Lensing signatures of self-interacting dark matter halos : an analytic approach

arXiv 2502.14964

JCAP 08 (2025) 048

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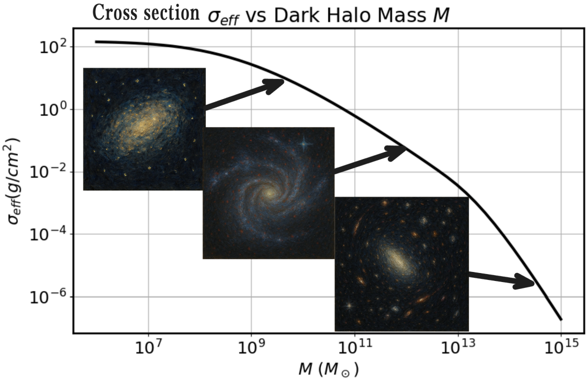
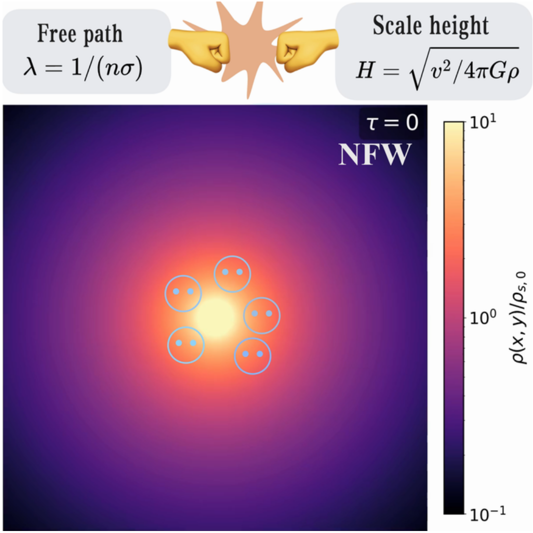
August 26, 2025, SiChuan

OUTLINE

1. Why an Analytic Model for SIDM Lensing?

SIDM halos: gravothermal evolution


Cross section σ_{eff} vs Dark Halo Mass M


Free path $\lambda = 1/(n\sigma)$

Scale height $H = \sqrt{v^2/4\pi G\rho}$

1️⃣ Core Formation $\lambda > H$

2️⃣ Self-similar Collapse $\lambda \sim H$

3️⃣ Post Self-similar Collapse $\lambda < H$

Generality–the gravothermal phase

Free path
$\lambda = 1/(n\sigma)$

Scale height
$H = \sqrt{v^2/4\pi G\rho}$

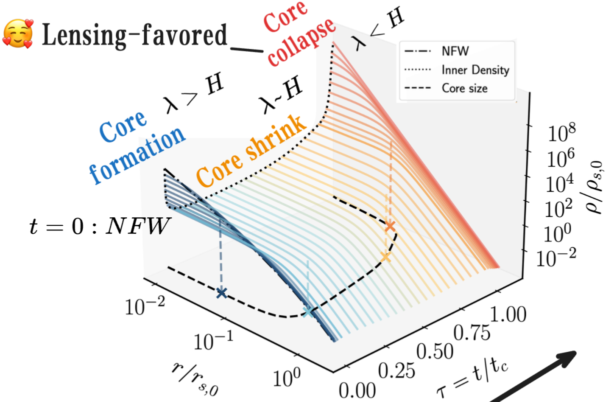
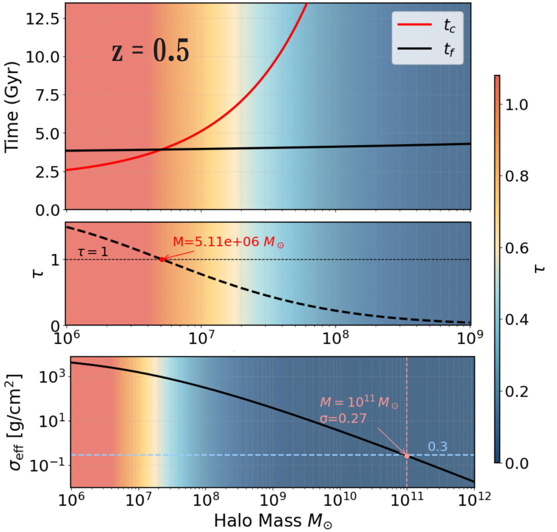


