Many-body atomic response functions for sub-GeV dark matter-electron interactions *

Cheng-Pang Liu
National Dong Hwa Univ.

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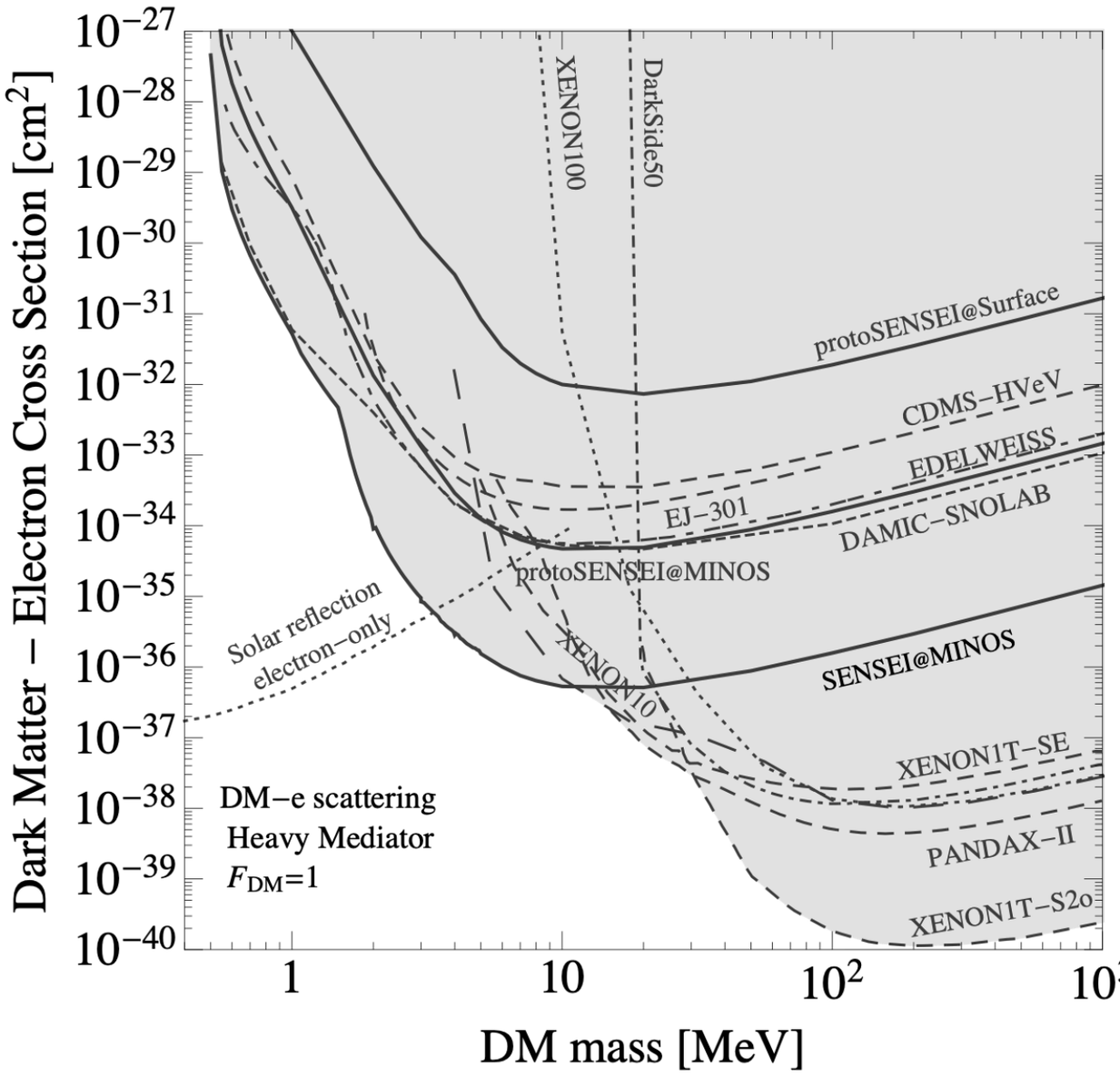
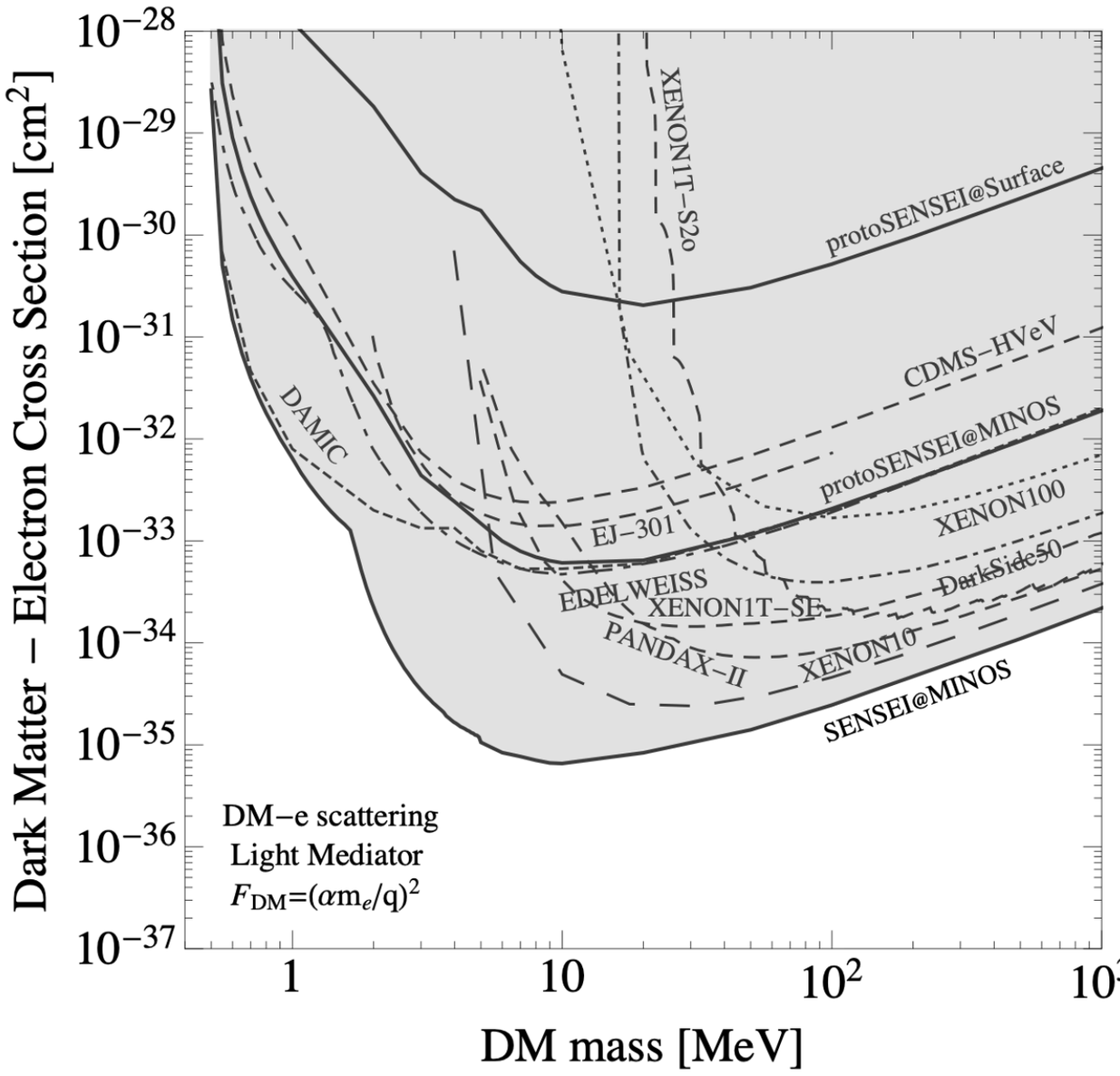
Direct Searches of Sub-GeV DM

