Gravitational Wave Birefringence in Fuzzy DM and Symmetron Cosmology

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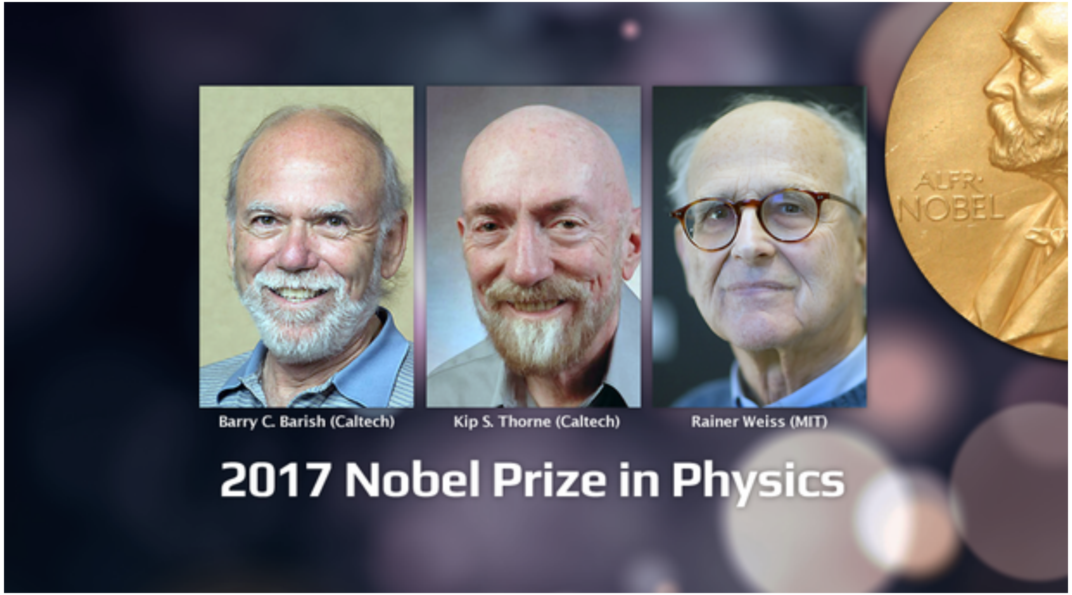
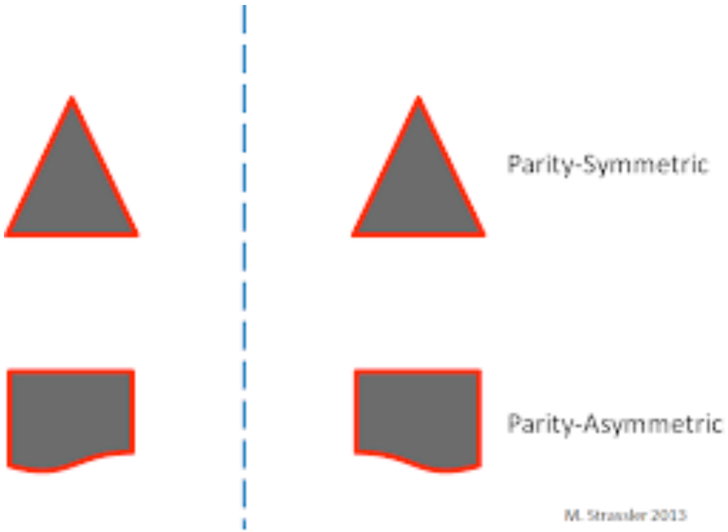
Introduction

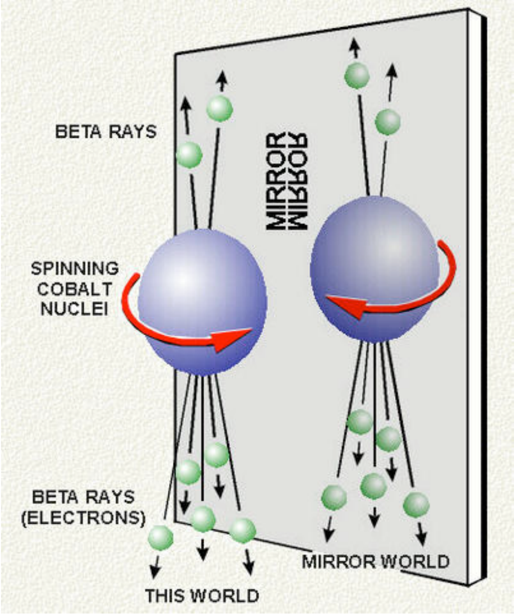
- Testing the nature of gravity is one of the key topics in the modern physics and astronomy.
- The direct detection of gravitational waves (GWs) by LIGO opened a new window to look into this important question.



Motivation

- It is well-known that there are four fundamental forces in nature, such as electromagnetic, strong, weak and gravitational interactions;
- Also, parity conservation is violated by the weak interaction;





Motivation

- **Question: Does gravity respect parity conservation?**
- In GR, parity reversal is a good symmetry;
- However, in many modified gravity theories, parity can be violated;
- GWs provide us a new tools to explore this fundamental symmetry in gravity.