Figure 1. SIDM density profile

$$t_c \sim 1/(\sigma v n)$$

Further background: Essig+ (PRL,2019) arXiv 1809.01144 · Outmezguine+ (2022) arXiv 2204.06568 · Yang+ (2023) arXiv 2305.16176 · Zhong+ (2023) arXiv 2306.08028



Efficiency–parametric SIDM model

arXiv 2305.16176

Nbody → **Semi-analytic techniques** → **Parametric modeling** → **SIDM Density Profile**

🚩 **Conducting-fluid** → **Semi-analytic techniques**

[This Work]

➡️ Input

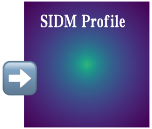
$$\rho_s = \rho_{s,0}\, f_\rho(\boxed{\tau})$$

$$r_s = r_{s,0}\, f_s(\tau)$$

$$r_c = r_{s,0} f_c(\tau)$$

$$\beta = f_\beta(\tau) \qquad \gamma = f_\gamma(\tau)$$

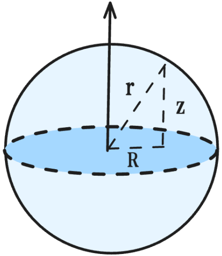
Output ➡️



🧐 Guess the form ⬇️

$$\rho(r) = \frac{\boxed{\rho_{s,0}}}{(r/\boxed{r_{s,0}})(1+r/r_{s,0})^2}$$

$$\rho_{\mathrm{PSIDM}}(r) = \frac{\boxed{\rho_s}}{\left[\left(\frac{r}{\boxed{r_s}}\right)^4 + \left(\frac{\boxed{r_c}}{r_s}\right)^4\right]^{\frac{\gamma}{4}} \cdot \left[1 + \left(\frac{r}{r_s}\right)^{\boxed{\beta}}\right]^{\frac{3-\boxed{\gamma}}{\beta}}}$$



📍 **Surface Density** → 📍 **Lensing Potential** → ✅ **Deflection Angle**

$$\Sigma(R) = \int_{-\infty}^{\infty} \rho_{SIDM}\, dz$$

$$\nabla^2\Psi(R) = 2\frac{\Sigma(R)}{\Sigma_{\mathrm{cr}}} = 2\kappa$$

$$\Sigma_{\mathrm{cr}} = \frac{c^2}{4\pi G}\frac{D_S}{D_L D_{LS}}$$

$$\nabla\hat{\Psi}(R) = \vec{\alpha}(R)$$

OUTLINE

2. Robustness

A parametric model for SIDM Lensing

$$\boxed{\Psi(R) = \frac{2}{\Sigma_{\rm cr}} \int_0^R s\Sigma(s) \ln\left(\frac{R}{s}\right) ds} \text{ — } \hat{\Psi} \equiv \frac{\Psi\Sigma_{\rm cr}}{\rho_{s,0} r_{s,0}^3}, \quad \hat{\Sigma} \equiv \frac{\Sigma}{\rho_{s,0} r_{s,0}^2} \longrightarrow \boxed{\hat{\Psi}(\hat{R}) = 2\int_0^{\hat{R}} s\hat{\Sigma}(s) \ln\left(\frac{\hat{R}}{s}\right) ds}$$

🧐 **Guess the form** ➡️

$$\boxed{\hat{\Psi}(\hat{R}) = a \ln\left(1 + b\hat{R} + c\hat{R}^2\right)^c - \ln\left(p\hat{R} + 1\right)^s}$$

