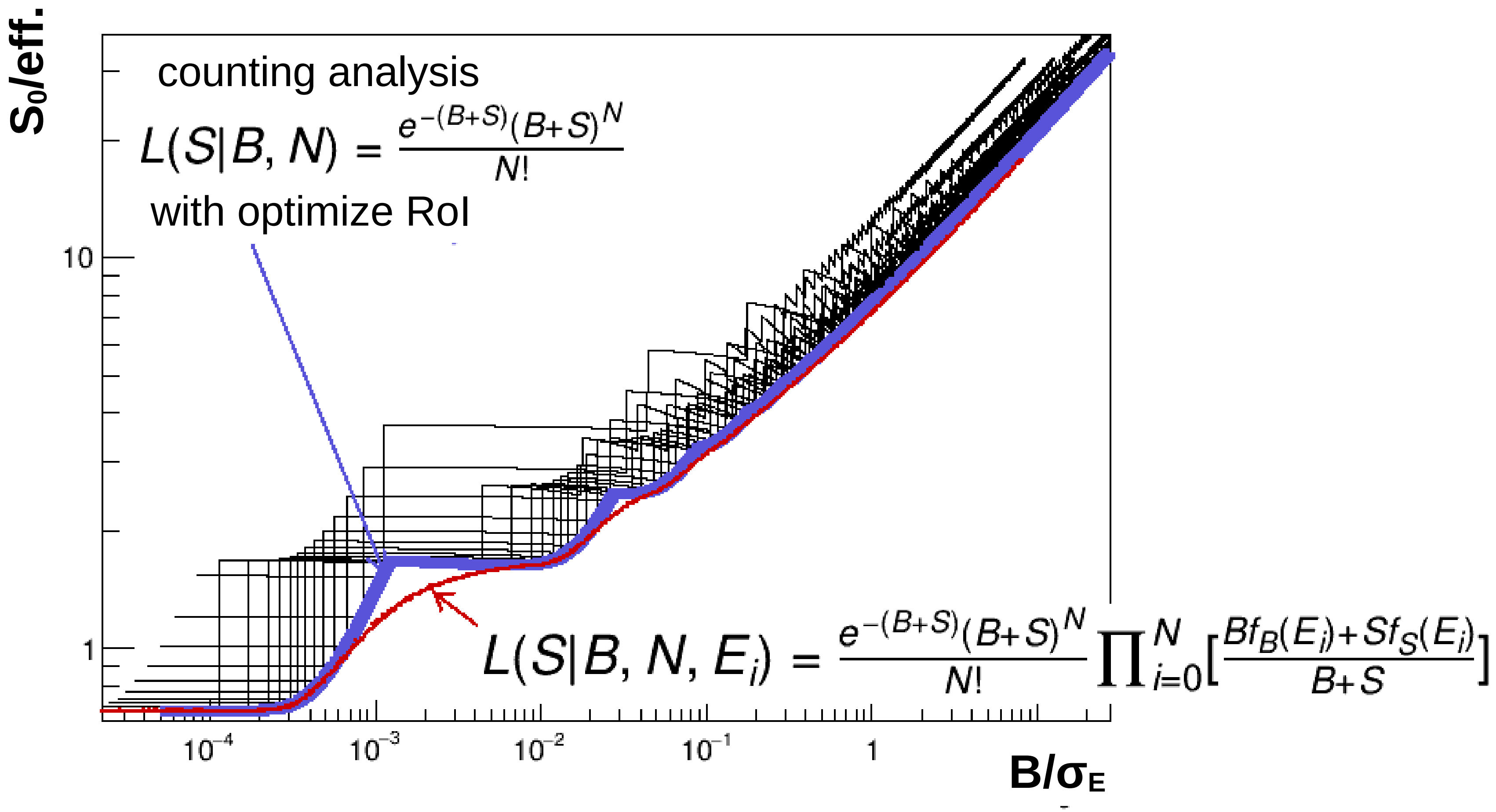
0.69

Pois(0|0.69) = 0.5

with energy information (E_i), sensitivities (50% >3σ) of signal strength is independent of RoI, depend on B/σ_E only.
(σ_E : energy resolution of detector)

for L = Poisson only, sensitivities depend on RoI of choice
suffer from "over-coverage" (=3σ is not always allowed for discreteness)

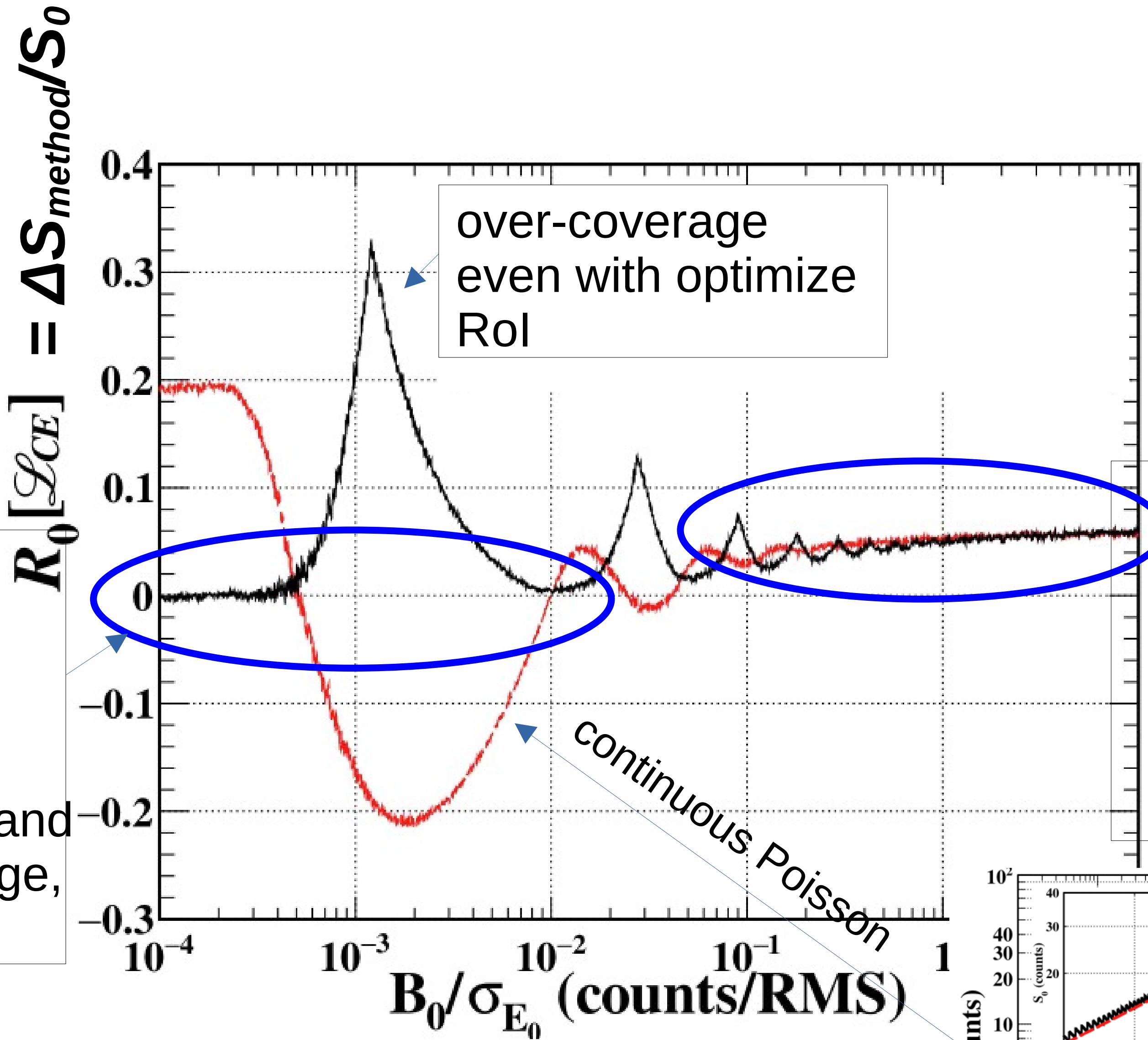
counting analysis



counting analysis : smaller RoI → smaller B → but suffer eff. lost

energy information : solve "over-coverage" problem of counting at low B
increase sensitivities at high B