GW Birefringence

Benchmark Model: Chern- Simons Modified Gravity

- One typical parity-violating gravity is the Chern-Simons Gravity

Chern-Simons Coupling

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\kappa R + \frac{\alpha}{4}\phi\, \tilde{R}^{\tau}{}_{\lambda\mu\nu}R^{\lambda}{}_{\tau}{}^{\mu\nu}\right],$$

Pontryagin density

$$R\tilde{R} \equiv \frac{1}{2}\tilde{\epsilon}^{\mu\nu\rho\sigma}R^{\tau}{}_{\lambda\mu\nu}R^{\lambda}{}_{\tau\rho\sigma}$$

Parity-odd

where $\kappa \equiv (16\pi G)^{-1}$ with G the Newton constant while α denotes the CS coupling with one length dimension.

- **ϕ** is an **axion field**, which can play a role of **dark matter** or **dark energy**;

- Due to the CS coupling, the axion background in the Universe behaves as the birefringence material for GW propagation.

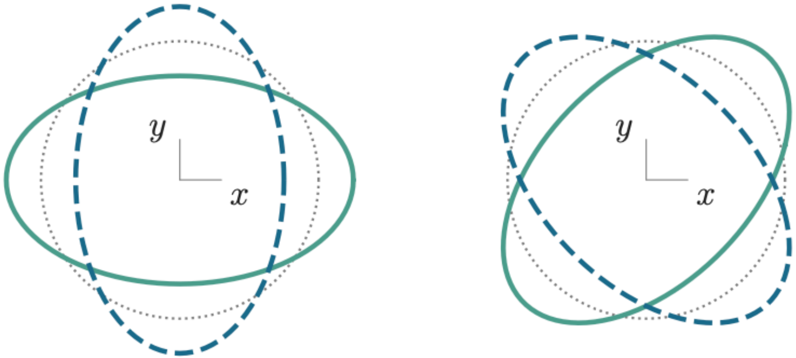
Jackiw & Pi (2003); Alexander & Yunes (2009)

Two GW Polarization Bases

- Linearly Polarized GWs: plus (+) and cross (×) polarizations

$$(h_{ij}) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

A GW moves along z-direction

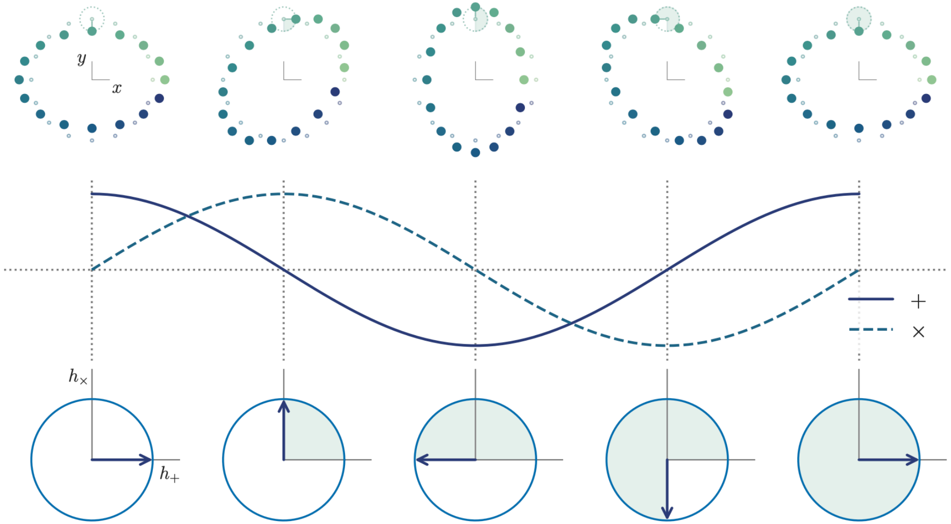


Two GW Polarization Bases

- Circularly Polarized GWs: Left-handed (L) and Right-handed (R)

$$\tilde{h}_{R/L} = \frac{1}{\sqrt{2}} \left(\tilde{h}_+ \mp i \tilde{h}_\times \right) ,$$

$$h^R_{ij}(t; f_0) = \int_{-\infty}^{+\infty} \tilde{h}^R_{ij}(f)\, e^{-i2\pi f t}\, \mathrm{d}f = \frac{1}{\sqrt{2}} (e^+_{ij} \cos \omega_0 t + e^\times_{ij} \sin \omega_0 t) ,$$



Right-handed

Credit: M. Isi(2022)