- Generically, m_χ in meV-GeV
- Detectable energy: $1/2\, m_\chi v_\chi^2$ ($v_\chi \sim 10^{-3}$) for scattering, larger if boosted, or m_χ for absorption
- ER windows: T in 10 eV - tens of keV for atomic ionization, 1 eV - tens of eV for electron-hole pair in semiconductor, …

To Exclude Like These:





arXiv:2203.08297

The landscape of low-threshold dark matter direct detection in the next decade

Counting Rate *dR/dT* Prediction

- AstroP./Cosmo.: DM velocity spectrum

$$\frac{dR}{dT} = n_\chi \int d^3 v_\chi f(\vec{v}_\chi) v_\chi \frac{d\sigma}{dT}$$

- HEP: DM-matter interaction (model-driven / EFT)

$$\begin{aligned}\mathcal{L}_{\chi e}^{(\mathrm{LO})} =& (c_1 + d_1/q^2)\chi^\dagger\chi\, e^\dagger e && \text{<= Spin-Independent}\\ &+ (c_4 + d_4/q^2)\chi^\dagger \vec{s}_\chi \chi \cdot e^\dagger \vec{s}_e e && \text{<= Spin-dependent}\end{aligned}$$

Short + Long-range

- NP/AMP/CMP: Differential cross section

$$\frac{d\sigma}{dT} = \frac{1}{2\pi v_\chi^2} \int_{q_{\min}}^{q_{\max}} q dq \left\{ (c_1 + d_1/q^2)^2 \mathcal{R}_{\mathrm{SI}}(T,q) + \frac{1}{12} s_\chi (s_\chi + 1)(c_4 + d_4/q^2)^2 \mathcal{R}_{\mathrm{SD}}(T,q) \right\}$$

Response Functions

$$\mathcal{R}_O(T,\vec{q}) = \sum_F \overline{\sum_I} \left| \left\langle F \left| \hat{O}(\vec{q}) \right| I \right\rangle \right|^2 \delta(E_F - E_I - T)$$

- Contains the full dynamical information of how a complex MB system reacts to external perturbation
- Operator $\hat{O}$ is prescribed by the DM interaction
- Biggest challenge: MB wave functions $|I\rangle$ and $|F\rangle$