effect of energy information



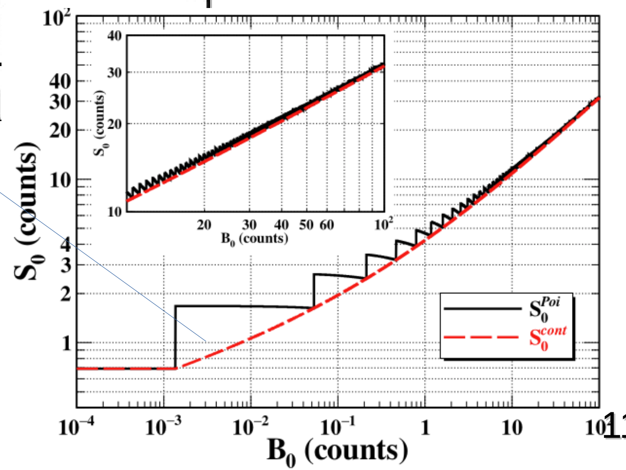
over-coverage even with optimize RoI

additional $\prod_{i=0}^{N}[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}]$ term improve the sensitivities by smear-out $P(q_0|H_x)$ and "solve" over-coverage, but still persist.

$\prod_{i=0}^{N}[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}]$

knowledge of the energy improve rhe sensitivities

continuous Poisson



Wilk and Wald

According to Wilk 1938, Walk 1943, at high statistic, and S away from boundary $P(q_0|H_0)$ and $P(q_0|H_1)$ could be approximated by:

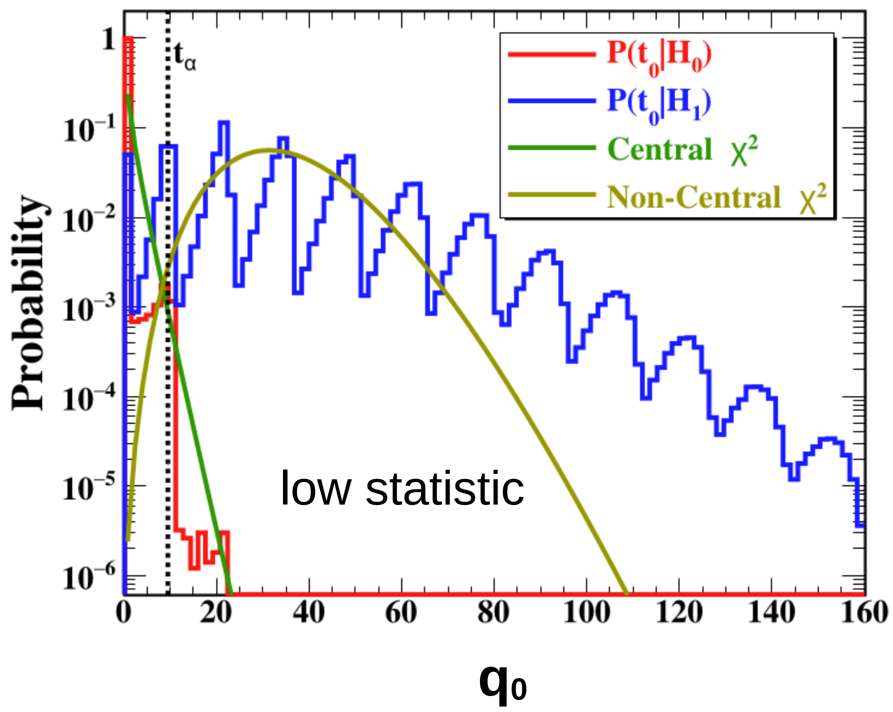
$$P(q_0|H_0) \approx \frac{1}{2}\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}e^{-q_0/2}$$

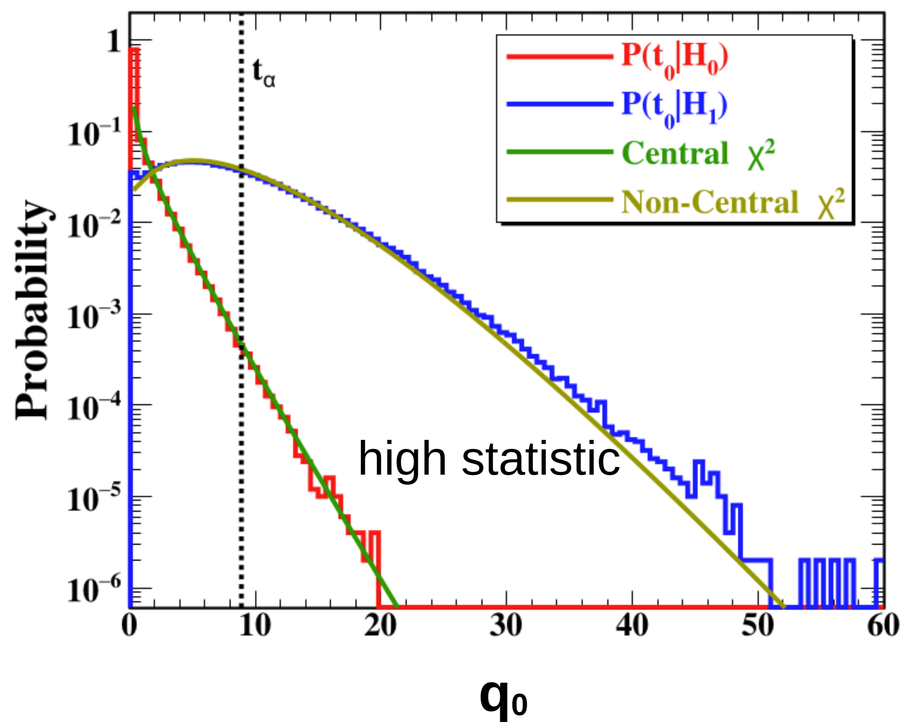
χ^2-distribution

$$P(q_0|H_1) \approx (1-\Phi(\sqrt{\Lambda}))\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}e^{-(\sqrt{q_0}-\sqrt{\Lambda})^2/2}$$