GW Birefringence in CS Gravity: Earlier Studies

- CS gravity can generate GW birefringence in the axion background: the left- and right-handed circular polarizations propagate differently
- Background: FRW metric + spatially homogeneous scalar field
- Modified GW Equations of Motion

$$ds^2 = a(\eta)^2[-d\eta^2 + (\delta_{ij} + h_{ij})]$$

$$\Box h_{\mathrm{R,L}} = -\frac{i\lambda_{\mathrm{R,L}}\alpha}{\kappa a^2}\left[-\frac{1}{a^2}(\phi'' - 2\mathcal{H}\phi')\partial_z h'_{\mathrm{R,L}} + \phi'\Box\partial_z h_{\mathrm{R,L}}\right]$$

$$\lambda_{\mathrm{R,L}} = \pm 1$$

Parity Violation

- Dispersion relation:

$$\omega^2 = k^2 - i\omega[2\mathcal{H} + 4\lambda_{\mathrm{R,L}}\alpha\phi' k\mathcal{H}/(\kappa a^2)]\,,$$

GW Birefringence in CS Gravity: Earlier Studies

- Dispersion relation:

$$\omega^2 = k^2 - i\omega[2\mathcal{H} + 4\lambda_{\rm R,L}\alpha\phi' k\mathcal{H}/(\kappa a^2)]\,,$$

- Amplitude Birefringence:

$$h^{\rm obs}_{R,L}(f) = h^{\rm GR}_{R,L}(f) \times \exp\left(\mp\kappa_A \times \frac{d_c}{\rm Gpc} \times \frac{f}{100\,\rm Hz}\right)$$

$$\kappa_A \equiv 4\pi\alpha\dot{\phi}_0 H_0/\kappa\,,$$

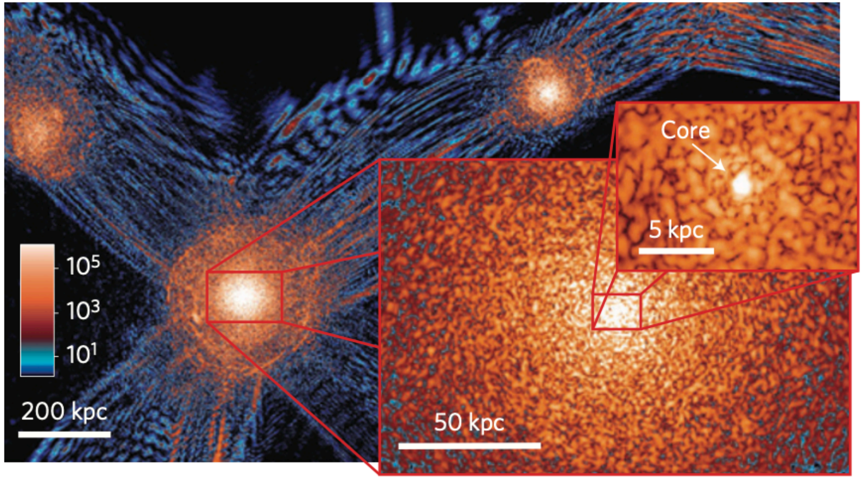
- We can constrain $\boldsymbol{\kappa}_A$ by modifying the GW waveform template with this birefringence factor and comparing with the observed GW events.

- GWTC-3 data: $\kappa = -0.019^{+0.038}_{-0.029}$ T.C.K.Ng, et al. (2023)
- GW170817 (multi-messanger): $\kappa_A = -0.12^{+0.60}_{-0.61}$ M. Lagos, et al. (2024)

GW Birefringence over Nontrivial Scalar Spatial Distributions

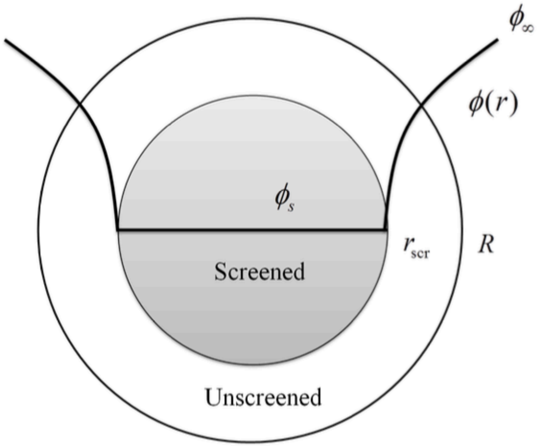
- In the real world, the ALP should have spatial dependence, no matter if it is **dark matter** or **dark energy** candidate.

Fuzzy DM



Solve small-scale structure problems

Symmetron



Screening to evade fifth force constraint

Fuzzy Dark Matter: GW Birefringence inside the Milky Way

- GW birefringence in an FDM profile with spatial variations in CS gravity

$$\Box h_{\mathrm{R,L}} \mp i(\alpha/\kappa)\partial^{\alpha}\left[\partial_z\phi\partial_\alpha\partial_t - \partial_t\phi\partial_\alpha\partial_z\right] h_{\mathrm{R,L}} = 0\,.$$

- Since we are mostly interested in effects around the galactic scale, we can ignore the cosmological expansion, so that we take the flat metric.

- Further assume GW wavelength are much smaller than the variation of the FDM background profile, $\partial_{t,z}\phi \ll \omega, k.$ we can apply the **Eikonal Approximation** to perform our calculation.

