Helicity Property of Relic Neutrinos and Implications on Their Detection

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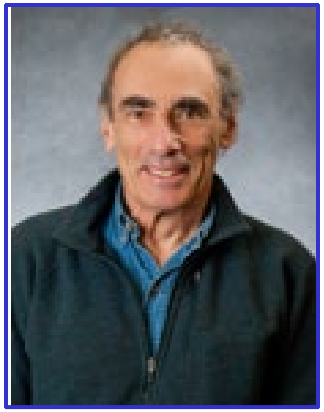
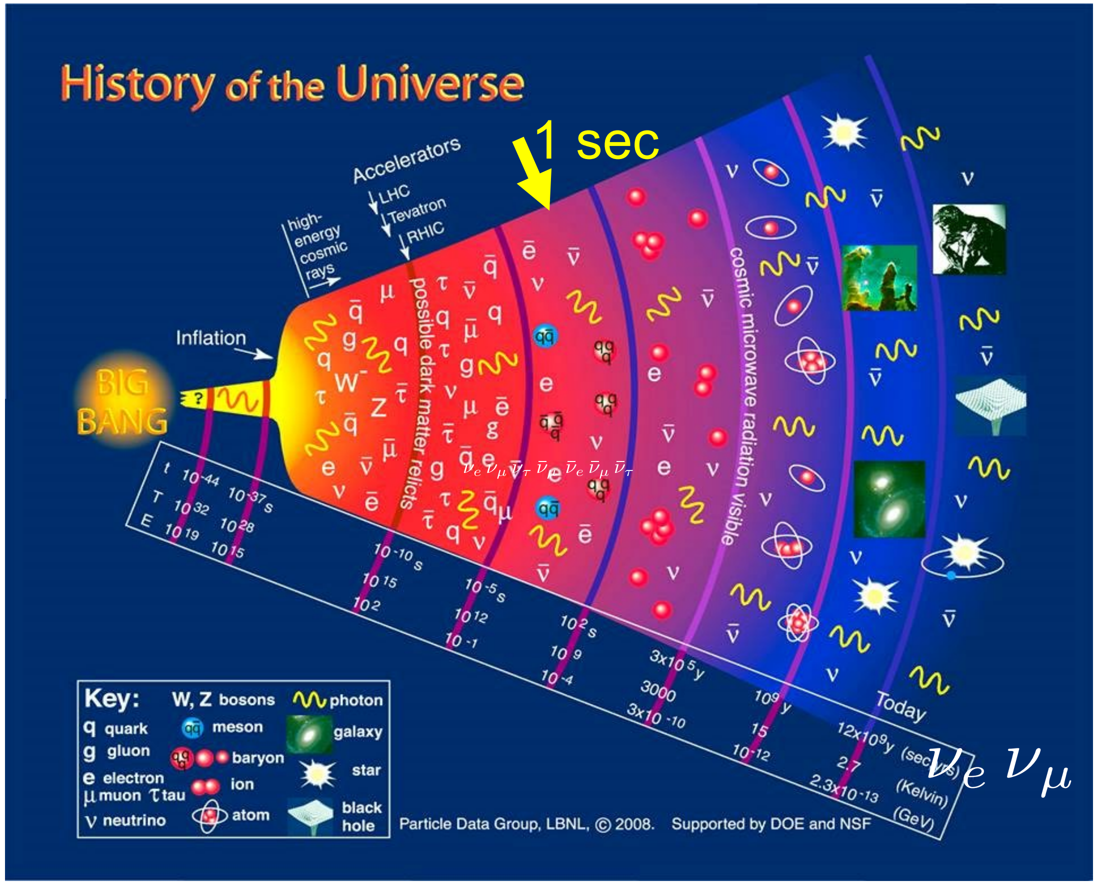
Based on three papers in collaboration with Gordon Baym

Phys. Rev. Letts. 126, 191803 (2021);
Phys. Rev. D 103, 123019 (2021);
Phys. Rev. D 106, 063018 (2022)



Relic neutrinos from the Big Bang forming the cosmic neutrino background (CνB)



Decoupling occurs at t ~1 sec, T ~1 MeV

CνB has never been observed !

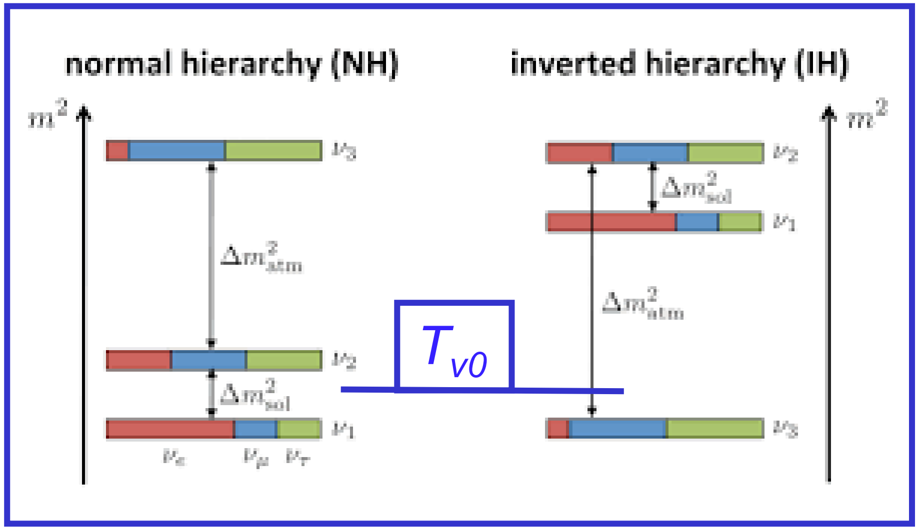
Cosmic neutrino background (CνB) versus cosmic microwave background (CMB)

	CMB	CνB	Relation
Temperature	2.73K	1.9 K (1.7 x 10^{-4} eV)	$T_\nu/T_\gamma = (4/11)^{1/3}$ $=0.714$
Decoupling at	3.8 x 10^5 years	~ 1 sec	
Density	~ 411 / cm^3	~ 336 / cm^3	$n_\nu = (9/11)\, n_\gamma$

- CνB took a snapshot of the Universe at a much earlier epoch than CMB
- At least two of the three neutrinos are non-relativistic
- ~20,000,000 of CνB inside you at this moment
- Density of CνB is ~100 times of solar neutrinos
- Decoupled as flavor eigenstates, now in mass eigenstates

At least 2 relic neutrino mass states are non-relativistic

(Current temperature: $T_{\nu 0} = 1.945\,\mathrm{K} = 1.676 \times 10^{-4}\,\mathrm{eV}$)



$$\Delta m^2_{21} = 7.50 \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m^2_{31,N} = 2.52 \times 10^{-3}\,\mathrm{eV}^2$$

$$\Delta m^2_{31,I} = -2.51 \times 10^{-3}\,\mathrm{eV}^2$$

$$T_{\nu 0} = 1.945\,\mathrm{K} = 1.676 \times 10^{-4}\,\mathrm{eV}$$

At least two neutrino masses are larger than 100 K with $m_i \gg T_{\nu 0} = 1.945\ \mathrm{K} = 1.676 \times 10^{-4}\ \mathrm{eV}$

Normal Hierarchy: If $m_1 = 0$, $\beta_1 = 1,\ \beta_2 \sim 1/50,\ \beta_3 \sim 1/300$

Inverted Hierarchy: If $m_3 = 0$, $\beta_3 = 1,\ \beta_1 \sim \beta_2 \sim 1/300$