non-center χ^2-distribution

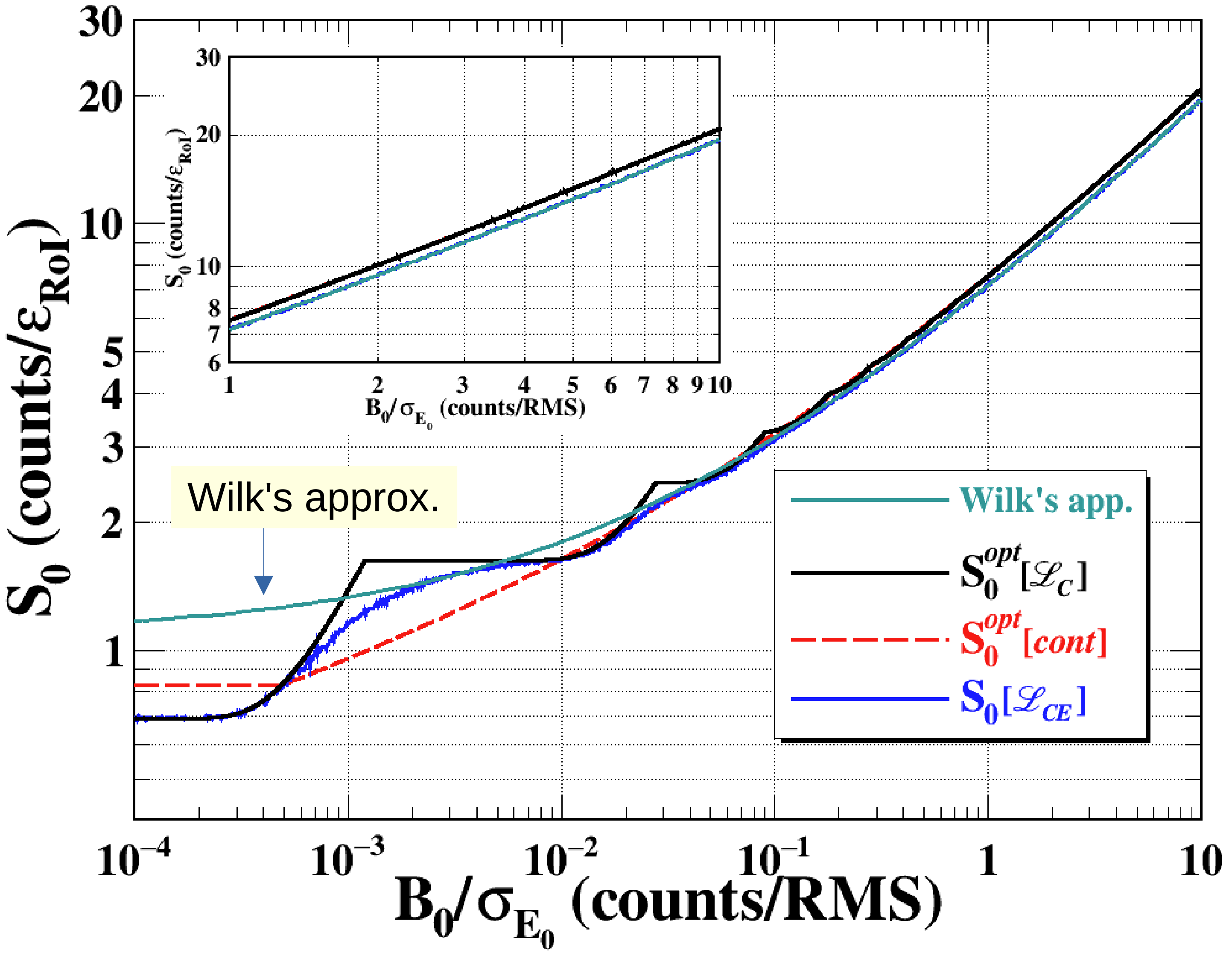
the non-center parameter Λ : q_0 of most probable (Asimov) data-set.





Approximation at large B

Large B and S required a lot of PC time to get sensitivities, Wilk's approximation match well with simulation.

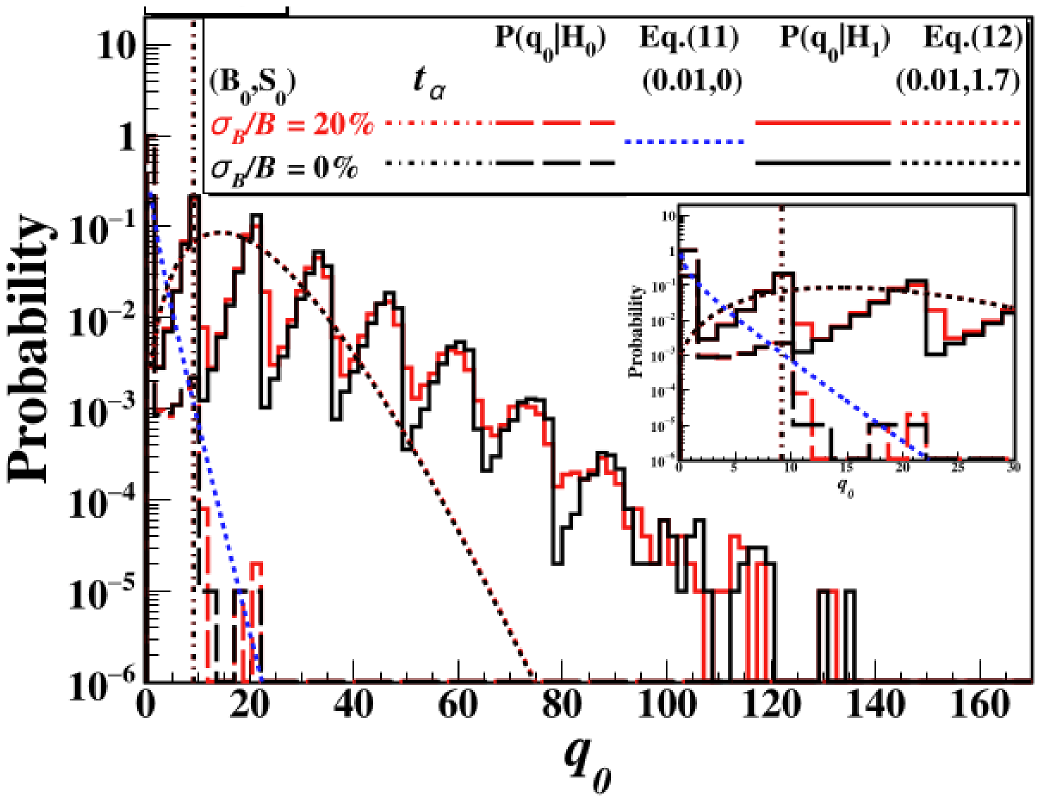


uncertainties of B

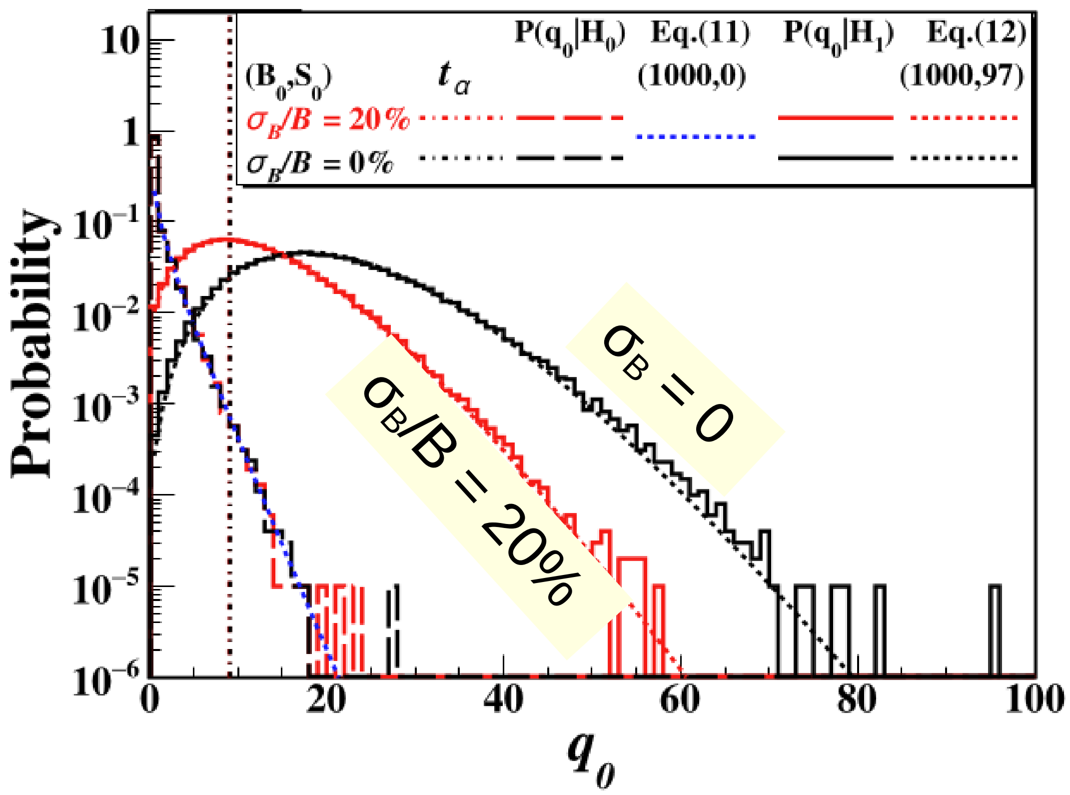
$$L(S|B,N,E_i) = \frac{e^{-(B+S)}(B+S)^N}{N!}\prod_{i=0}^{N}\left[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}\right]\times\boxed{\frac{e^{-(\tau B)}(\tau B)^{n_0}}{n_0!}}$$