Eikonal Approximation

- GW waveform: $h_{\mathrm{R,L}} = h^0_{\mathrm{R,L}} e^{iS}$, where the dominant evolution comes from the phase S, while $h^0_{\mathrm{R,L}}$ is slowly varying.

$$\omega = -\partial_t S\,, \quad k = \partial_z S$$

- Dispersion relations:

$$D^{\pm} = (\omega^2 - k^2)\left[1 \mp \frac{\alpha}{\kappa}(\omega\partial_z\phi + k\partial_t\phi)\right] \mp \frac{i\alpha}{\kappa}\left[(\omega^2 + k^2)\partial_t\partial_z\phi + \omega k(\partial_z^2\phi + \partial_t^2\phi)\right] = 0$$

Real part $\gg$ Imaginary part $\quad \partial_{t,z}\phi \ll \omega, k.$

- Real part: dispersion → Velocity birefringence

- Imaginary part: dissipation → Amplitude birefringence

Eikonal Approximation: Dispersion

- Leading-order dispersion relation:

$$D^{\pm} = (\omega^2 - k^2)\left[1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi)\right] = 0\,.$$

- w=k is always the solution, which means that both polarizations of GWs move in the speed of light even there is an axion background.
- We can check this by examining the GW paths:

$$\frac{dx^i}{dt} = -\frac{\partial D^{\pm}/\partial k_i}{\partial D^{\pm}/\partial\omega} = \frac{2k\left[1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi)\right] \pm (\alpha/\kappa)(\omega^2 - k^2)\partial_t\phi}{2\omega\left[1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi)\right] \mp (\alpha/\kappa)(\omega^2 - k^2)\partial_z\phi}\delta^i_z \approx \delta^i_z\,,$$

$$\frac{dk_i}{dt} = \frac{\partial D^{\pm}/\partial x^i}{\partial D^{\pm}/\partial\omega} = \frac{\mp(\alpha/\kappa)(\omega^2 - k^2)(\omega\partial_z\partial_i\phi + k\partial_i\partial_t\phi)}{2\omega\left[1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi)\right] \mp (\alpha/\kappa)(\omega^2 - k^2)\partial_z\phi} = 0\,,$$

$$\frac{d\omega}{dt} = -\frac{\partial D^{\pm}/\partial t}{\partial D^{\pm}/\partial\omega} = \frac{\pm(\alpha/\kappa)(\omega^2 - k^2)(\omega\partial_z\partial_t\phi + k\partial_t^2\phi)}{2\omega\left[1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi)\right] \mp (\alpha/\kappa)(\omega^2 - k^2)\partial_z\phi} = 0\,,$$

➡ The frequency and wavenumber of GW keep the same, and GWs still travel along a straight line for both polarizations.

Eikonal Approximation: Dissipation

- Dissipation effects:

$$D^{\pm} = (\omega^2 - k^2)\left[1 \mp \frac{\alpha}{\kappa}(\omega\partial_z\phi + k\partial_t\phi)\right] \mp \frac{i\alpha}{\kappa}\left[(\omega^2 + k^2)\partial_t\partial_z\phi + \omega k(\partial_z^2\phi + \partial_t^2\phi)\right] = 0$$

- Here w and k can take complex values

$$\begin{aligned}
\frac{dx^i}{dt} &= -\frac{\partial D^{\pm}/\partial k_i}{\partial D^{\pm}/\partial\omega} = \frac{k\delta^i_z\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \pm i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (\omega/k)(\partial_z^2\phi + \partial_t^2\phi)]\right\}}{\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \{1 \pm i(\alpha/\kappa)[2\partial_t\partial_z\phi + (\partial_t^2\phi + \partial_z^2\phi)]\}\delta^i_z\,, \\
\frac{dk_i}{dt} &= \frac{\partial D^{\pm}/\partial x^i}{\partial D^{\pm}/\partial\omega} = \frac{\mp i(\alpha/\kappa)\left[(\omega^2 + k^2)\partial_t\partial_z\partial_i\phi + \omega k\partial_i(\partial_z\phi + \partial_t^2\phi)\right]}{2\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \mp i(\alpha\omega/2\kappa)\left[2\partial_t\partial_z\partial_i\phi + \partial_t^2\partial_i\phi + \partial_z^2\partial_i\phi\right]\,, \\
\frac{d\omega}{dt} &= -\frac{\partial D^{\pm}/\partial t}{\partial D^{\pm}/\partial\omega} = \frac{\pm i(\alpha/\kappa)\left[(\omega^2 + k^2)\partial_t^2\partial_z\phi + \omega k(\partial_t^3\phi + \partial_t\partial_z^2\phi)\right]}{2\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \pm i(\alpha\omega/2\kappa)\left[2\partial_t^2\partial_z\phi + \partial_t\partial_z^2\phi + \partial_t^3\phi\right]\,,
\end{aligned} \tag{36}$$

Eikonal Approximation: Dissipation

- Define
$$\dot{\phi} \equiv \frac{d\phi}{dt} = \partial_t \phi + \partial_z \phi \frac{dz}{dt} \approx \partial_t \phi + \partial_z \phi \,,$$