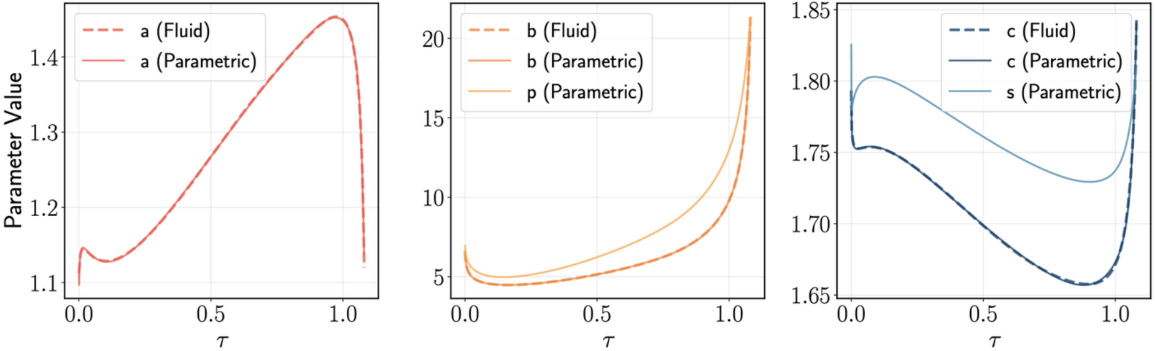


Figure 3. The lensing model parameters a, b, c, p, and s as functions of τ

Accruacy of the model

$$\hat{\alpha}(\hat{R}) = \frac{\partial \hat{\Psi}(\hat{R})}{\partial \hat{R}} = \frac{a \cdot c\left(b + 2c\hat{R}\right)\ln^{c-1}\left(b\hat{R} + c\hat{R}^2 + 1\right)}{b\hat{R} + c\hat{R}^2 + 1} - \frac{p \cdot s\ln^{s-1}\left(p\hat{R} + 1\right)}{p\hat{R} + 1} \longrightarrow \hat{\Sigma}(\hat{R}) = \frac{1}{2\hat{R}}\frac{\partial}{\partial \hat{R}}\left[\hat{R} \cdot \hat{\alpha}(\hat{R})\right]$$

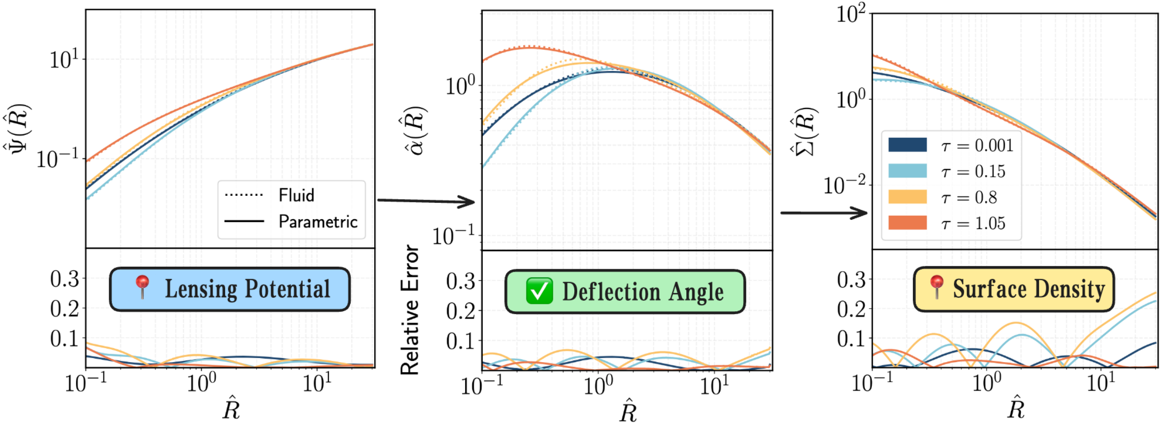
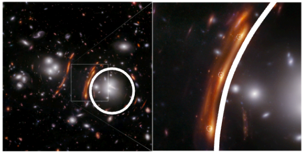


Figure 4. Comparison of simulated (dotted) and model-predicted (solid).