uncertainties of B
$\sigma_B = \sqrt{(\tau B)} / \tau$ → alternative measurement
τ: scale of data size from main channel

n_0 is a simulated number (as N, E_i) in hypothesis test



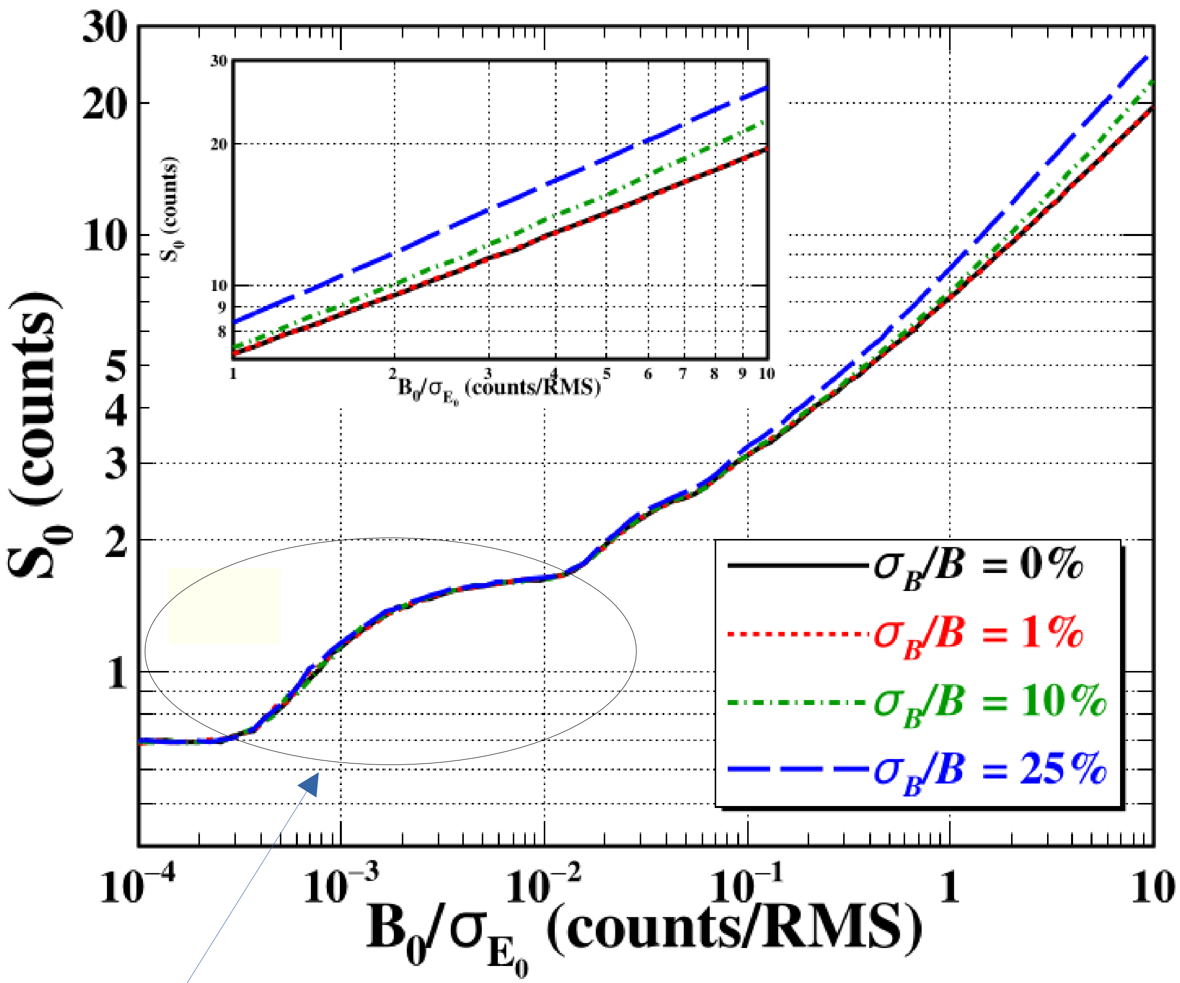
stat. fluctuation of B > sys. err. of B
at small B.



Wilk's approx. describe uncertainties
well at large B.