Capture of CνB on radioactive nuclei (positive Q value)

(S. Weinberg, 1962)

Tritium beta decay:

$$a)\ {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

3-body β-decay with Q-value of

$$Q_a = M({}^3\mathrm{H}) - M({}^3\mathrm{He}) - M(e^-) - M(\bar{\nu}_e)$$

Inverse tritium beta decay (ITBD):

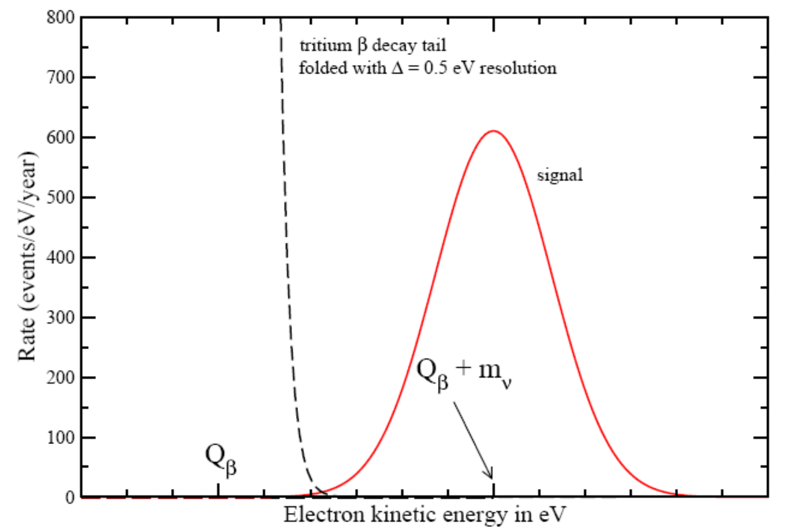
$$b)\ \nu_e + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

2-body reaction with the Q-value of

$$Q_b = M({}^3\mathrm{H}) - M({}^3\mathrm{He}) - M(e^-) + M(\bar{\nu}_e)$$

Therefore, $Q_b = Q_a + 2M(\bar{\nu}_e)$

Positive Q value implies low-energy relic neutrinos can be captured !



Look for a mono-energetic peak beyond the endpoint of tritium beta decay

PTOLEMY experiment for this search

Helicity dependence of the ITBD

$(\nu_e + {}^3H \to {}^3He + e^-)$

- ITBD for neutrino in mass eigenstate i and helicity h:

$$\sigma_i^h = \frac{G_F^2}{2\pi v_i} |V_{ud}|^2 |U_{ei}|^2 F(Z, E_e) \frac{m({}^3He)}{m({}^3H)} E_e p_e A_i^h (\bar{f}^2 + 3\bar{g}^2)$$

- The helicity-dependent factor, A_i^h, is given as

$$A_i^{\pm} = 1 \mp \beta_i; \quad \text{where } \beta_i = v_i / c$$

- For relativistic neutrinos, $\beta_i \to 1$, we have

$$A_i^+ \to 0 \quad \text{and} \quad A_i^- \to 2$$

- For non-relativistic neutrinos, $\beta_i \to 0$, we have

$$A_i^+ \to 1 \quad \text{and} \quad A_i^- \to 1$$

- ITBD rate depends on the helicity, h, of neutrinos

What are the helicities of relic neutrinos?

Helicity versus chirality for massive neutrino (where does the 1±β factor come from?)

For a Dirac spinor of momentum p along the z-axis with negative helicity (h = -1) we have

$$u^-(p) = \begin{pmatrix} 0 \\ \sqrt{E+m} \\ 0 \\ -\sqrt{E-m} \end{pmatrix}; \quad P_R = \frac{1+\gamma^5}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}; \quad P_L = \frac{1-\gamma^5}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$u^-(p) = u_L^-(p) + u_R^-(p) = P_L\, u^-(p) + P_R\, u^-(p)$$

$$u_L^-(p) = \frac{1}{2}\begin{pmatrix} 0 \\ \sqrt{E+m}+\sqrt{E-m} \\ 0 \\ -\sqrt{E+m}-\sqrt{E-m} \end{pmatrix}; \quad u_R^-(p) = \frac{1}{2}\begin{pmatrix} 0 \\ \sqrt{E+m}-\sqrt{E-m} \\ 0 \\ \sqrt{E+m}-\sqrt{E-m} \end{pmatrix}$$

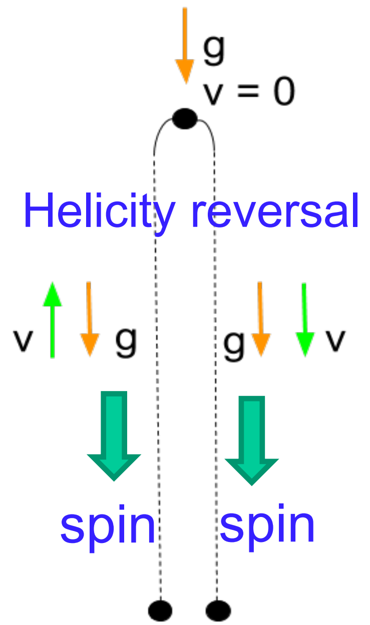
$$R = \frac{\sqrt{E+m}-\sqrt{E-m}}{\sqrt{E+m}+\sqrt{E-m}} = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}};$$

R is the relative amplitude for a negative helicity neutrino to be right-handed

Evolution of relic neutrino helicity
(from t ~ 1 sec to t ~ 13.8 billion years)

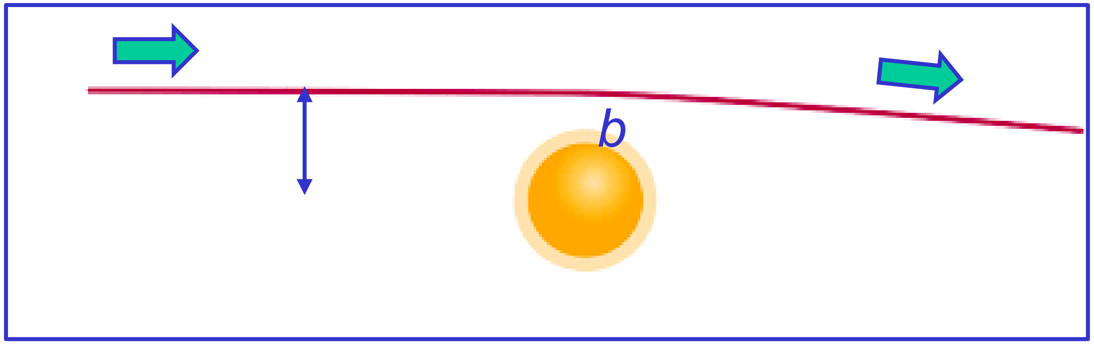
- Relic neutrinos decoupled at a temperature of ~1 MeV, and were highly relativistic. Neutrinos were produced essentially in $h=-1$ state, and antineutrinos in $h=+1$ state.
- Rotation of neutrino spin due to transverse matter source is less than the rotation of neutrino momentum (gravitational lensing of neutrino), changing neutrino helicity.
- Dirac neutrino with non-zero magnetic moment will precess in galactic or cosmic magnetic fields, changing neutrino helicity.