Isolated Halo–Critical Curves and Caustics



$$\det\left(\frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}}\right)=0$$

source plane

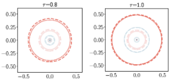
lens plane

For Fluid → **Using FFT** $k^2\tilde{\psi}(k)=2\tilde{\kappa}(k)$

Pseudo ellipsoid in surface density 👉

$$\hat{R}=R\sqrt{1-e\cos(2(\phi_0-\phi))}$$

$0.019 \lesssim \tau \lesssim 0.45$ below Σ_{cr}



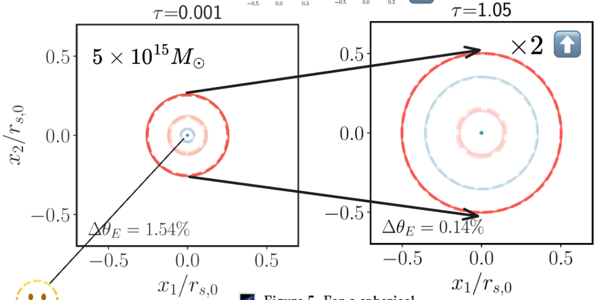
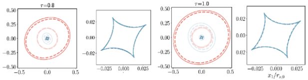


Figure 5. For a spherical



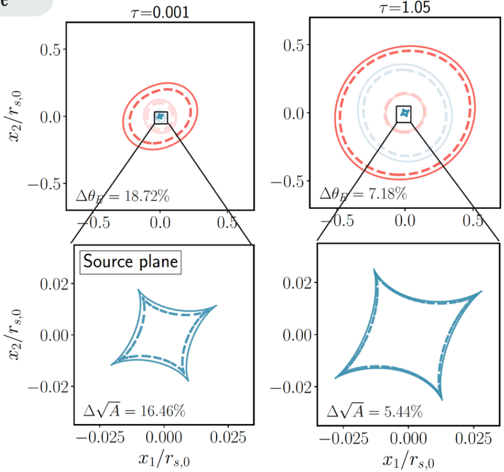


Figure 6. With a constant ellipticity e = 0.6.

🟦 Tangential Caustics ⬜ Radial Caustics 🟥 Tangential Critical ▭ Radial Critical —— Parametric ---- Fluid

Isolated Halo–Lens equation and Einstein radius

Lens equation– $x_s = x_l - \frac{D_{LS}}{D_S}\alpha_i$

🥳 **It could potentially boost GGSL events!**

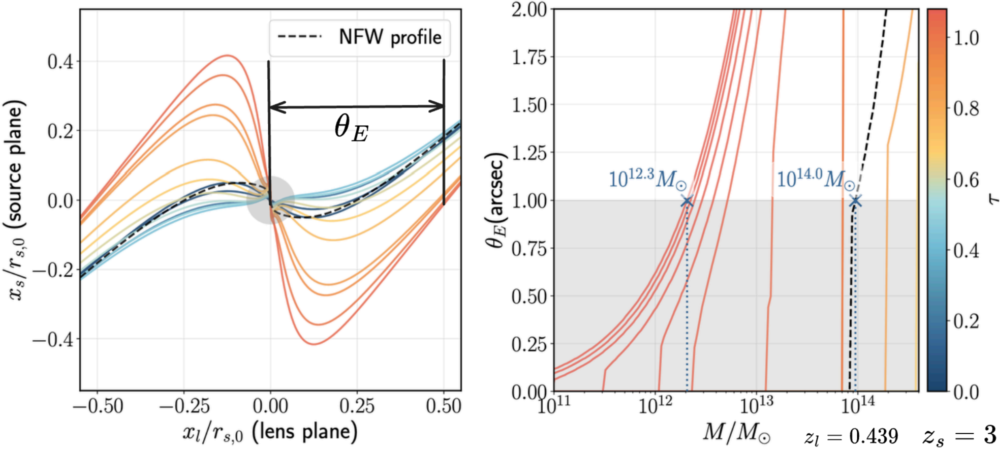
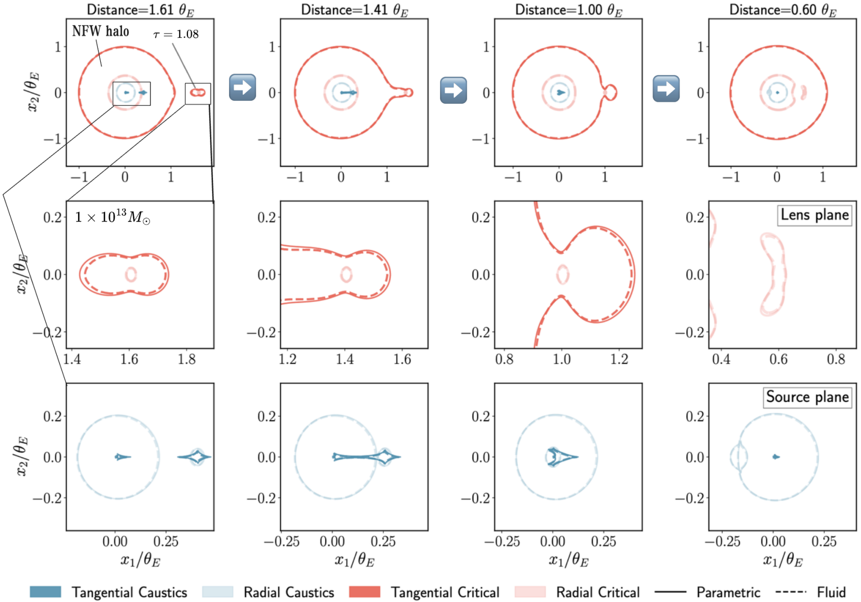