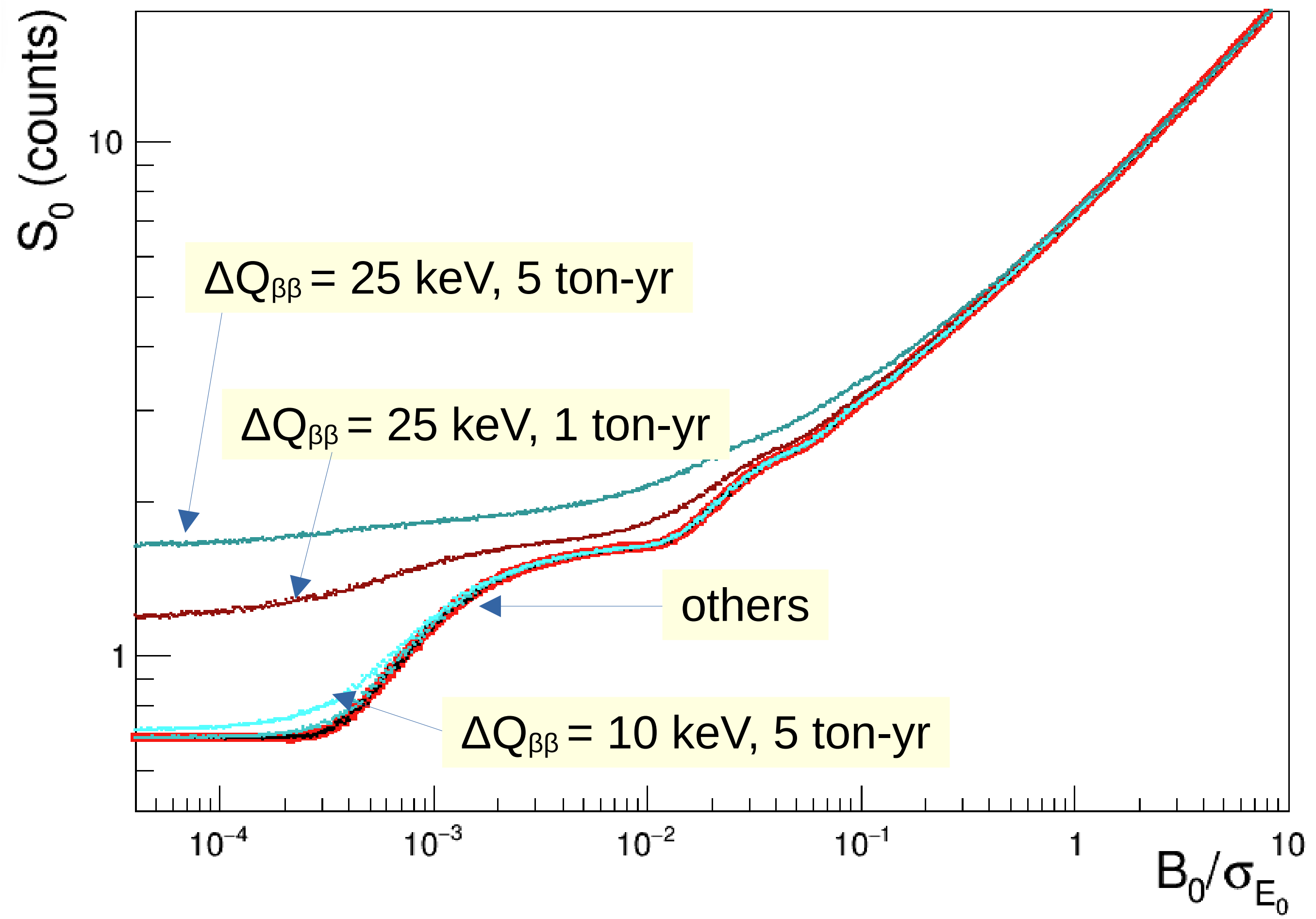
uncertainties of B

$$L(S|B,N,E_i) = \frac{e^{-(B+S)}(B+S)^N}{N!}\prod_{i=0}^{N}\left[\frac{Bf_B(E_i)+Sf_S(E_i)}{B+S}\right]\times \boxed{\frac{e^{-(\tau B)}(\tau B)^{n_0}}{n_0!}}$$



fix σ_B/B → no effect at low B due to statistic fluctuation dominant over σ_B

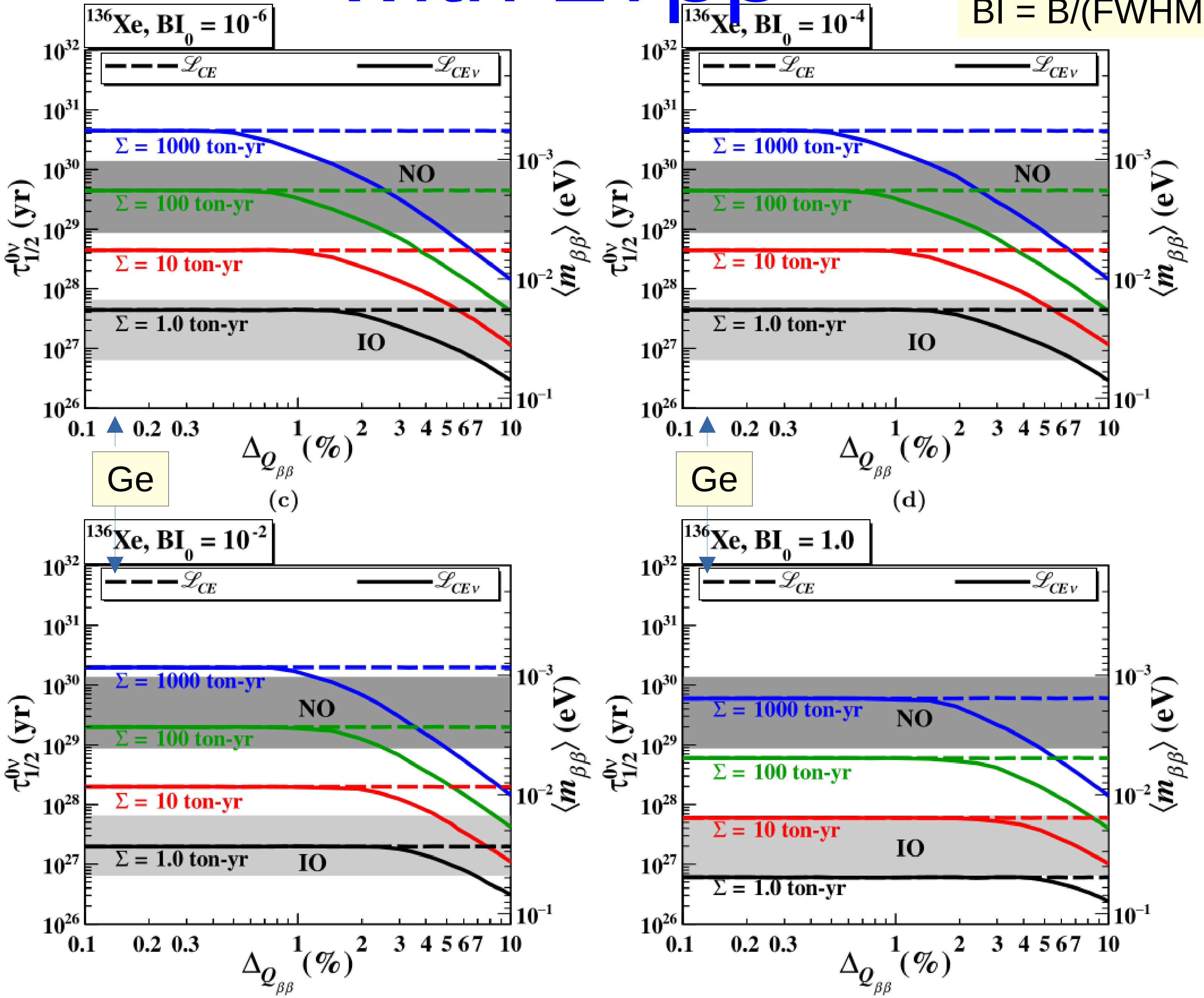
with 2νββ



2νββ become a dominant background when flat background is small.
negligible for $\Delta q_{\beta\beta}$ < 10 keV and exposure < 1 ton-yr