- Integration of above equations gives:
$$\Delta k_i = \mp i \alpha \omega \partial_i \dot{\phi}/(2\kappa) \,,$$
$$\Delta \omega = \pm i \alpha \omega \partial_t \dot{\phi}/(2\kappa) \,.$$

- The phase of the GW varies:
$$\Delta S = -\int_{t_e}^{t_o} dt \Delta\omega + \int_{\mathbf{x}_e}^{\mathbf{x}_o} dx^i \Delta k_i = \mp i \alpha \omega (\dot{\phi}_o - \dot{\phi}_e)/(2\kappa) \,,$$

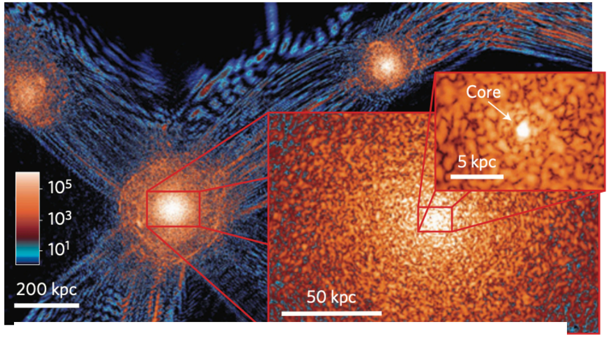
- GW amplitude birefringence:
$$h_{\mathrm{R,L}} = h^0_{\mathrm{R,L}} \exp(i\Delta S) = h^0_{\mathrm{R,L}} \exp\left(\pm \alpha \omega (\dot{\phi}_o - \dot{\phi}_e)/(2\kappa)\right) \,,$$

Fuzzy Dark Matter: GW Birefringence inside the Milky Way

- Locally, the fuzzy DM profile is given by

$$\phi(t, \mathbf{x}) = \frac{\sqrt{2\rho_{\rm NFW}}}{m_\phi} \cos\left(m_\phi t + \alpha(\mathbf{x})\right) .$$

A. Khmelnitsky & V. Rubakov (2014)



- Due to $\partial_t \boldsymbol{\phi} \gg \partial_z \boldsymbol{\phi}$ for $m_\phi \sim 10^{-22}$ eV

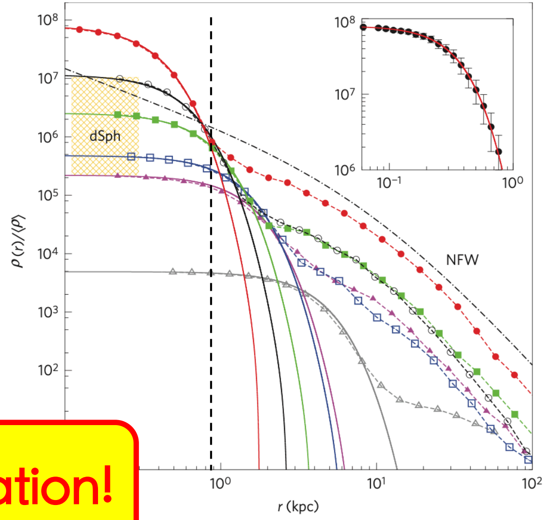
$$\dot{\phi}_o \approx \partial_t \phi = \sqrt{2\rho_{\rm NFW}} \cos\left(m_\phi t + \alpha(\mathbf{x})\right)$$

- The magnitude birefringence can be given by

$$h^{\rm obs}_{R,L}(f) = h^{\rm GR}_{R,L}(f) \times \exp\left(\pm\kappa'_A \times \frac{f}{100\ {\rm Hz}}\right)$$

$$\kappa'_A \equiv \pi(\alpha/\kappa)\sqrt{2\rho_\odot}\,\sin(m_\phi t + \alpha_0) .$$

Time Modulation!



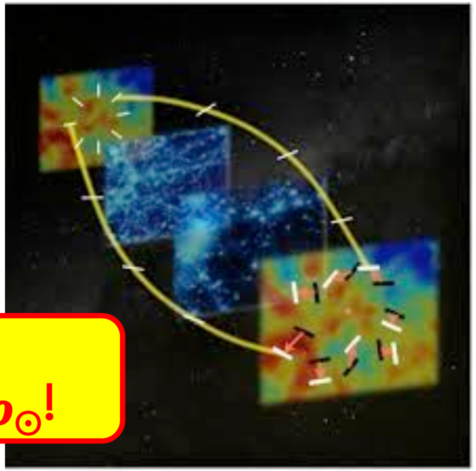
Z.-X. Xiong & **DH**: 2406.13394

Fuzzy Dark Matter: Extra-Galactic GW Birefringence

- FDM Field in Cosmology: $\phi(t) = \phi_0 \, (1/a)^{3/2} \cos(m_\phi t + \alpha_c) \,,$ $\phi_0 \; = \; \sqrt{2\rho_{\rm DM}}/m_\phi$
- The field amplitude and phase should change at different spatial points, with the typical scale as the inverse of de Broglie wavelength $m_\phi v$. For $v \ll 1$, such variations can be ignored compared with m_ϕ.
- GW propagation: $\Box h_{\rm R,L} = \pm \frac{i\alpha}{\kappa a^2} \left[\frac{1}{a^2} (\phi'' - 2\mathcal{H}\phi') \partial_z h'_{\rm R,L} - \phi' \Box \partial_z h_{\rm R,L} \right] ,$
- GW birefringence:

$$h^{\rm ex}_{\rm R,L}(f) = h^0_{\rm R,L}(f) \exp \left(\pm \frac{\kappa^{\rm ex}_A}{1 \ {\rm Gpc}} \times \frac{f}{100 \ {\rm Hz}} \right) ,$$

$$\kappa^{\rm ex}_A \equiv \alpha \pi \sqrt{2\rho_{\rm DM}} \sin(m_\phi t + \alpha_c)/\kappa \, .$$



Ignored for $\rho_{\rm DM} \ll \rho_\odot$!

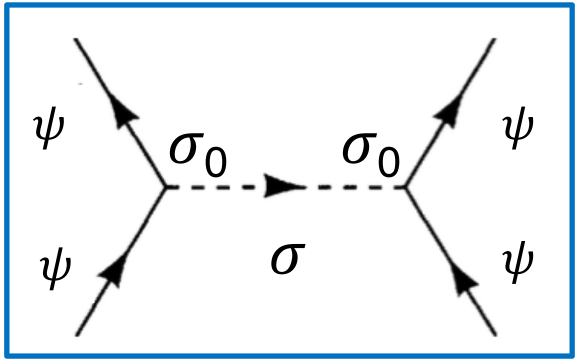
 Z.-X. Xiong & **DH**: 2406.13394

- **Symmetron** can be a **dark energy** candidate, which can avoid the strong fifth force constraints in the solar system by screening the light scalar field.
- Symmetron Scalar σ + Z_2 Symmetry: $\sigma \rightarrow -\sigma$

$$S_0 = \int d^4x\sqrt{-g}\left[\kappa R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma)\right] + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_{\rm m}(\psi, \tilde{g}_{\mu\nu})$$

- To guarantee the Weak Equivalence Principle, the matter fields ψ should couple universally to the Jordan-frame metric $\tilde{g}_{\mu\nu} \equiv A^2(\sigma)g_{\mu\nu}$
- Due to the Z_2 symmetry, the coupling function A(σ) is

$$A(\sigma) = 1 + \frac{\sigma^2}{2M^2},$$



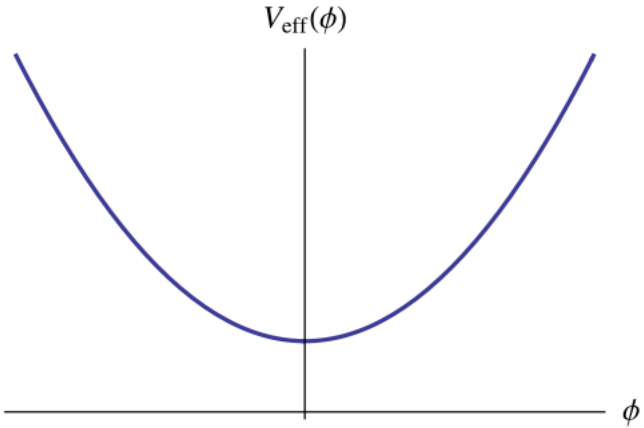
The symmetron-matter coupling ∝ VEV of σ

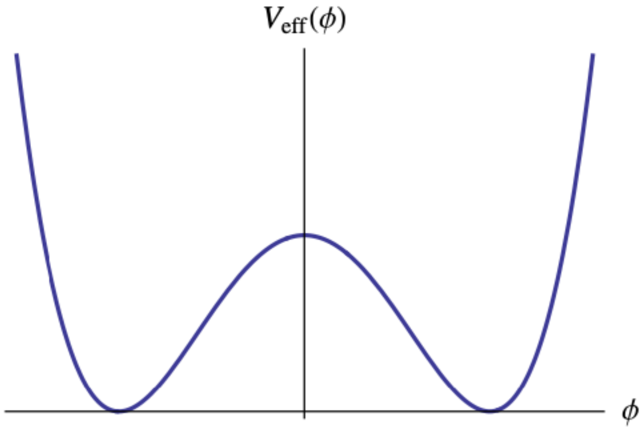
K.Hinterbichler & J. Khoury, et al. (2010,2011)

Introduction to Symmetron

- Symmetron Effective Potential $V_{\text{eff}}(\sigma) = V(\sigma) + \sum_i A^{1-3w_i}(\sigma)\rho_i$

$$V(\sigma) = -\frac{1}{2}\mu^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\mu^4}{4\lambda} = \frac{\lambda}{4}\left(\sigma^2 - \frac{\mu^2}{\lambda}\right)^2$$