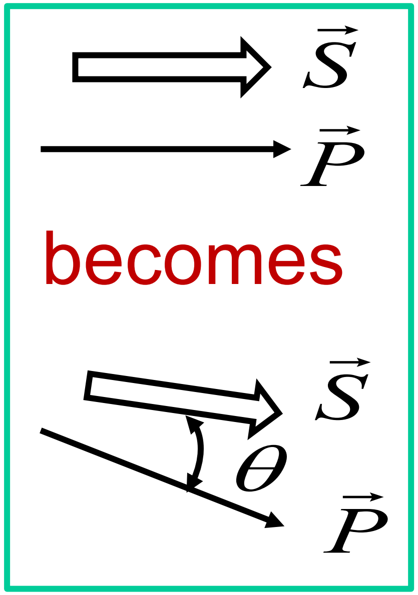
How would gravity modify the neutrino helicity?



If a neutrino with negative helicity is emitted upward from the Earth, it could fall back to the Earth having a positive helicity, affecting its weak interaction rate!

How would gravity modify the neutrino helicity?





Momentum bending: $\Delta\theta_P = \dfrac{2MG}{b\mathrm{v}^2}(1+\mathrm{v}^2)$

Spin bending: $\Delta\theta_S = \dfrac{2MG}{b}\dfrac{2\gamma+1}{\gamma+1}; \quad (\gamma = 1/\sqrt{1-\mathrm{v}^2})$

$$\theta \equiv \Delta\theta_S - \Delta\theta_P = -\frac{2MG}{b\gamma\mathrm{v}^2}$$

(spin bending lags momentum bending)

$\theta \to 0$ as $\mathrm{v} \to 1$

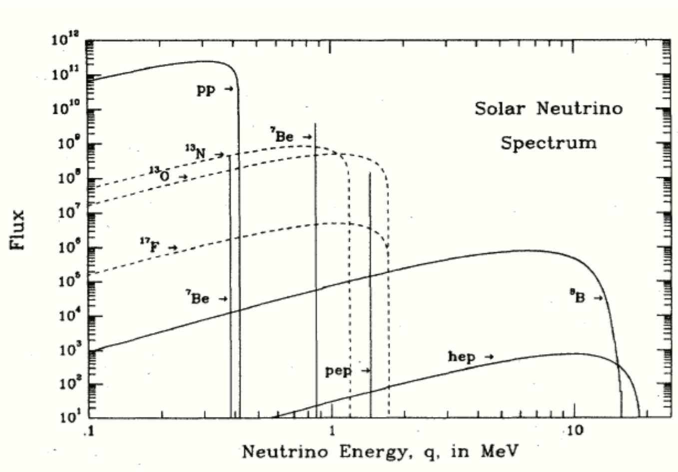
θ is large as $\mathrm{v} \to 0$

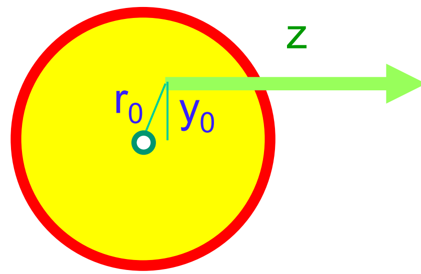
An angle θ between the spin and momentum directions means

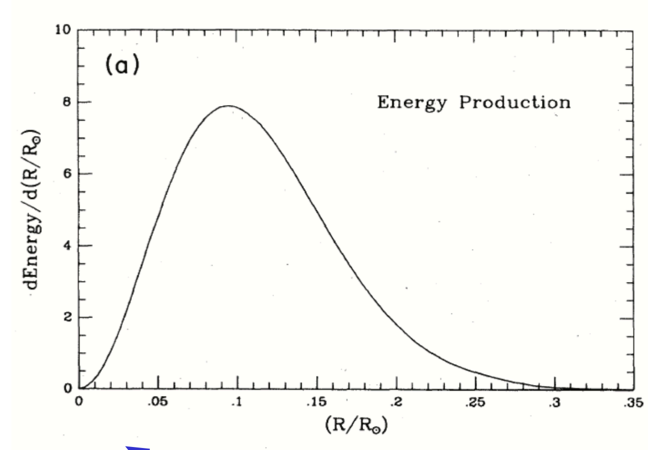
$$|h=+1\rangle \to \cos(\theta/2)|h=+1\rangle + \sin(\theta/2)|h=-1\rangle$$

Probability for $h=-1$ is $\sin^2(\theta/2)$

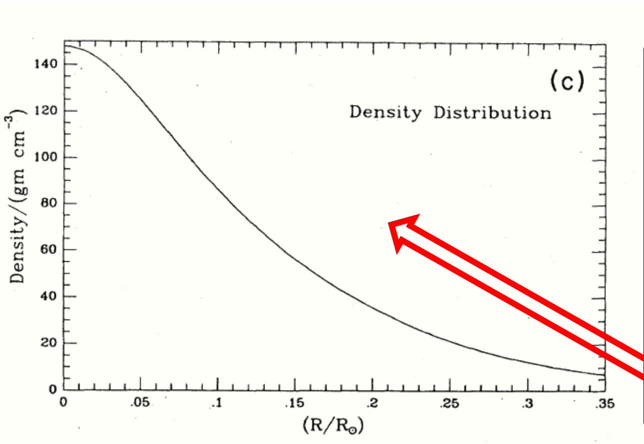
Helicity modification of solar neutrinos by Sun's gravity



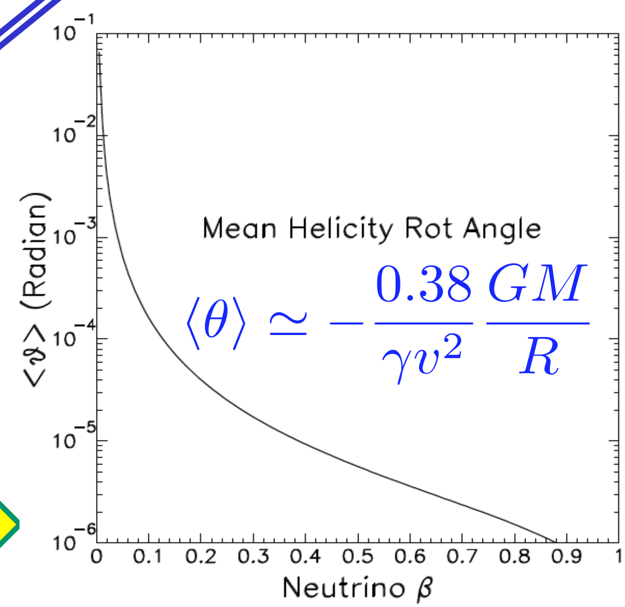




$$\theta(y_0, r_0) = -\frac{1}{\gamma v^2}\int_{z_0}^{\infty} dz \frac{GM(r)y_0}{r^3}$$



Averaged over spatial distribution of solar neutrino emission and mass distribution in Sun



Significant helicity modification of heavy particles with spin, e.g., dark photons, from Sun

Rotation of neutrino spin and momentum by scalar inhomogeneities

Gravitation potential Φ rotates momentum and spin:

$$\left(\frac{d\hat{p}}{dt}\right)_\perp = -\left(\mathrm{v}+\frac{1}{\mathrm{v}}\right)\vec{\nabla}_\perp \Phi; \qquad \left(\frac{d\vec{S}}{dt}\right)_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{\mathrm{v}}\,\vec{\nabla}_\perp \Phi$$

Spin bending lags momentum bending: $\left(h\frac{d\hat{S}}{dt}-\frac{d\hat{p}}{dt}\right)_\perp = \frac{m}{p}\vec{\nabla}_\perp \Phi$

Neutrinos undergoes a random walk through the inhomogeneities.