Hamiltonian to Solve

$$H_A = \sum_{i=1}^{Z} t_i - \frac{Ze^2}{|\vec{r}_i|} + \frac{1}{2}\sum_{i\neq j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \ldots$$

- Non-relativistic: $t = \vec{p}^2/2m_e$
- Relativistic: $t = \vec{\alpha} \cdot \vec{p} + \beta m_e$
- MB problem: *e*-*e* interaction

Central Field Approximation

$$H_A \approx \sum_{i=1}^{Z} t_i - V_i(r_i)$$

- All electrons are independent
- Many ways to formulate the CF potential (H-like, Hartree approximation, frozen core, etc.)
- *N* (# of orbitals) uncoupled differential equations

Hartree-Fock Approximation

$$H_A \approx \sum_{i=1}^{Z} t_i - \frac{Ze^2}{r_i} + V^{(D)}(\vec{r}_i) + \sum_j V^{(E)}(\vec{r}_i, \vec{r}_j)$$

- Independent electrons with exchange effect
- *N* coupled integro-differential equations (unco. DE if using local exchange)
- Missing the correlation effect from $V_{ee} - (V^{(D)} + V^{(E)})$

Our Approach: MCDF+MCRRPA

An *ab initio* method improved upon Hartree-Fock theory

- DF/R [Dirac-Fock]: Solving MB Dirac, instead of Schrödinger, equations. [note. $Z\alpha$~0.4 (Xe)]

- RPA [random phase approximation]: correlation is partially included [note. via so-called ring diagrams] to all orders

- For open-shell atoms, g.s. is multi-configuration (MC) [note. Ge $C_1|4p_{3/2}^2\rangle + C_2|4p_{1/2}^2\rangle$]