High density → σ_0=0 → suppressed scalar force

Low density → $\sigma_0 \neq 0$ → scalar force is effective

K. Hinterbichler & J. Khoury, et al. (2010,2011)

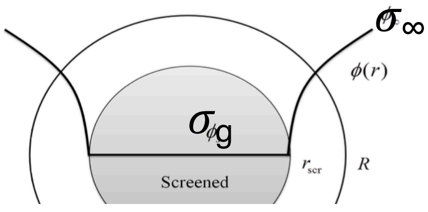
Symmetron: GW Birefringence inside the Milky Way

Z_2 Symmetry → $S_{\rm CS} = \int dx^4 \sqrt{-g}(\alpha/4)\sigma^2 R^{\tau}{}_{\lambda\mu\nu}\widetilde{R}^{\lambda}{}_{\tau}{}^{\mu\nu}$

- Different from original CS gravity, this new term explicitly breaks parity
- Directly apply the results in the conventional CS gravity by $\phi \rightarrow \sigma^2$.
- Amplitude Birefringence by eikonal approximation

$$h_{\rm R,L}(f) = h^{\rm GR}_{\rm R,L}(f)\exp\left(i\int_{\mathbf{x}_{\rm out}}^{\mathbf{x}_{\rm in}} \Delta k_i dx^i\right) = h^{\rm GR}_{\rm R,L}(f)\exp\left(\lambda_{\rm R,L}\kappa'_A \frac{f}{100\rm Hz}\right)$$

where $\kappa'_A \equiv (2\pi\alpha/\kappa)(\sigma_{\rm in}\partial_z\sigma_{\rm in} - \sigma_{\rm out}\partial_z\sigma_{\rm out})$



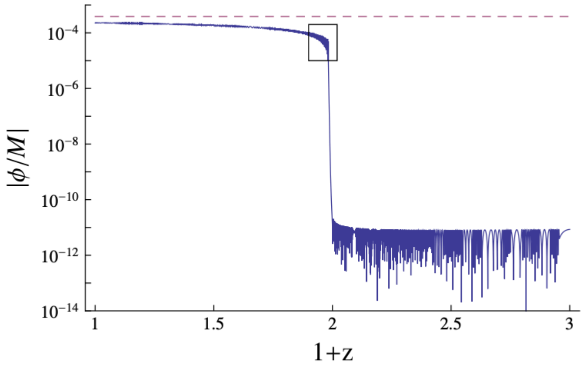
$\partial_z\sigma_{in} \approx \partial_z\sigma_{out} \approx 0$

Screening → Suppress GWB!

Symmetron: Extra-Galactic GW Birefringence

- Cosmological Symmetron Profile: adiabatic solution to its effective potential

$$V_{\rm eff} = V(\sigma) + A(\sigma)\rho_m + A^4(\sigma)\rho_{\rm de}$$
$$= V(\sigma) + \rho_c\left[A(\sigma)\Omega_m a^{-3} + A^4(\sigma)(1-\Omega_m)\right]$$



$$\Rightarrow \quad \sigma^2 = \frac{1}{\lambda}\left[\mu^2 - \frac{\rho_c(\Omega_m a^{-3} + 4\Omega_\Lambda)}{M^2}\right]$$

$$\Box h_{\rm R,L} = -\frac{i\lambda_{\rm R,L}\alpha}{\kappa a^2}\left[-\frac{1}{a^2}\left[(\sigma^2)'' - 2\mathcal{H}(\sigma^2)'\right]\partial_z h'_{\rm R,L} + (\sigma^2)'\Box\partial_z h_{\rm R,L}\right] \qquad \kappa_A \equiv \frac{72\pi\alpha\Omega_m H_0^4}{\lambda M^2}$$

- Amplitude Birefringence: $h_{\rm R,L}(f) = h^{\rm GR}_{\rm R,L}e^{i\Delta S_c} \approx h^{\rm GR}_{\rm R,L}\exp\left(-\lambda_{\rm R,L}\kappa_A\frac{f}{100\ {\rm Hz}}\frac{d_c}{1\ {\rm Gpc}}\right)$
  - GWTC-3 data: T.C.K.Ng, et al. (2023)