Averaged bending angle is zero, but RMS grows with time

Relate field fluctuations to density fluctuations: $\delta(\vec{x}) \equiv \delta\rho(\vec{x})/\bar{\rho}$

$$\langle \delta(\vec{x})\delta(\vec{x}')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} P(k)$$

$$\langle (\Delta\theta_p)^2 \rangle = \frac{2}{\pi}\int du \left(4\pi G\bar{\rho}a^2\right)^2 \int dk \frac{P(k)}{k}$$

Gravitational spin rotation relative to momentum

For massive relic neutrinos, after including matter and dark energy in $\bar{\rho}(a) = \rho_M / a^3 + \rho_V$:

$$\left\langle (\Delta\theta_p)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4\right)^{3/2} \mathrm{v}\left(\frac{1}{\mathrm{v}} + \mathrm{v}\right)^2$$

$$\left\langle (\Delta\theta_s)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4\right)^{3/2} \mathrm{v}^3\left(\frac{2\gamma+1}{\gamma+1}\right)^2$$

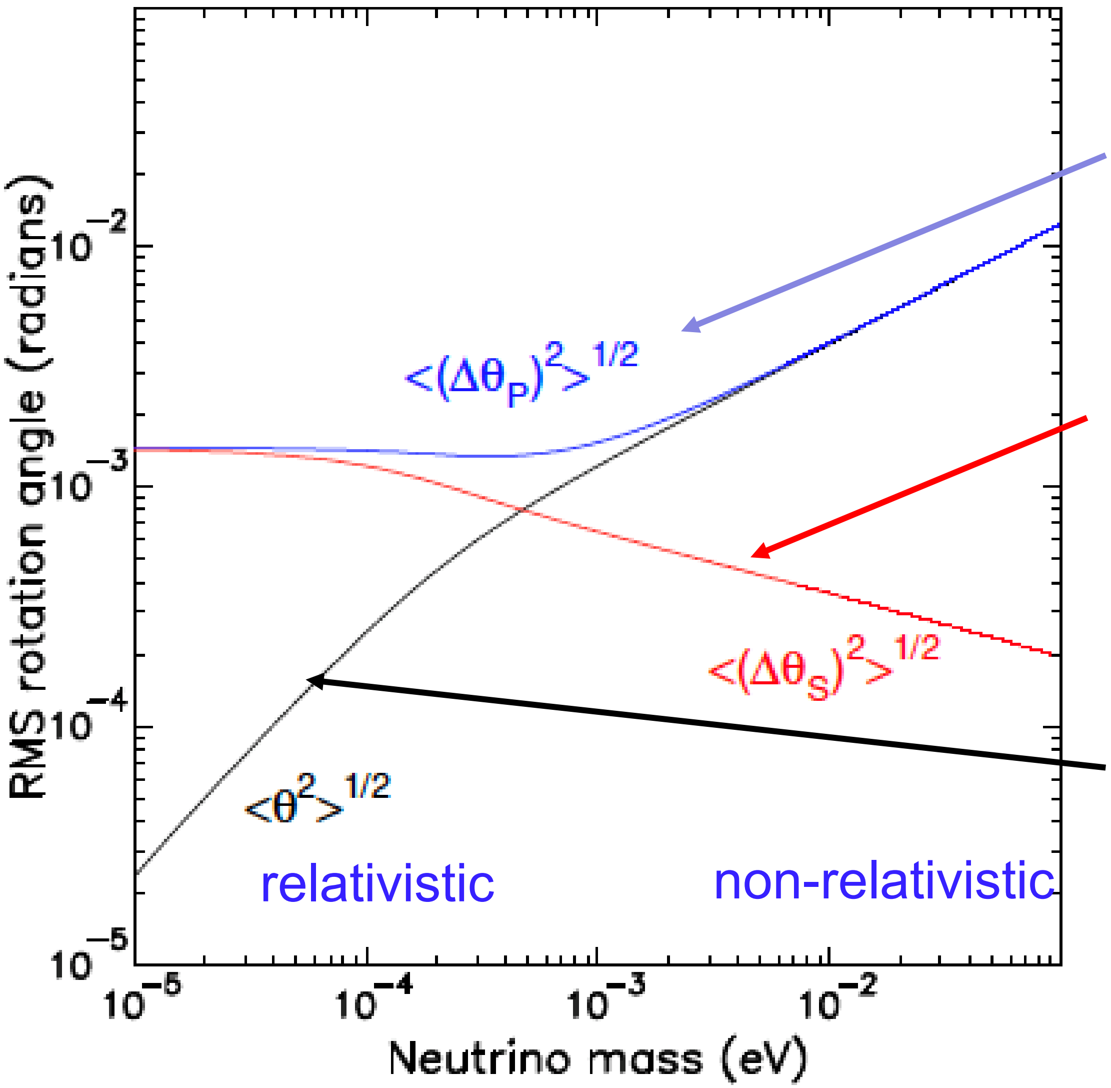
$$\left\langle \theta^2 \right\rangle \equiv \left\langle (\Delta\theta_p)^2 \right\rangle - \left\langle (\Delta\theta_s)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4\right)^{3/2} \left(\frac{1}{\mathrm{v}} - \mathrm{v}\right)$$

(where Ω_M = matter fraction, Ω_V =dark energy fraction)

Main effect is from matter dominated era (redshift $\sim 10^4$ to now)

(For detailed derivation, see Baym and Peng, PRD 103 (2021))

Spin rotation relative to momentum rotation due to gravity for relic neutrino mass state
(depending on neutrino's mass)

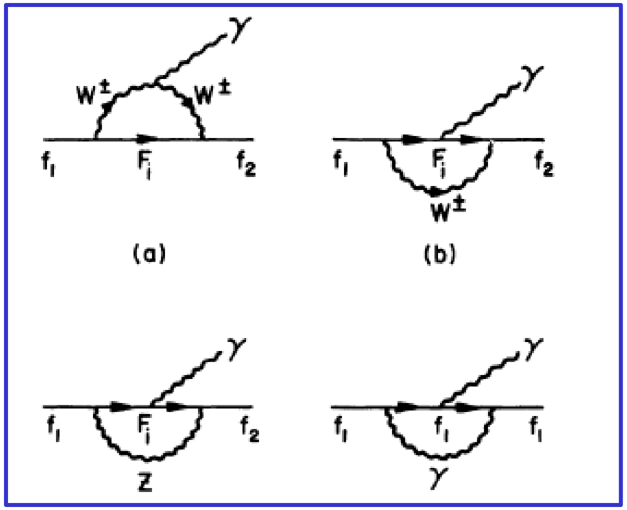


RMS for $\Delta\theta_P$:
rotation angle for momentum

RMS for $\Delta\theta_S$:
rotation angle for spin

RMS for θ :
rotation angle for spin relative to momentum

Rotation of neutrino spins in magnetic fields via neutrino magnetic moment



Standard model processes lead to a non-zero neutrino magnetic moment

$$\mu_\nu^{SM} \simeq \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \simeq 3\times10^{-21} m_{-2}\,\mu_B$$

Fujikawa-Schrock, *PRL* 1980

$$\mu_B = \text{Bohr magneton} = e/2m_e$$

$$m_{-2} = m_\nu / 10^{-2}\,\text{eV}$$

The magnetic moment could be much larger (BSM physics)