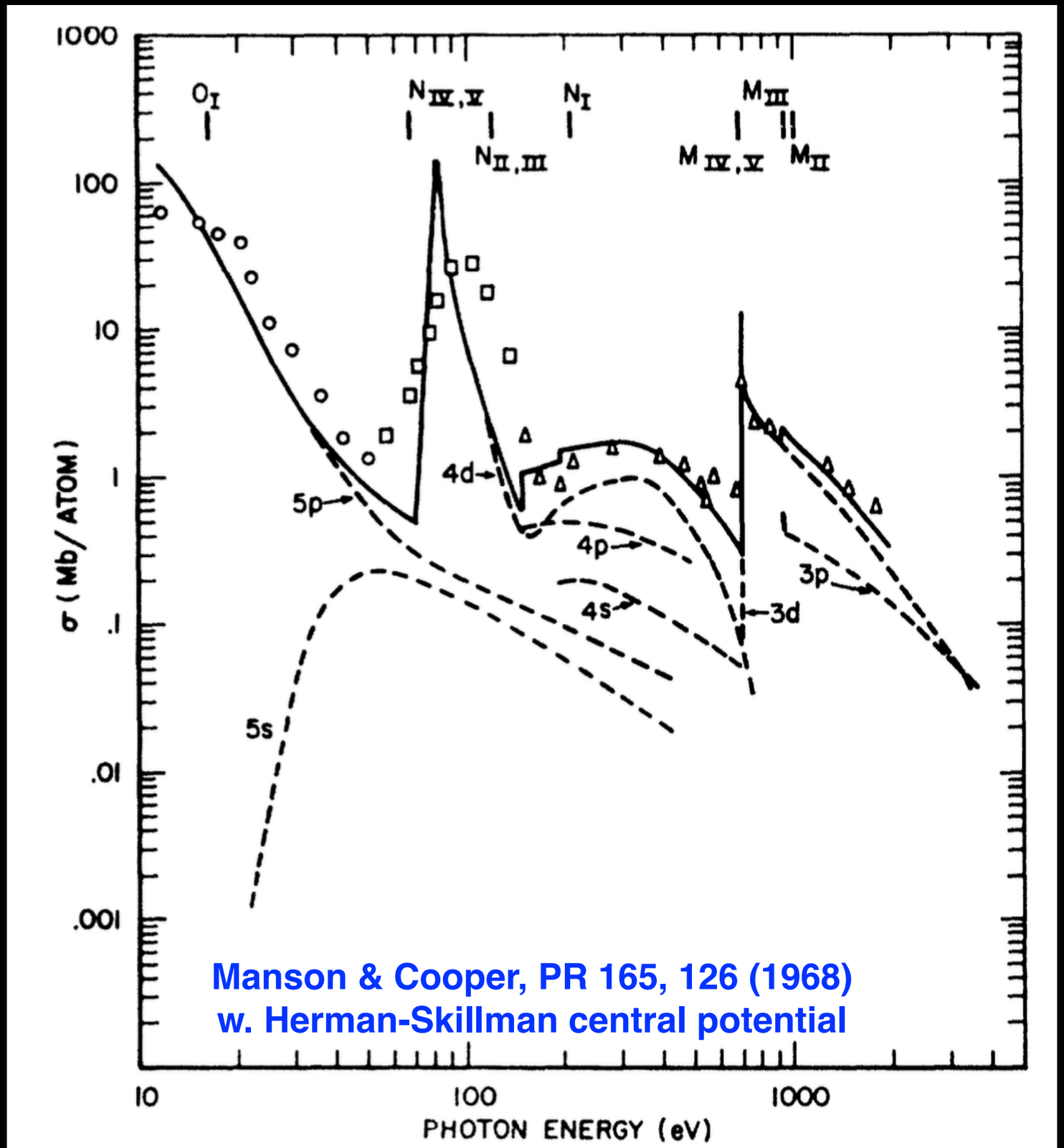
Benchmark: Photoabsorption

$$\chi + \mathrm{A} \rightarrow \chi + \mathrm{A}^{+} + e^{-} \text{ (yet unknown)}$$

$$\gamma + \mathrm{A} \rightarrow \mathrm{A}^{+} + e^{-} \text{ (been measured!)}$$

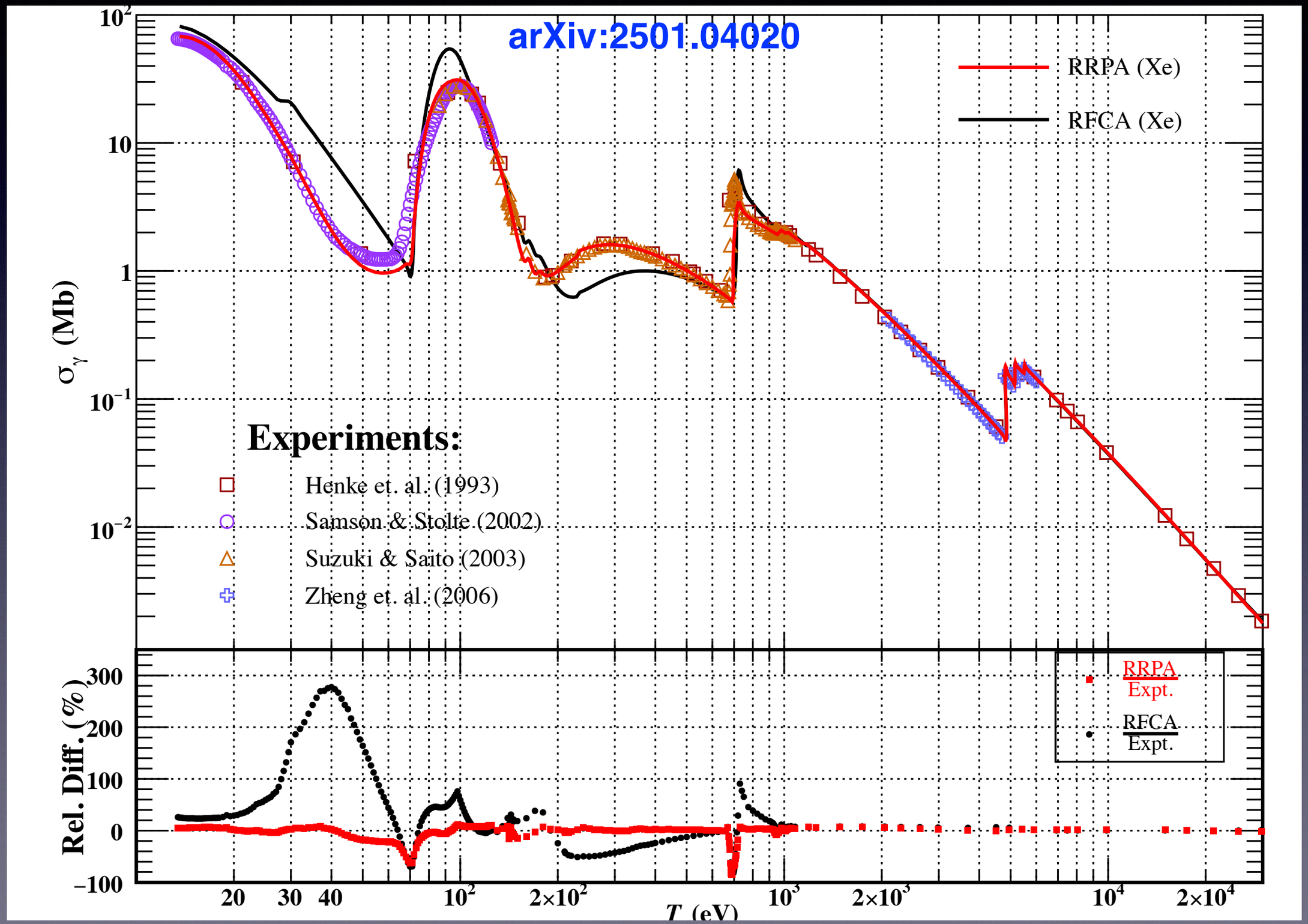
- Important test for the quality of the initial $|A\rangle$ and final state $|A^{+}, e^{-}\rangle$ wave functions
- Correlation effects are known to be important for excited states