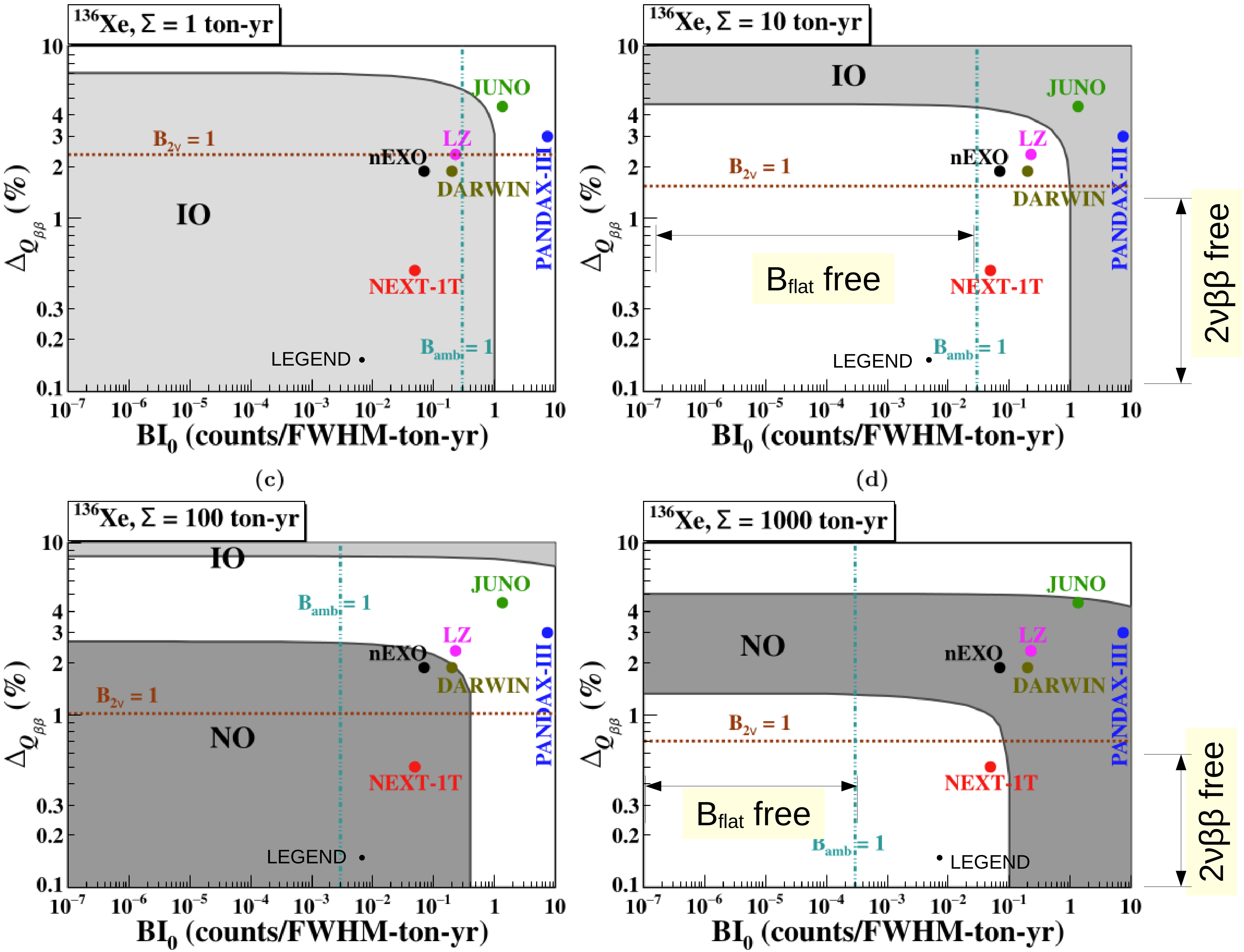
with 2νββ

BI = B/(FWHM×ton×yr)



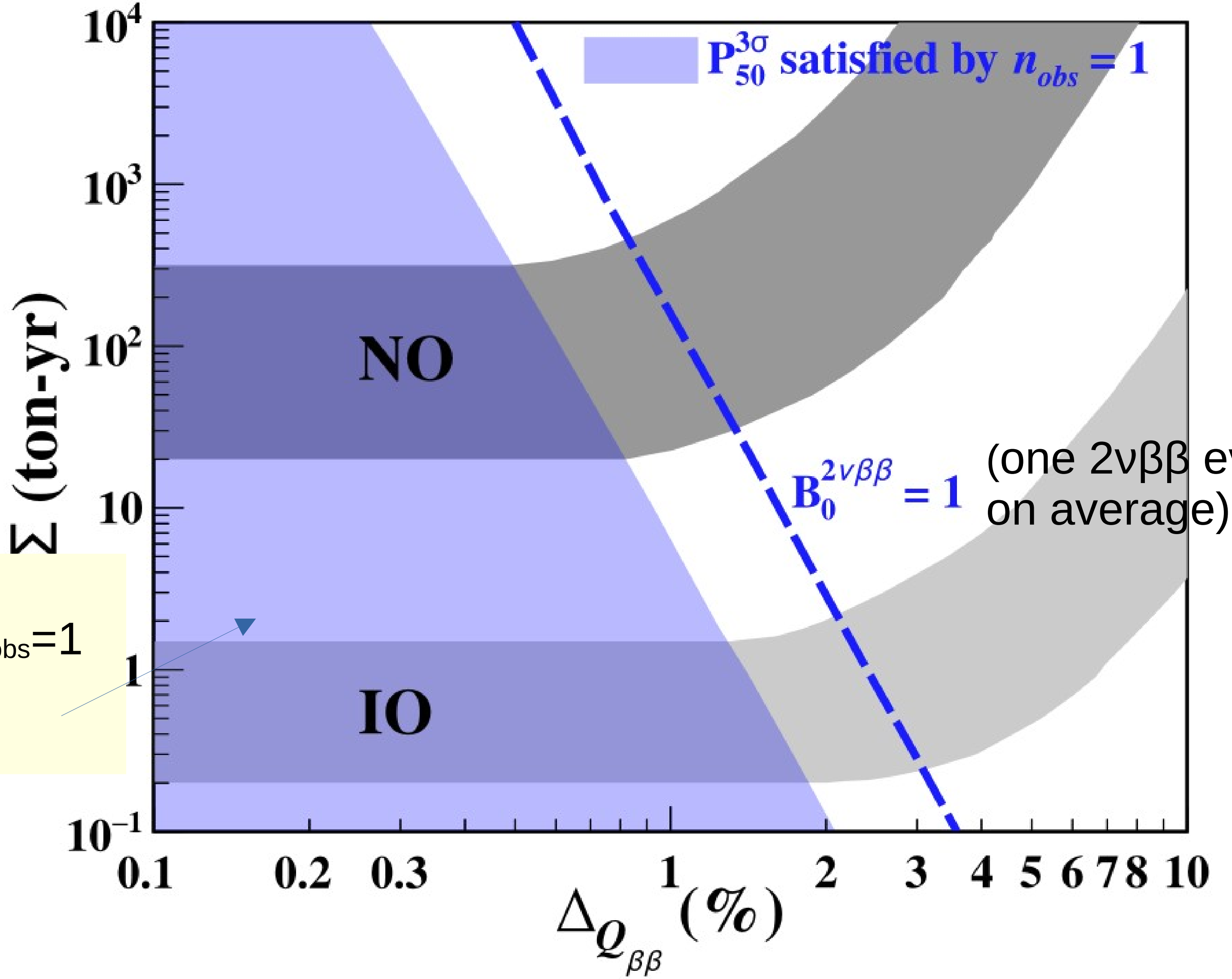
2νBB background affect low-B large exposure experiment more.

with 2νββ



136Xe, Σ = 1 ton-yr
IO
JUNO
LZ
nEXO
DARWIN
PANDAX-III
NEXT-1T
LEGEND
B2ν = 1
Bamb = 1
ΔQββ (%)
BI0 (counts/FWHM-ton-yr)
136Xe, Σ = 10 ton-yr
Bflat free
2νββ free
(c)
(d)
136Xe, Σ = 100 ton-yr
NO
136Xe, Σ = 1000 ton-yr

2νββ free

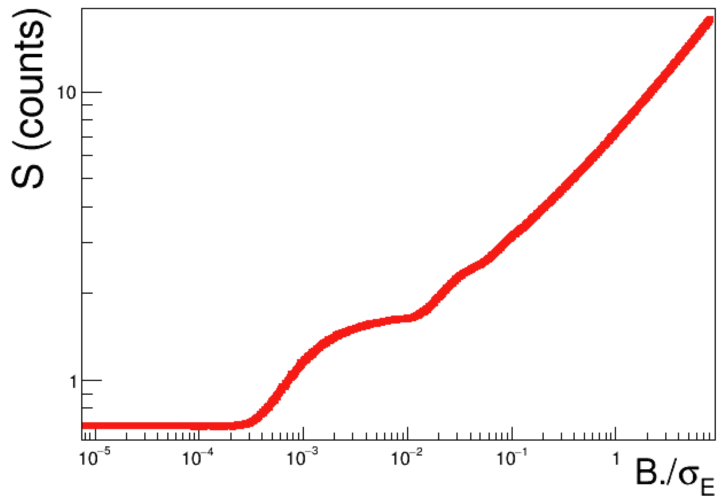


(one 2νββ event on average)

2νββ free
region where N_{obs}=1 claims observe
($P^{3\sigma}_{50}$ criteria)

summary

- we example projected sensitivities of $0\nu\beta\beta$ by hypothesis test with profile likelihood ratio at both low statistics and high statistic region. and cross-check with other approximation method.
- example parameter space where Wilk's approximation is valid.
- effect of discreteness of Poisson still preserve at low B.
- more work on other systematic uncertainties: $G^{0\nu}$, g^4_A, $|M^{0\nu}|^2$, $2\nu\beta\beta$ spectum.