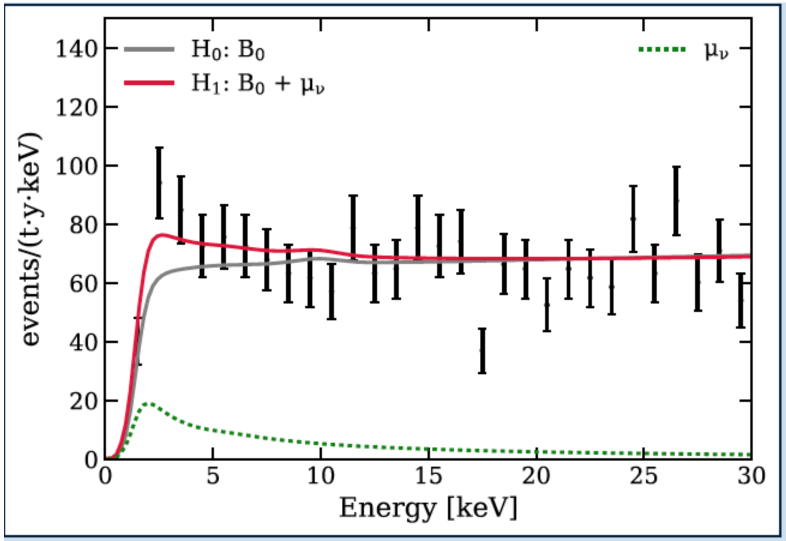
Upper bounds: $\mu_\nu < 2.9\times10^{-11}\mu_B$ GEMMA (2010)

$\mu_\nu < 7.4\times10^{-11}\mu_B$ TEXONO (2007)

$\mu_\nu < 2.8\times10^{-11}\mu_B$ Borexino (2017)

Naturalness upper bound: $\mu_\nu \le 10^{-16} m_{-2}\mu_B$ *Bell et al. PRL* 2005

XENON1T low energy electron event excess



Excess of low energy electron events 1-7 keV over expected background???

Aprile et al. PR D 102, 072004 (2020)

Possible explanations:

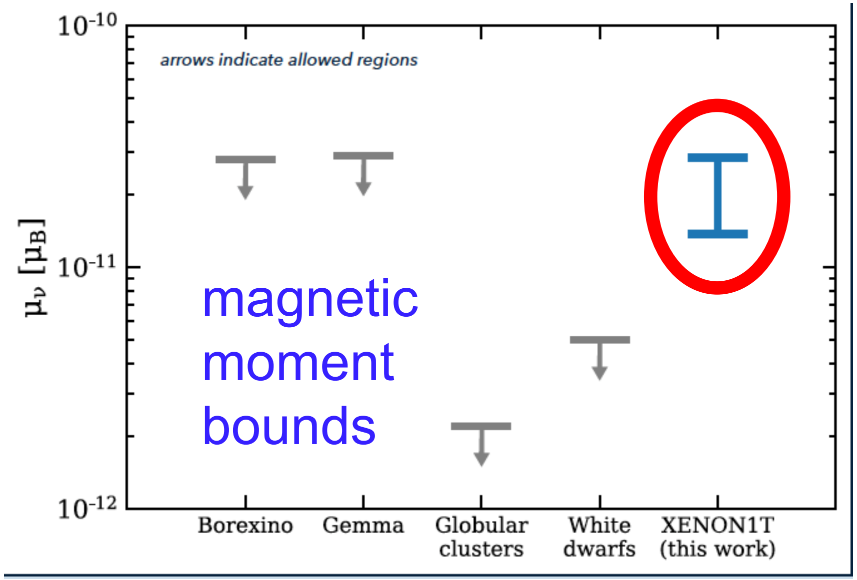
Large neutrino magnetic moment (3.2σ)

Solar axions (3.5σ)

Tritium (in Xe) beta decays

Excess consistent with neutrino magnetic moment:

$$\mu_{\nu,1T} \sim 1.4 - 2.9 \times 10^{-11} \mu_B$$

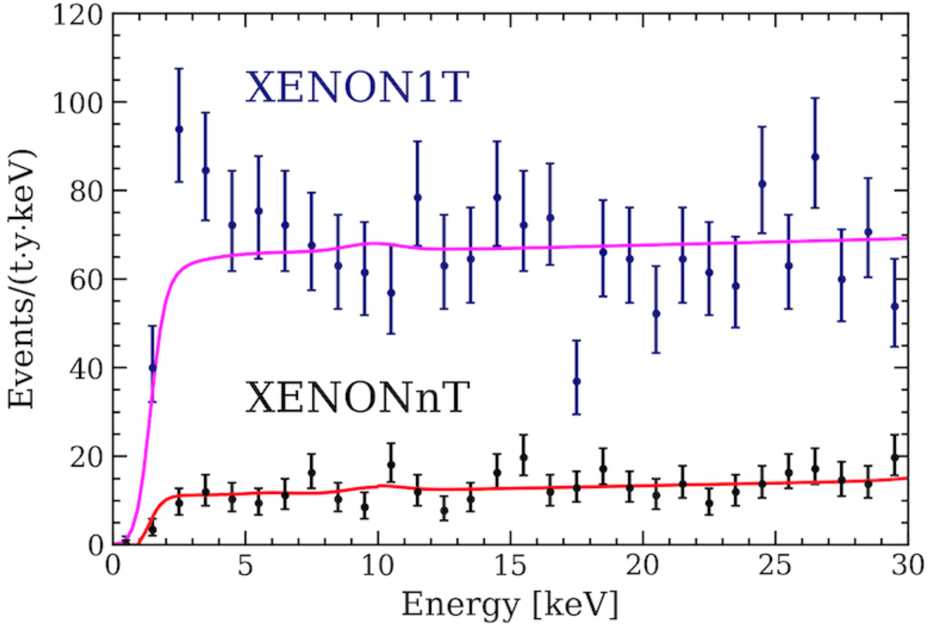


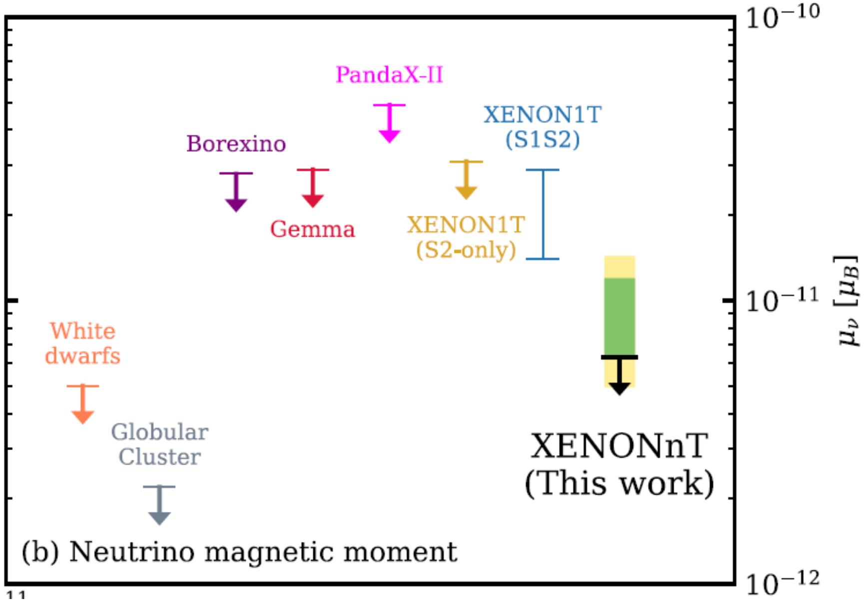
Beyond Standard Model physics??

Excess now tracked to tritium contamination

E. Aprile et al, PRL: 129, 161805 (2022)

XENONnT = 6 tons of Xe





No indication of BSM neutrino magnetic moment

Neutrino's spin precesses in B field, but momentum does not

(neutrinos are electrically neutral)

Magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Define spin in rest frame of neutrino.