Typical $\sigma_{\mathrm{abs}}(E_\gamma)$ w. CFA & HF

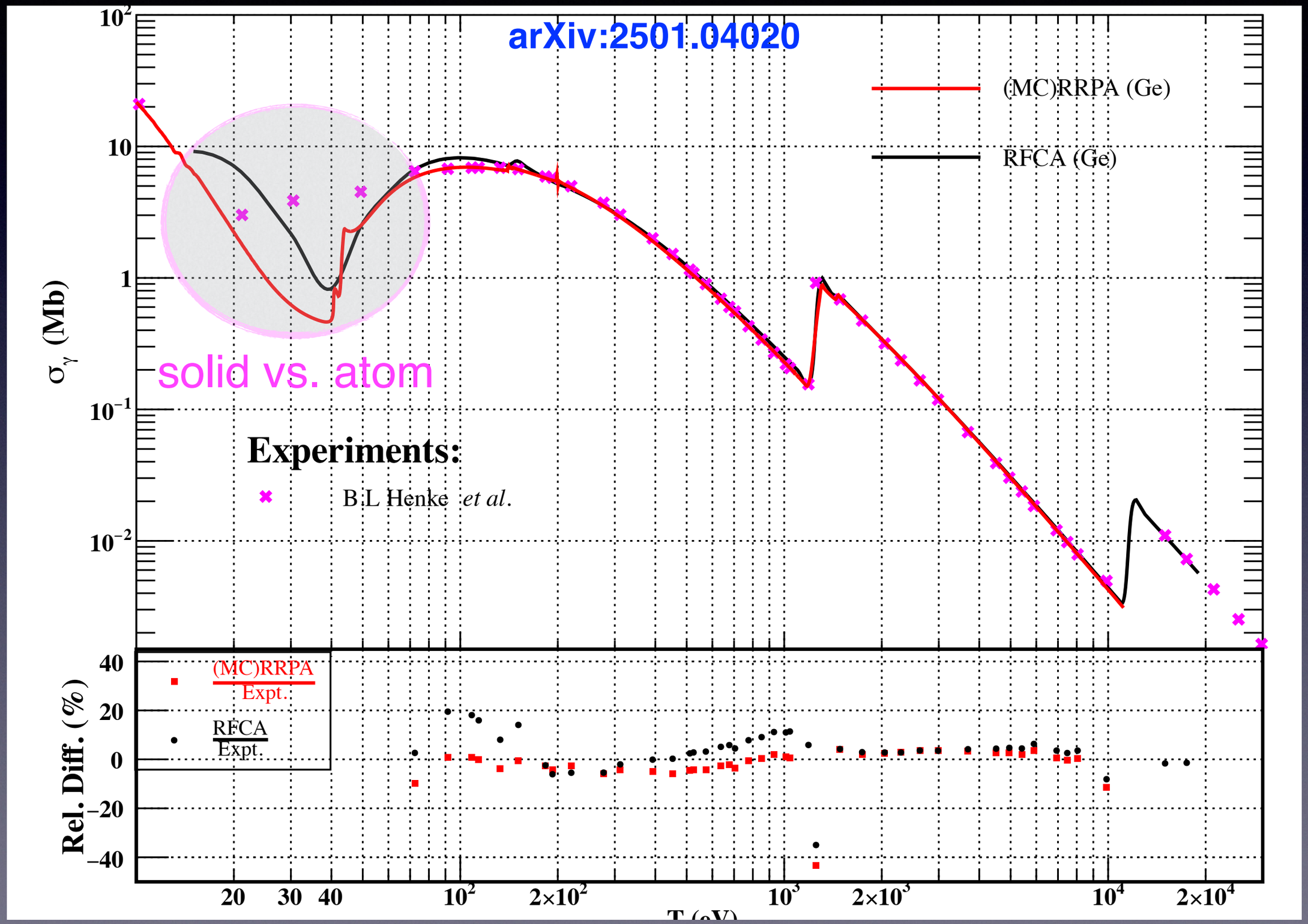


Manson & Cooper, PR 165, 126 (1968)
w. Herman-Skillman central potential

Xe Photoabsorption



Ge Photoabsorption



Computing Price

For electric dipole (E1) transition in photoionization of Xe:

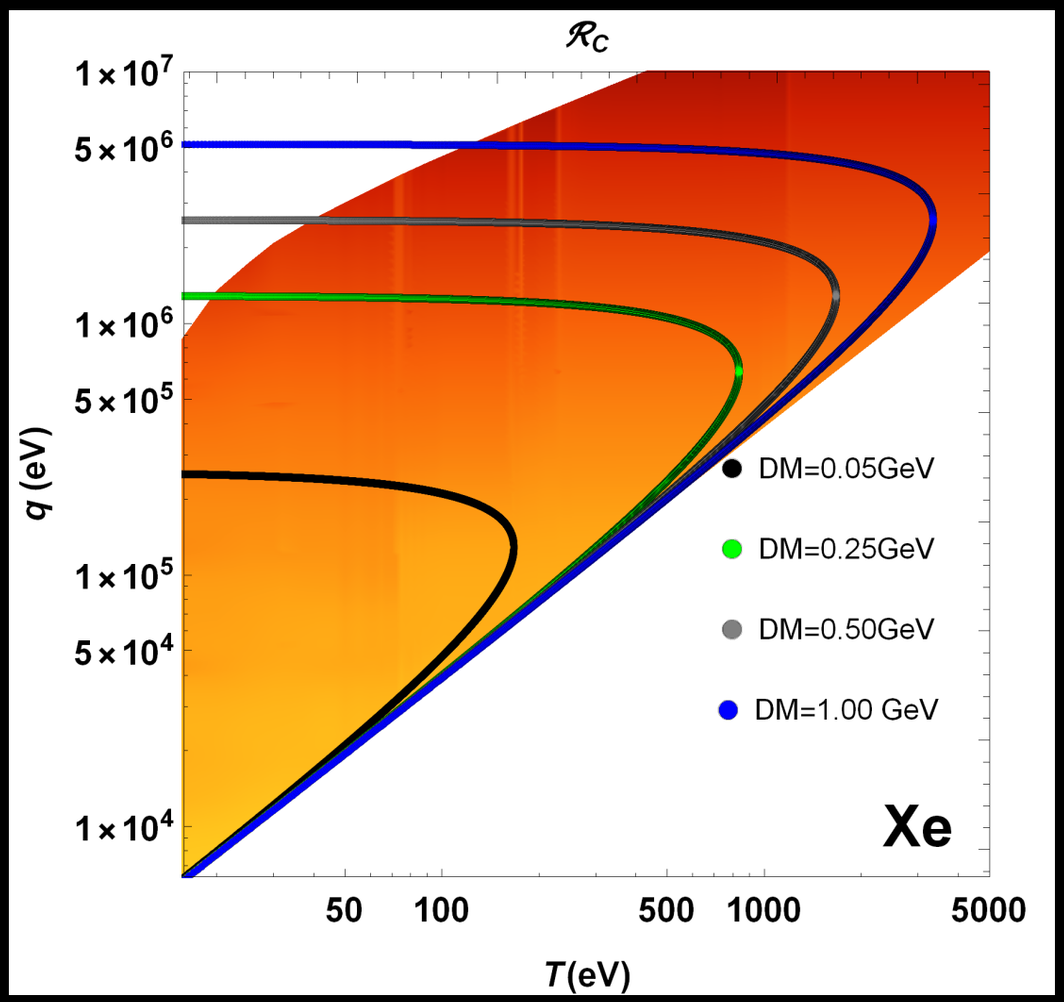
- CFA / HF(local): 17 uncoupled diff. equations
- HF: 17 coupled integro-diff. equations
- RRPA: 84 coupled integro-diff. equations

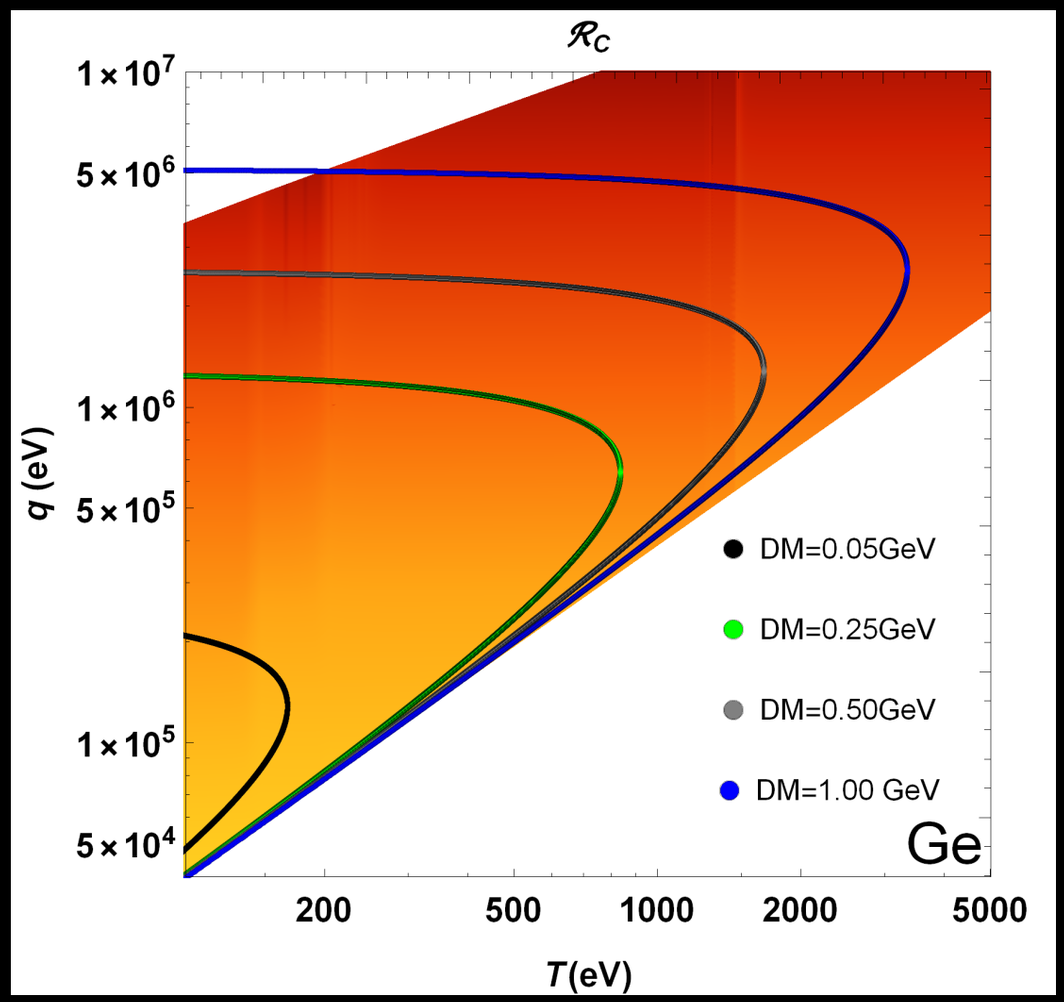
For sub-GeV DM:

- SI: C_J (J = 0 to 6)
- SD: L_J^5 (J = 0 to 6), E_J^5 & M_J^5 (J = 1 to 6)

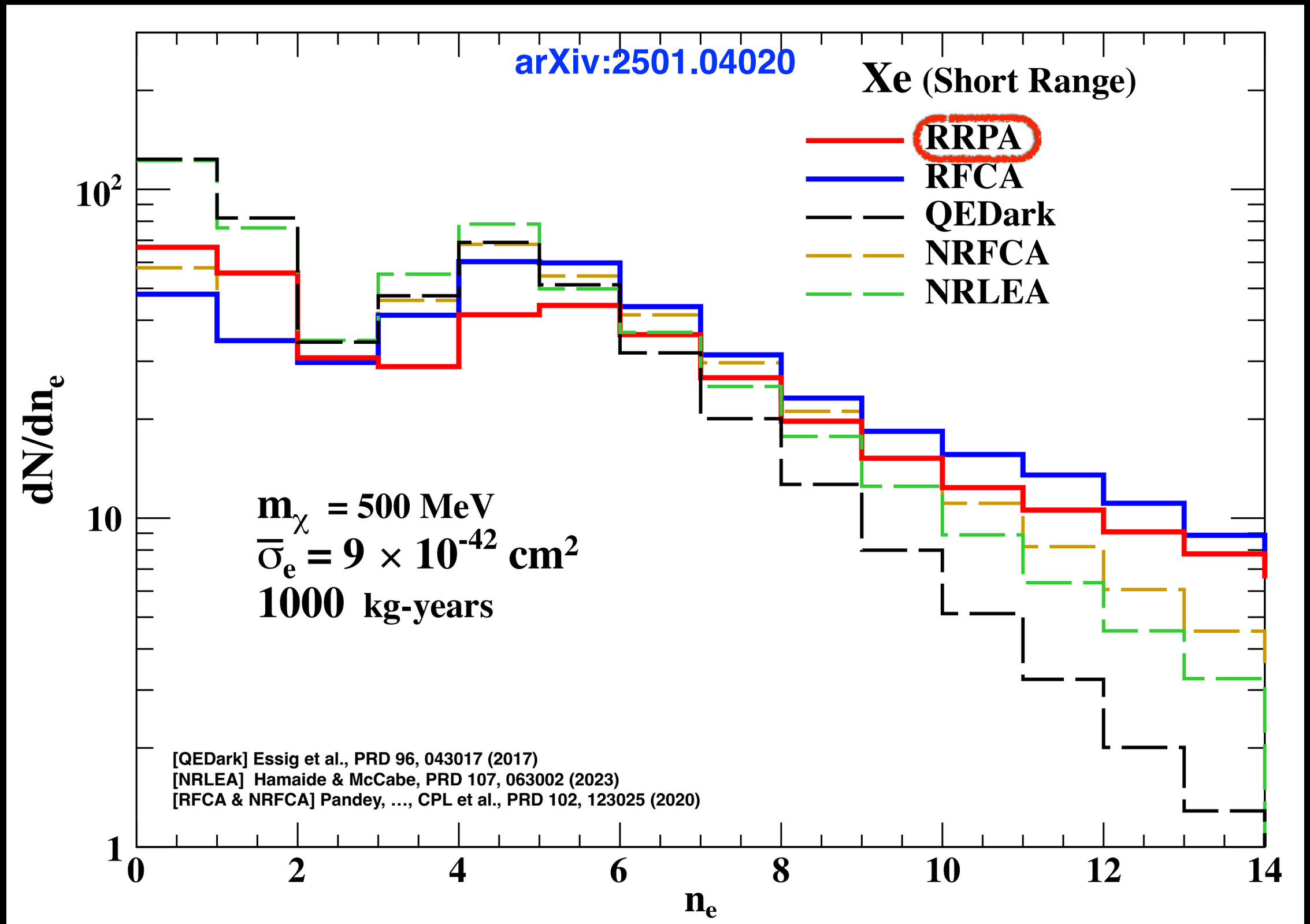
EG: Resp. Func. for SI Interact.