Figure 7. Lensing equation of isolated spherical SIDM halos and Einstein Radial vs. Mass

Host halo + Subhalo–Critical Curves vs. separation with subhalo $\tau = 1.08$



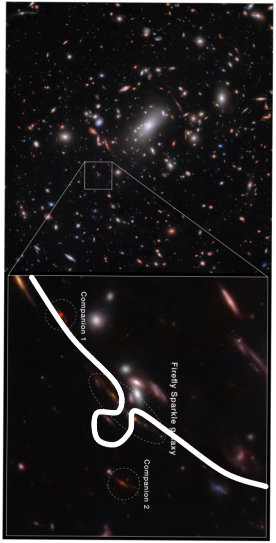


Figure 8. Subhalo embed in the Main halo vs. distance

Host halo + Subhalo–Critical Curves and Caustics vs. τ at fixed separation

Figure 9 GGSL probability ↗

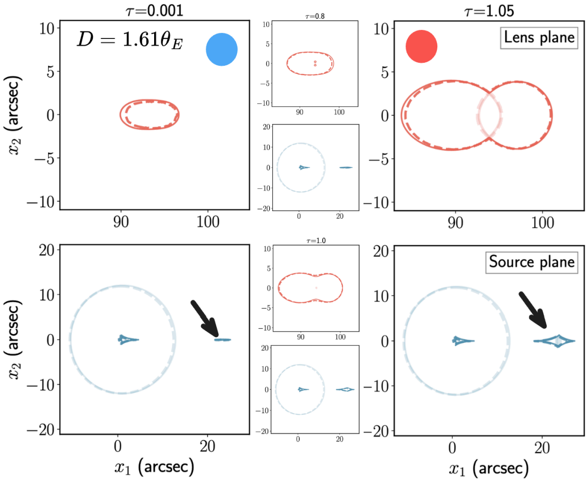
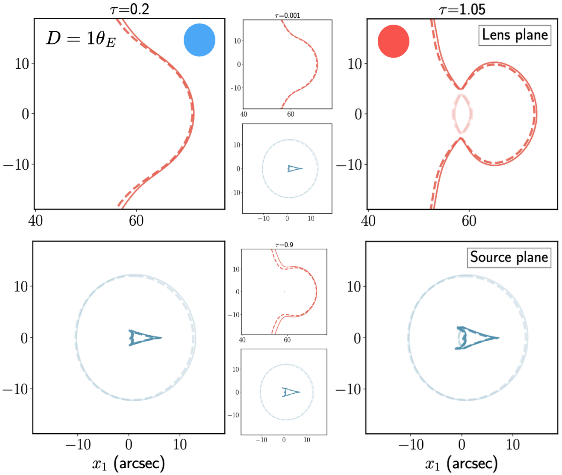


Figure 10 ✅ Very good agreement !