Rest frame precession :

$$\frac{d\vec{S}}{d\tau} = 2\mu_\nu \vec{S} \times \vec{B}_R \qquad B_R = \text{magnetic field in rest frame}$$

In terms of "lab" frame magnetic field: $B_{\parallel R} = B_\parallel,\ \ B_{\perp R} = \gamma B_\perp$

Bargmann-Michel-Telegdi (BMT) equation of motion:

$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right)$$

Apply to both galactic and cosmic magnetic fields

Cosmic magnetic field rotation of neutrino spin

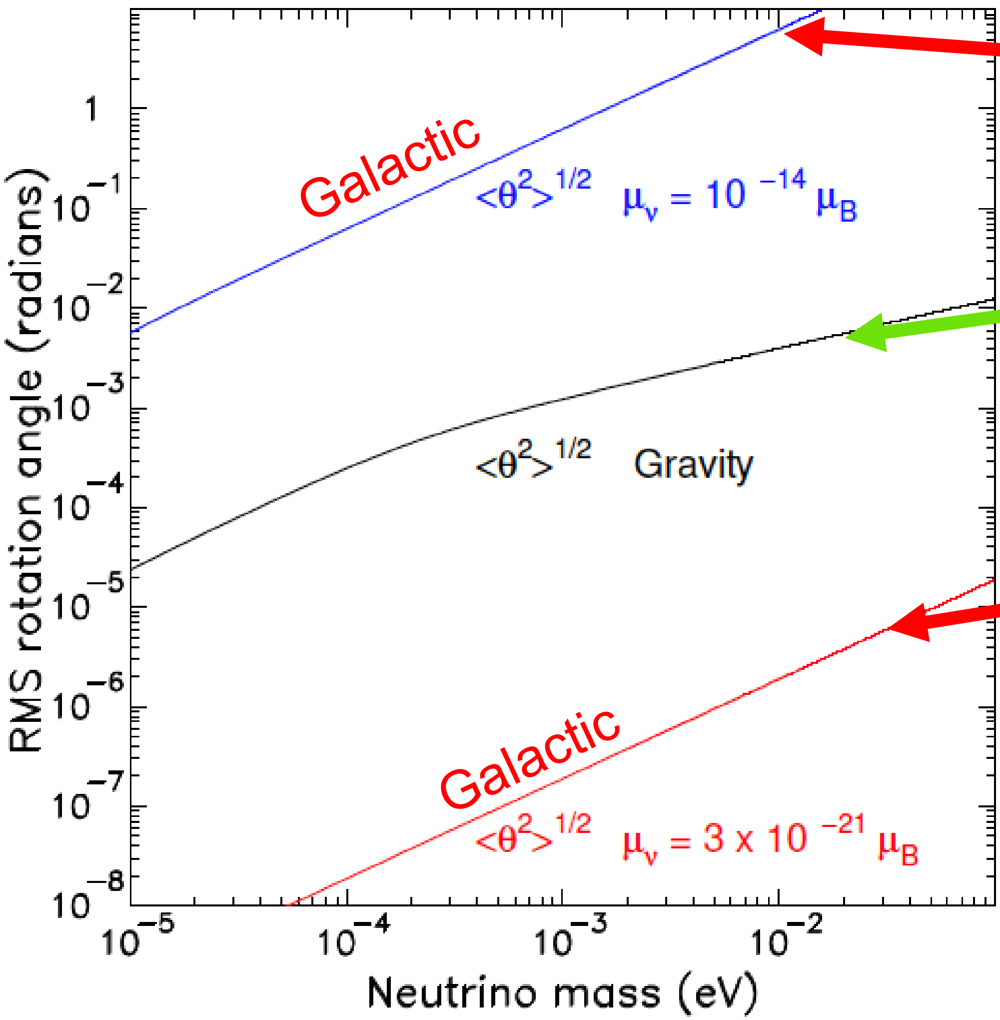
$$\left\langle \theta^2 \right\rangle_{\text{Galaxy}} \sim 4\times10^{29}\, m_{-2}^{2} \left(\frac{\Lambda_g}{1kpc}\right)\left(\frac{B_g}{10\,\mu G}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

$$\left\langle \theta^2 \right\rangle_{\text{Cosmic}} \sim 2\times10^{27} \left(\frac{\Lambda_0}{1Mpc}\right)\left(\frac{B_0}{10^{-12}\,G}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Λ_0 = coherence length of cosmic magnetic field

To within uncertainties in magnetic fields, coherence lengths, and neutrino masses, spin rotation in cosmic magnetic fields ~ galactic fields

Spin rotation from gravitational vs. magnetic fields



Rotation in Milky Way with magnetic moment ~100 times smaller than current upper limit

Gravitational rotation
GB+JCP PRD

Rotation in Milky Way with standard model magnetic moment

ITBD rate depends on the helicity, mass and type of relic neutrinos

- $\sigma_i^h = \frac{G_F^2}{2\pi v_i} |V_{ud}|^2 |U_{ei}|^2 F(Z,E_e) \frac{m(^3He)}{m(^3H)} E_e p_e A_i^h (\bar{f}^2 + 3\bar{g}^2)$
- Define A_{eff} as the sum of $|U_{ei}|^2 A_i^h$ over mass state i and helicity h:

$$A_{eff} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T$$

- Helicity-dependent factor, A_i^h, is $A_i^\pm = 1 \mp \beta_i$; where $\beta_i = v_i / c$
- T denotes the thermal average over the present momentum distribution, $f(p)$, of relic neutrinos: $f(p) = [e^{p/T_0} + 1]^{-1}; T_0 = 0.1676\ m\text{eV}$

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$

- For Majorana type, both neutrinos and antineutrinos contribute

$$A_{eff,M} = (1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) + (1 - \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) = 2$$

ITBD rate for Dirac neutrinos without helicity flip

- For Majorana type, both neutrinos and antineutrinos contribute

$$A_{eff,M} = (1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) + (1 - \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) = 2$$

- For Dirac type, only neutrinos (not antineutrinos) contribute

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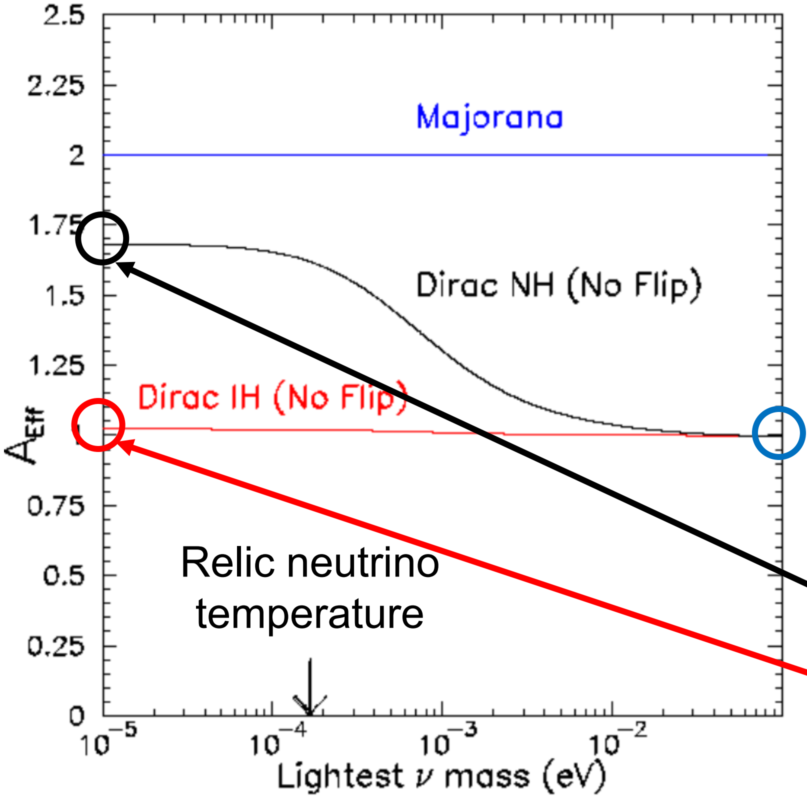