We provide data to make plots like below, and codes to calculate $d\sigma/dT$ and dR/dT :

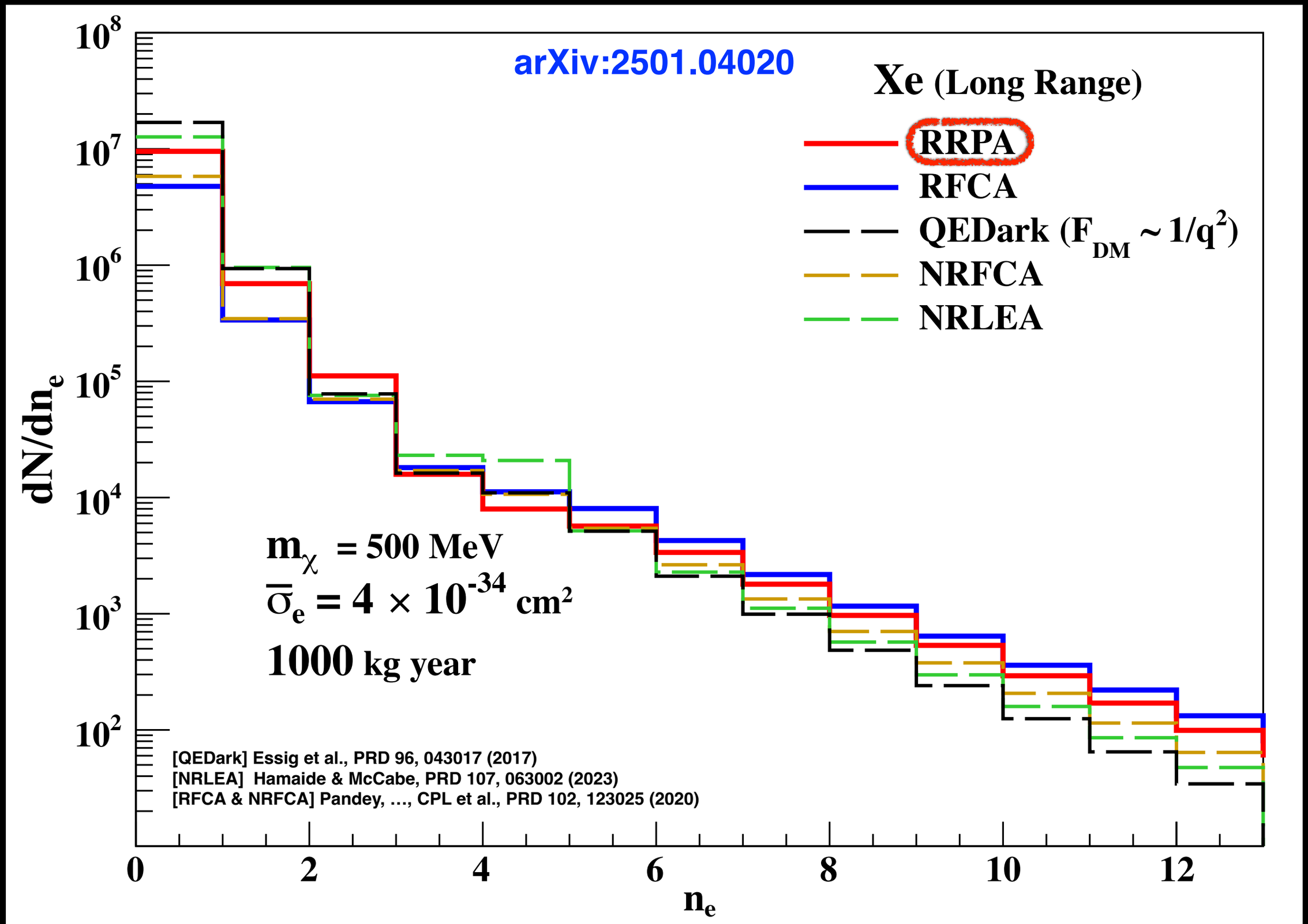




Converting dR/dT to dN/dn_e



Converting dR/dT to dN/dn$_e$



Progress & Outlook

- 1st version of the (MC)RRPA response functions for Xe (T>12eV) and Ge (T>80eV) with LO DM-electron interactions have been released.
- Other atomic species (noble gas etc.), DM-electron interaction terms (NLO, NNLO), parameter space of (T,q), will be gradually added into the database.

STAY TUNE!

Theoretical Dark Matter Collaboration (TDMC)

☆ Introduction

☆ Group Members

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2. Prof. Hsin-Chang Chi
3. Prof. C. -P. Liu (Spokesperson)
4. Dr. Mukesh K. Pandey
5. Dr. Chih-Pan Wu

Adjoint members @TEXONO

1. Prof. Henry T. Wong
2. Asst. Prof. Lakhwinder Singh

☆ Recent Works

☆ Publications

- **Click to find the data and codes**
- **Questions, comments, and requests are welcome!**

☆ Atomic Responses and Computational Tools

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