Thanks

test statistic

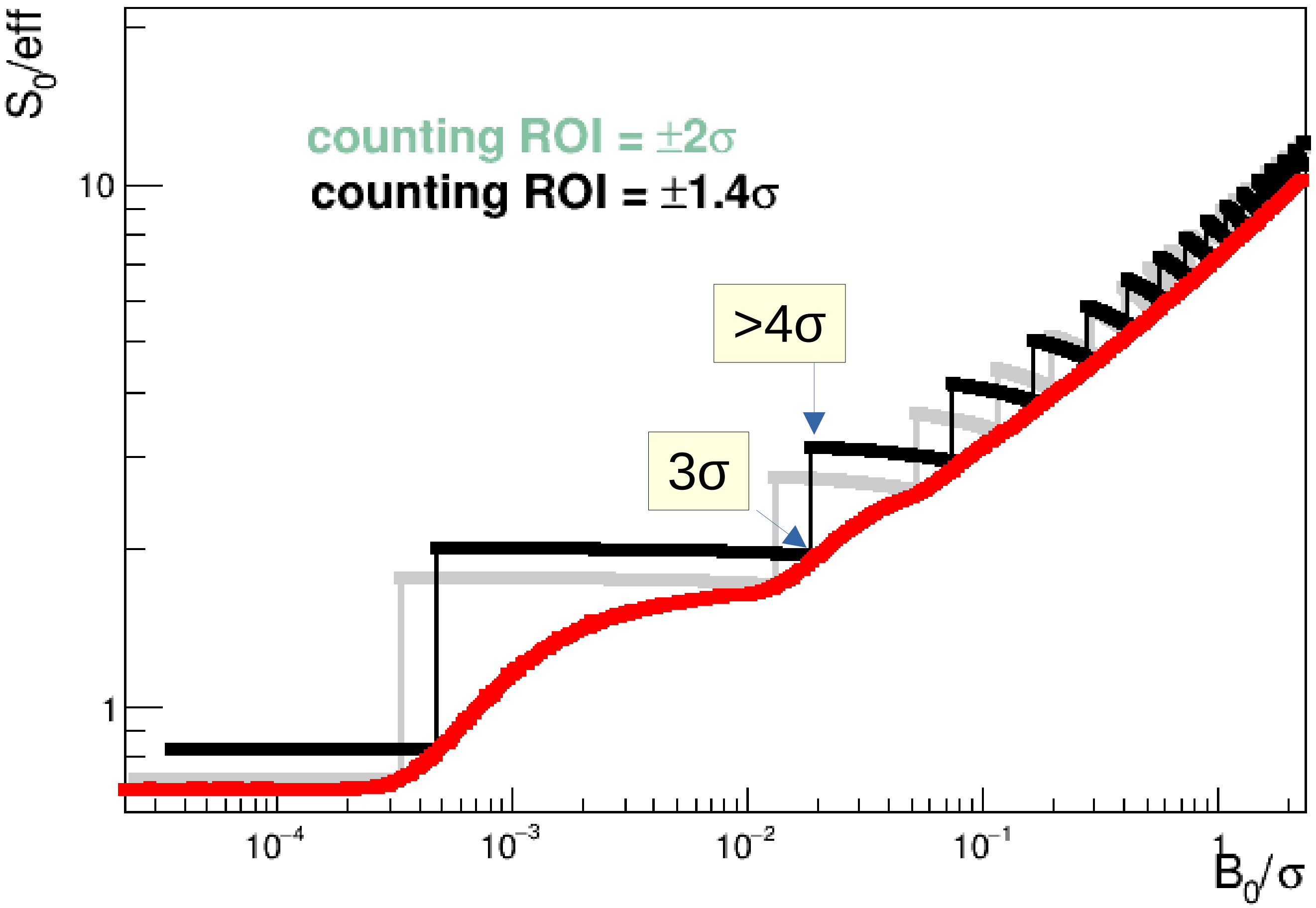
The question: **what is $T^{0\nu}_{1/2}$ (or $S^{0\nu}_{obs}$) to define positive claim of 0νββ**?
→ depend on 2νββ background → exposure + energy resolution of detector.
→ depend on flat background → exposure.
→ depend on uncertainties of backgrounds.
→ depend on uncertainties of $G^{0\nu} g^4_A |M^{0\nu}|^2$.
→ depend on acceptance region (?-σ) and "power of test".
→ depend on choice of "test statistic".

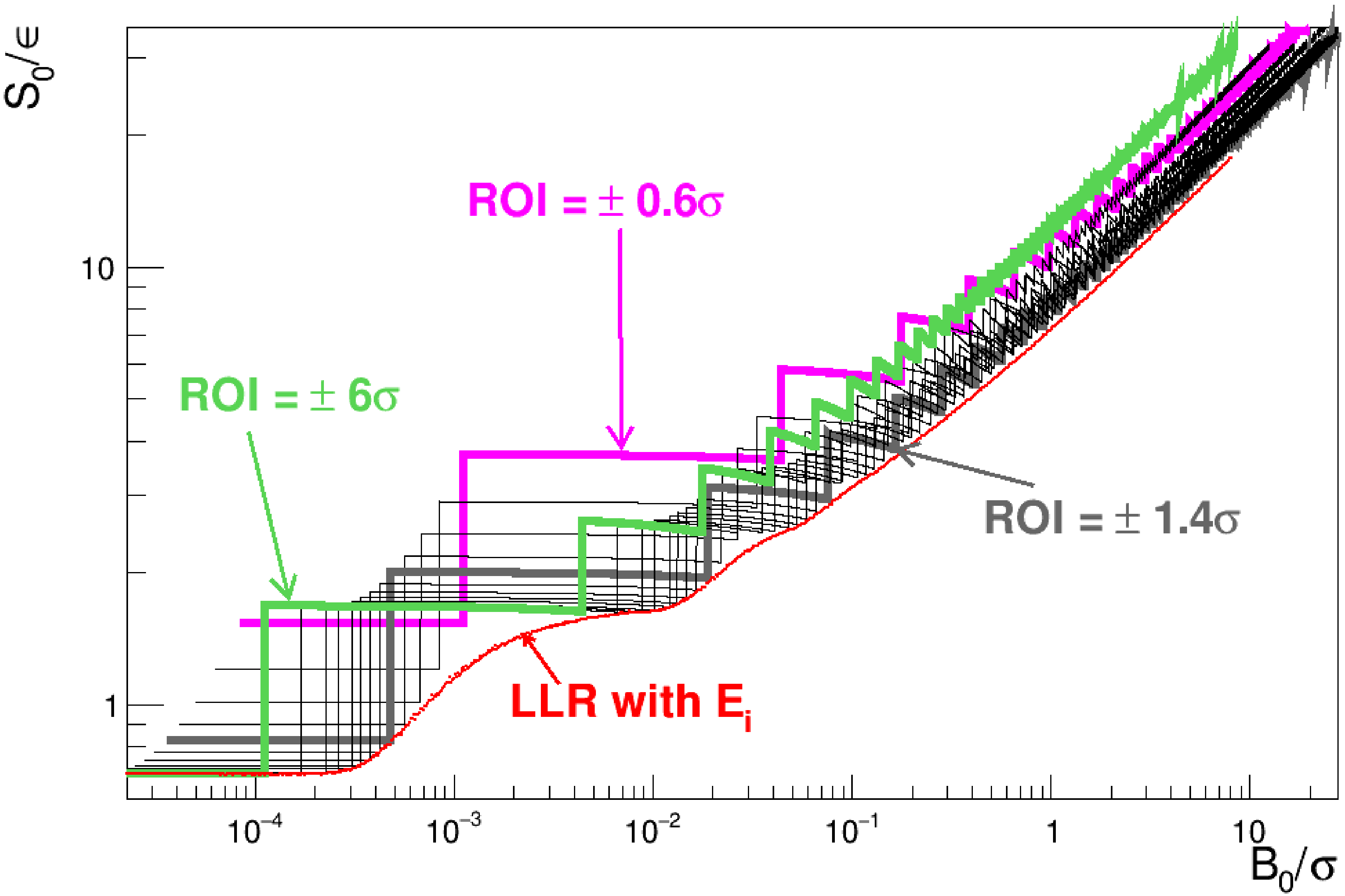
Test statistic : mapping from "experimental outcome" → "real number"
→ we need this or we can not compare two outcome with multiple values.
Neyman-Pearson Lemma → "likelihood ratio" optimized for hypothesis separation.
→ "profile likelihood ratio" when nuisance parameters exist.