- For Dirac neutrinos without helicity flip ($\cos\theta_i = 1$)

$$A_{eff,D} = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \rangle_T$$

- If all neutrinos are non-relativistic, $\beta_i \to 0$, then

$$A_{eff,D} = 1$$

- If the lightest neutrino is relativisitic, then

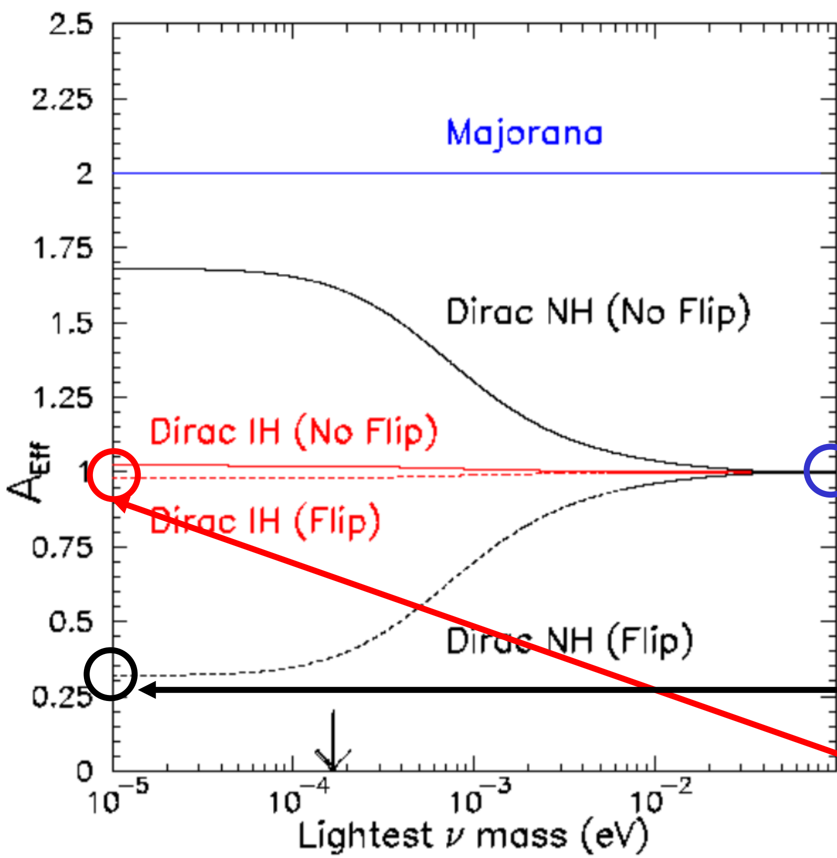
$A_{eff,D} = 1 + |U_{e1}|^2 = 1.68$ for normal mass hierarchy

$A_{eff,D} = 1 + |U_{e3}|^2 = 1.02$ for inverted mass hierarchy

ITBD rate for Dirac neutrinos with helicity flip

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$



- Dirac neutrinos with helicity flip ($\cos\theta_i = -1$)

$$A_{eff,D} = 1 - \sum_i |U_{ei}|^2 \langle \beta_i \rangle_T$$

- If all neutrinos are non-relativistic, $\beta_i \to 0$,

$$A_{eff,D} = 1$$

- If the lightest neutrino is relativisitic,

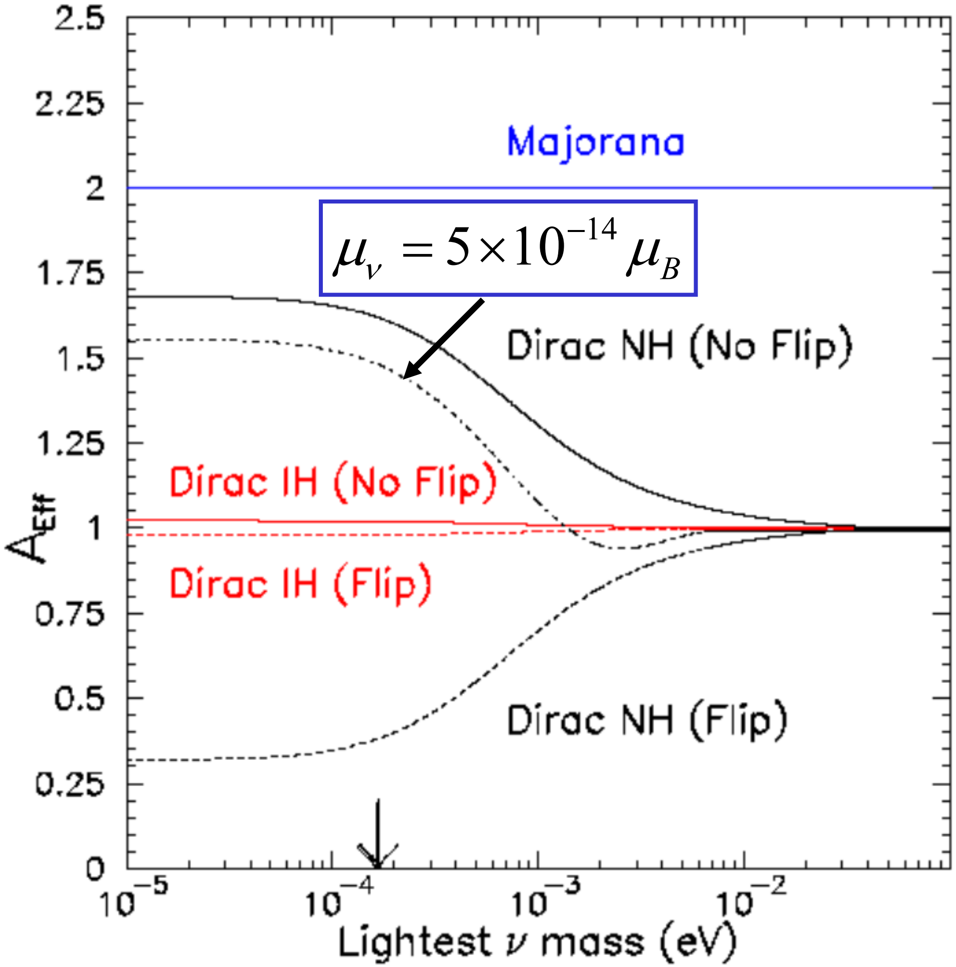
$$A_{eff,D} = 1 - |U_{e1}|^2 = 0.32 \quad \text{normal hierarchy}$$

$$A_{eff,D} = 1 - |U_{e3}|^2 = 0.98 \quad \text{inverted hierarchy}$$

ITBD rate for Dirac neutrinos with partial helicity flip

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$



- For Dirac with NH, ITBD rate is modified even with a modest μ_ν of $5\times10^{-14}\mu_B$
- For Dirac with IH $A_{eff,D} \simeq 1$ insensitive to μ_ν
- For Majorana neutrinos $A_{eff,M} = 2$, independent of μ_ν

Baym and Peng, PRL 126, 191803 (2022)

The ITBD has never been observed yet !