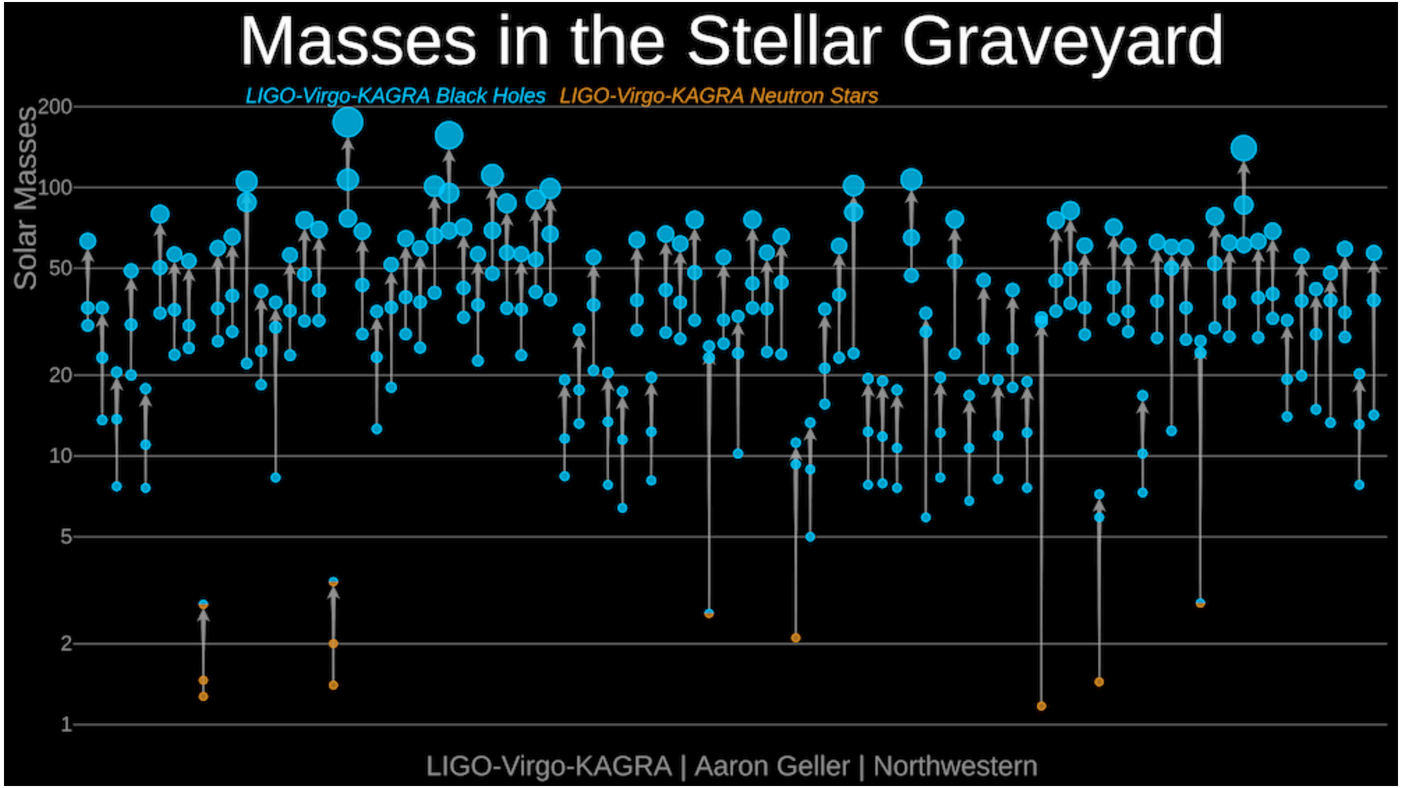
$$\kappa = -0.019^{+0.038}_{-0.029} \quad \Rightarrow \quad \alpha(M/M_{\rm Pl})^4 \lesssim 8\times10^{-22}\ {\rm Gpc}^2$$

Conclusions

- Observation of **GW birefringence** is a remarkable way to test parity violation in gravity;
- We have studied the GW birefringence over a nontrivial spatial distribution of the fuzzy DM and symmetron in the Milky Way ;
- It is found that both GW circular polarizations moves with the speed of light in both models, while their relative amplitudes would be changed, generating the amplitude birefringence!
- For the fuzzy DM, its galactic distribution produces the dominant effect, which shows a remarkable time modulation.
- In the symmetron model, we introduce a new Z_2-symmetric CS-like interaction, which generates the GWB. It is interesting to note that the galactic contribution is suppressed due to its screening mechanism.

THANK YOU !

- Up to O3 run, the LVK Collaboration observed 90 GW events.



Directional Dependence of GW Birefringence in the MW

- Subdominant Spatial Dependence: $\partial_z \boldsymbol{\phi} \ll \partial_t \boldsymbol{\phi}$ for $m_{\boldsymbol{\phi}}$ ~10^{-22} eV

$$\kappa'_A = \pi(\alpha/\kappa)\partial_r\phi(R_\odot)\cos\langle\mathbf{k},\mathbf{r}\rangle\,.$$

where $\langle\mathbf{k},\mathbf{r}\rangle$ represents the angle between the incident GW and the radial direction of the Solar system in the Milky Way. The spatial variation of the Solar system r is given by

$$\partial_r\phi(R_\odot) = -\frac{1}{m_\phi R_\odot}\sqrt{\frac{\rho_\odot}{2}}\left(\frac{1+3R_\odot/r_s}{1+R_\odot/r_s}\right)\cos(m_\phi t+\alpha_0)\,, \tag{57}$$

- For $m_{\boldsymbol{\phi}}$ ~10^{-22} eV & $R_\odot$ = 8 kpc

$$\frac{\partial_z\phi}{\partial_t\phi} \sim \frac{1}{m_\phi R_\odot} \sim \mathcal{O}(10^{-5})\,.$$