🟦 Tangential Caustics ⬜ Radial Caustics 🟥 Tangential Critical ▨ Radial Critical —— Parametric ---- Fluid

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3. Discussion and Application

Core collapsed halos in the self-similar regime

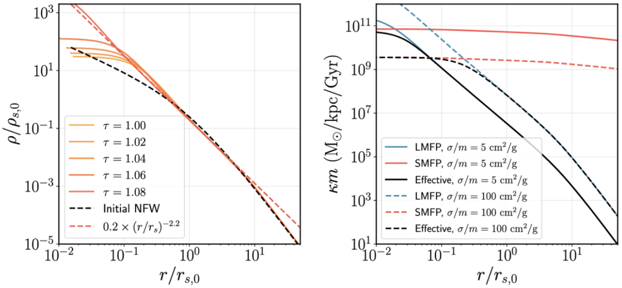


Figure 11. In the self-similar regime density profile

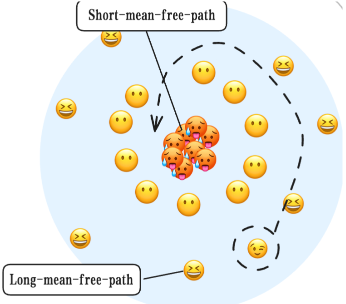
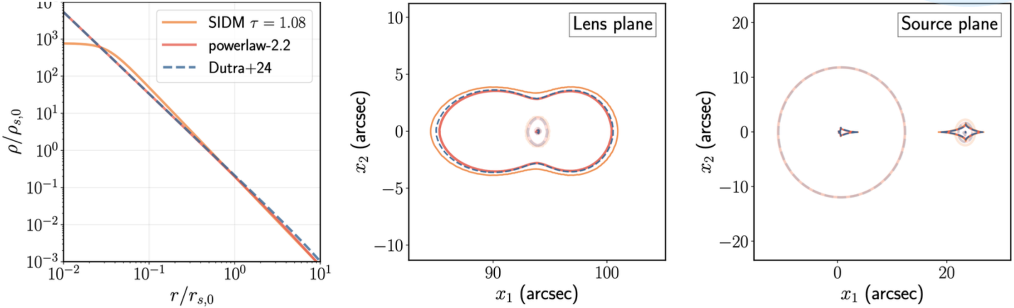
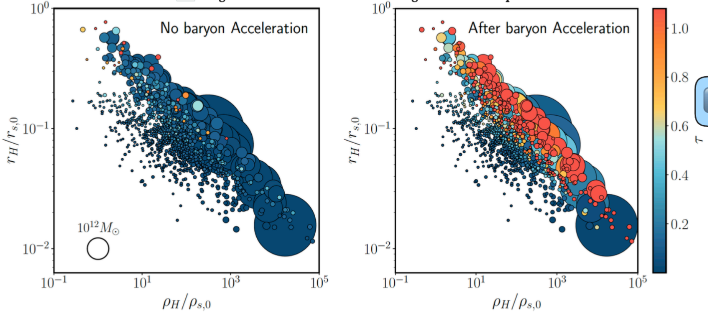


Figure 12. Comparison of lensing features of core-collapsed halos



Impact of baryons

Figure 13. The distribution of gravothermal phases



📌 Data from TNG-50-1 simulation

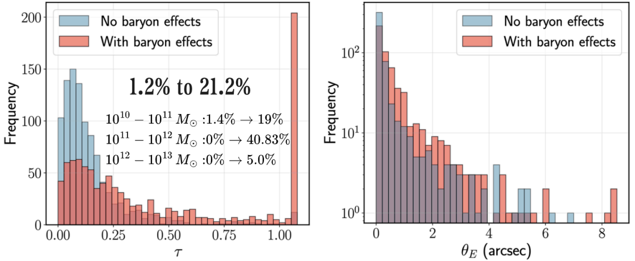
1000 halos

1 Collapse acceleration arXiv 2405.03787

$$t_{c,b} = t_{c,0}\underset{\uparrow}{\mathcal{F}_t}(\hat{\rho}_H, \hat{r}_H),$$