To detect the ITBD, use known sources of electron neutrinos

Peng and Baym, PRD 106, 063018 (2022)

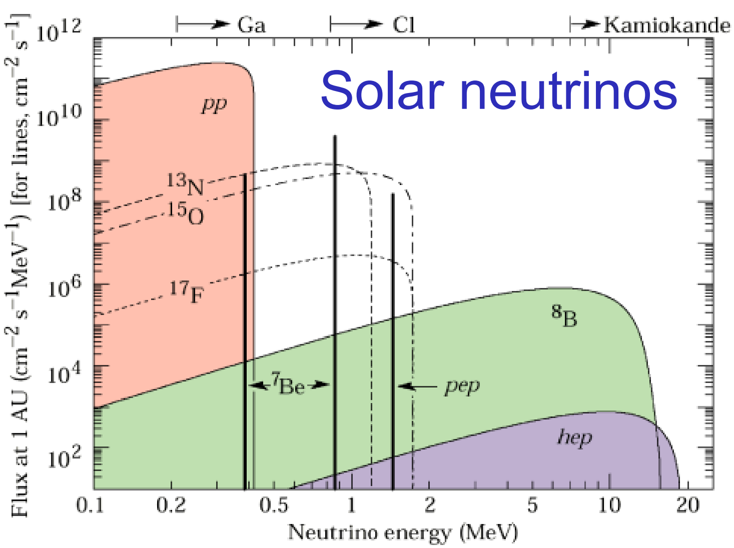
Solar Neutrinos and ^{51}Cr sources

$$^{51}\mathrm{Cr} \rightarrow {}^{51}\mathrm{V} + e^{+} + \nu_e$$

Experiment	Isotope	Strength	Production Process
GALLEX [3]	^{51}Cr	1.69 MCi	Thermal neutron capture on ^{50}Cr
SAGE [2]	^{51}Cr	0.517 MCi	Epithermal neutron capture on ^{50}Cr
GALLEX [1]	^{51}Cr	1.87 MCi	Thermal neutron capture on ^{50}Cr
SAGE [4]	^{37}Ar	0.409 MCi	Fast neutron $^{40}\mathrm{Ca}(n,\alpha)^{37}\mathrm{Ar}$
BEST [5]	^{51}Cr	3.4 MCi	Thermal neutron capture on ^{50}Cr

Table 1: Mega-Curie-scale electron capture neutrino sources that have been produced.

Coloma et al. (Snowmass 2020)



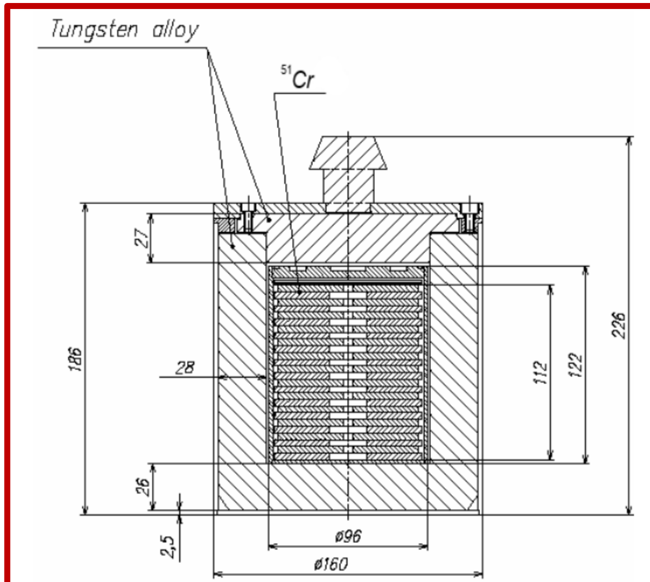


Figure 2. Neutrino source based on ^{51}Cr, all dimensions are in mm.

3.4 MCi ^{51}Cr source for the experiment BEST

Expected ITBD rates from various sources

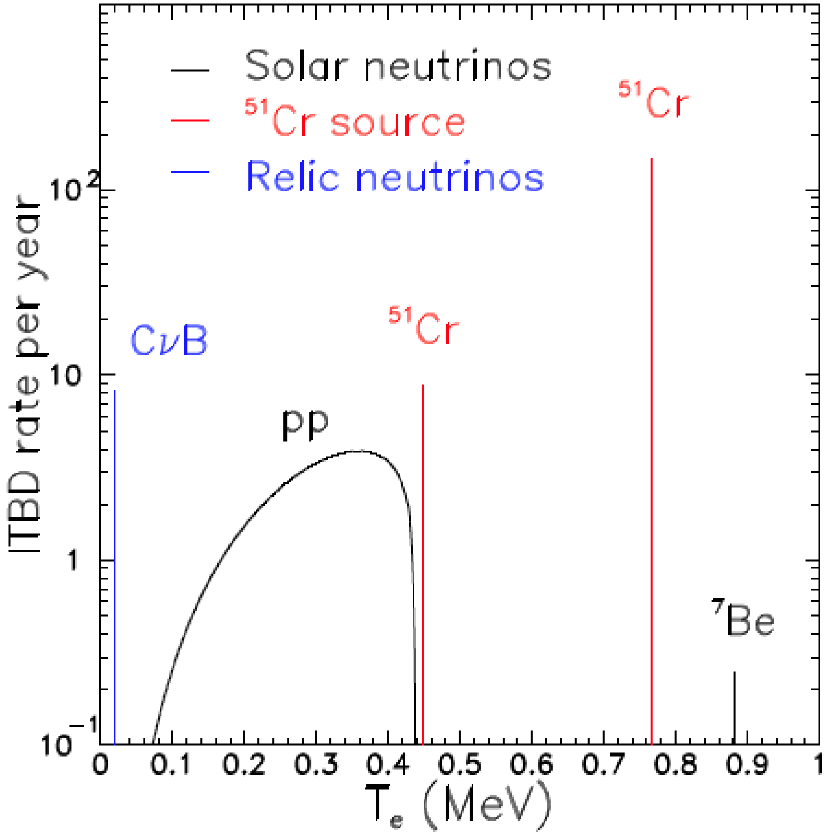
Assuming a 100 g tritium target

Peng and Baym, PRD 106, 063018 (2022)

3.0-MCi ^{51}Cr at 50 cm away from 100 g tritium target

TABLE I. ITBD rate for various sources of electron neutrinos, together with the electron kinetic energies, T_e. The relic neutrinos are assumed to be Majorana in the rate calculation.

Source	T_e (MeV)	Rate (1/year)
^{51}Cr 0.427 + 0.432 MeV ν_e	0.447	8.8
^{51}Cr 0.747 + 0.752 MeV ν_e	0.767	147.0
Solar pp ν_e	0.0186 to 0.44	0.8
Solar ^{7}Be ν_e	0.881	0.23
Relic $\nu_e/\bar{\nu}_e$	0.018	8.2



Conclusion

- Relic neutrino helicities could be modified by gravity and magnetic fields
- Detection rate of relic neutrinos via the ITBD reaction is sensitive to the Dirac/Majorana nature of neutrino, and to the masses of neutrinos
- For Dirac neutrino with normal hierarchy, the ITBD rate also depends on neutrino helicity, which is sensitive to neutrino magnetic moment
- Detection of relic neutrinos can reveal fundamental properties of neutrinos and the Early Universe


Thank you!