We adopted frequentist statistic in this report:

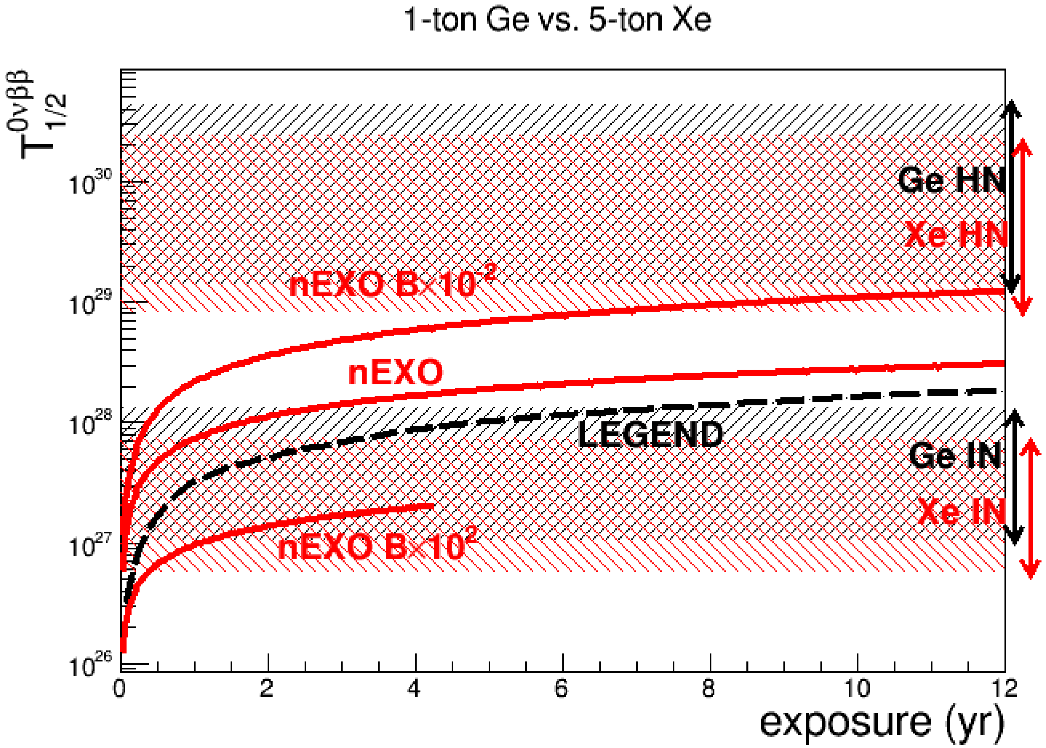
- probabilities of parameters (of interested or nuisance) are not used.
- uncertainties of parameters formulated from probabilities of measurements.
- C.I. is defined as "coverage" of measurement (outcome) → Neyman construction.

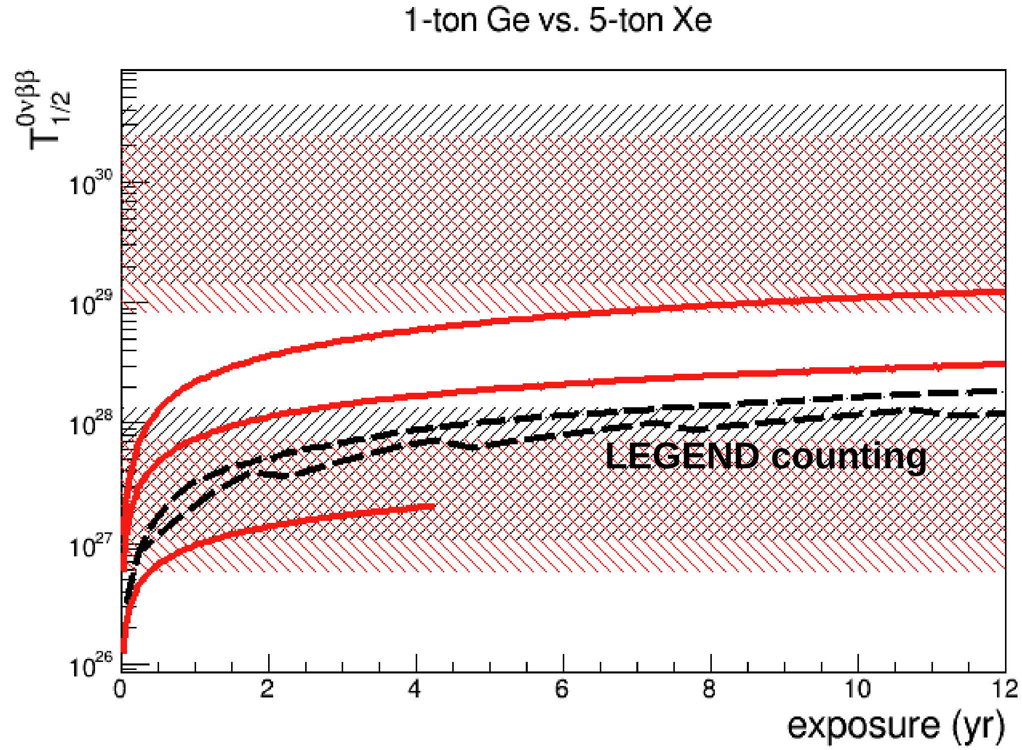
over-coverage of counting





S_0/ϵ
10
1
ROI = $\pm$ 0.6σ
ROI = $\pm$ 6σ
ROI = $\pm$ 1.4σ
LLR with E_i
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
B_0/σ


1-ton Ge vs. 5-ton Xe
$T^{0\nu\beta\beta}_{1/2}$
Ge HN
Xe HN
nEXO B×10^{-2}
nEXO
LEGEND
Ge IN
Xe IN
nEXO B×10^{2}
exposure (yr)


1-ton Ge vs. 5-ton Xe
$T^{0\nu\beta\beta}_{1/2}$
LEGEND counting
exposure (yr)