💡 Form factor including baryonic dependencies

2 Core contraction $r_c(\tau) = r_{s,0} f_c(\tau)(\mathcal{F}_t)^2$



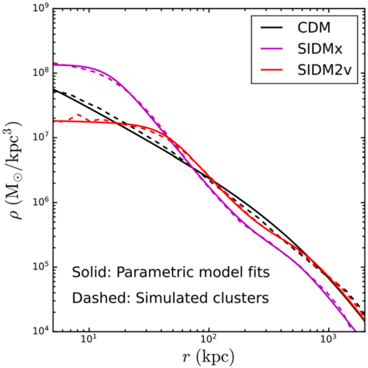
Baryon effects
- ❌ 40%
- ✅ 62%

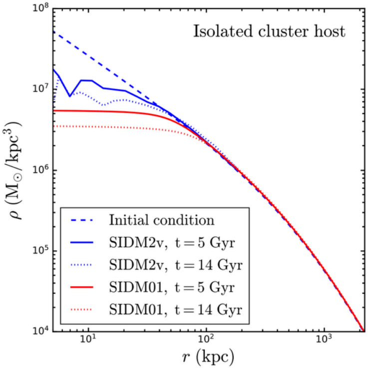
$\theta_E > 0.2\ arcsec$

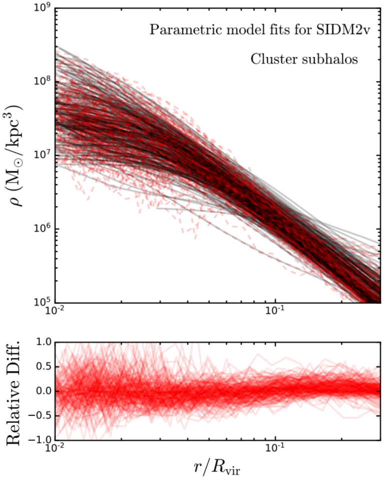
Figure 14. Distribution of τ and Einstein radius

Application in 2-component SIDM

arXiv 2506.14898







✅ Accurate fits across CDM, SIDMx, and SIDM2v halos.

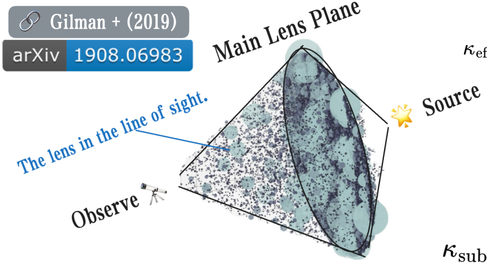
✅ Robust results for a large population of cluster subhalos

Fast Lensing of Galaxy-Scale Substructure in SIDM

The plot is from pyHalo.

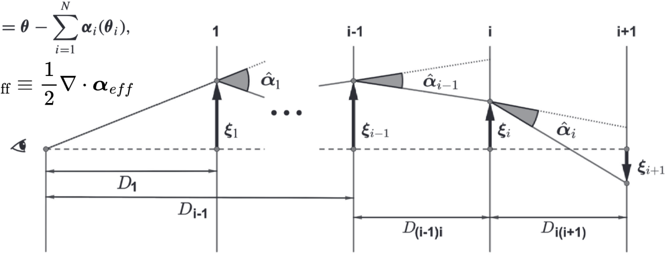
🔗 Gilman + (2019)

arXiv 1908.06983



$$\boldsymbol{\beta} = \boldsymbol{\theta} - \sum_{i=1}^{N} \boldsymbol{\alpha}_i(\boldsymbol{\theta}_i),$$

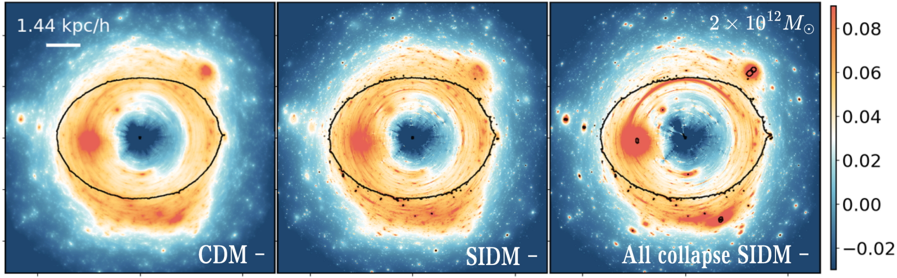
$$\kappa_{\rm eff} \equiv \frac{1}{2}\nabla \cdot \boldsymbol{\alpha}_{eff}$$



$$\kappa_{\rm sub(eff)} = \kappa_{\rm eff} - \kappa_{\rm macro} - \langle \kappa_{\rm sub} \rangle$$

🔗 Nan Li + (2020)

arXiv 2006.08540



30,000 halos
+ high−resolution ray tracing

Numerical: 🐌 1 week
↓
This work: 🚀 10 min

SUMMARY

🙁 CDM is challenged: GGSL, lensing substructure anomalies, ...

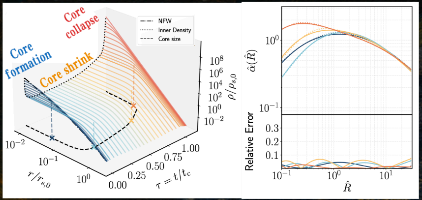
💡 Collapsed SIDM halos can produce more lensing effects

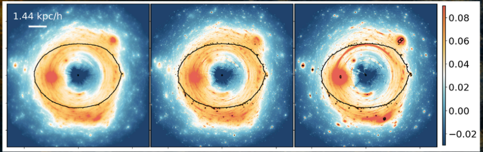
- ✅ More substructure disturbances
- ✅ More GGSL
- ✅ Possible existence of more compact subhalos

🥳 We need SIDM

😢 SIDM lacks an analytical lensing model

🎉 Our parametric model





Thank you for Listening !

🌐 https://github.com/HouSiyuan2001/SIDM_Lensing_Model

arXiv 2502.14964

Why we need time normalization?

Heat conduction breaks time reversal invariance

Arrow of time dependent on SIDM (the collision term)

$$\frac{\partial}{\partial r}M = 4\pi r^2 \rho,$$

– Mass distribution

$k_B T \propto m v^2$

Dynamic pressure

$$\frac{\partial}{\partial r}(\rho \nu^2) = -\frac{G M \rho}{r^2},$$

– Equilibrium condition

$$\frac{\partial}{\partial r}\left(r^2 \kappa m \frac{\partial \nu^2}{\partial r}\right) = r^2 \rho \nu^2 \frac{D}{Dt} \ln \frac{\nu^3}{\rho}$$

– Energy transport

$$\kappa_{LMFP} \propto \frac{\sigma}{m} \frac{\rho \nu^3}{m G}$$

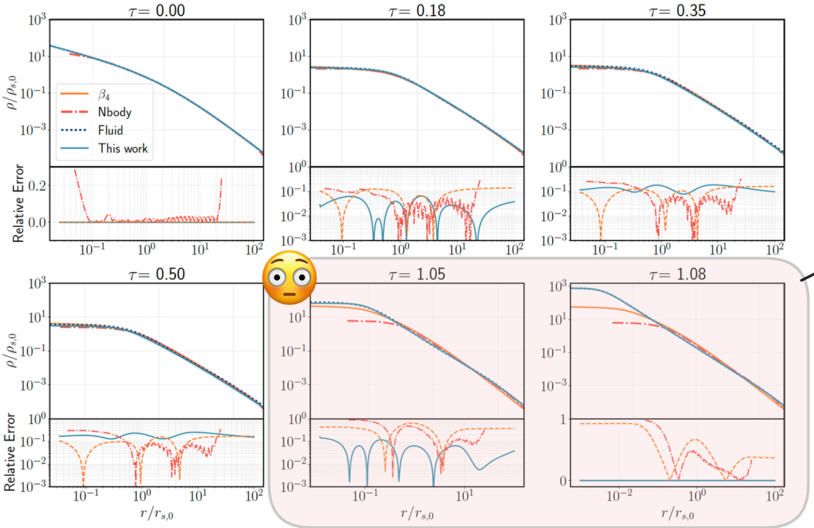
SIDM effect can be absorb into t

$$\frac{\partial}{\partial r}\left(\frac{r^2 \rho_{DM} \nu_{tot}^3}{G} \frac{\partial \nu_{tot}^2}{\partial r}\right) \propto r^2 \rho_{tot} \nu_{tot}^2 \frac{D}{D(t\sigma/m)} \ln \frac{\nu_{tot}^3}{\rho_{tot}}$$

arXiv 2405.03787

arXiv 2305.16176

Why we choose fluid simulation?



❌ N−body simulations cannot achieve sufficient accuracy during the core collapse phase

✅ Fluid simulations are needed.

Figure 2. Comparison of SIDM halo density profiles from simulations and model predictions at representative τ values.