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Detection of Phase Transition Gravitational Waves

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Aug.28th 2025

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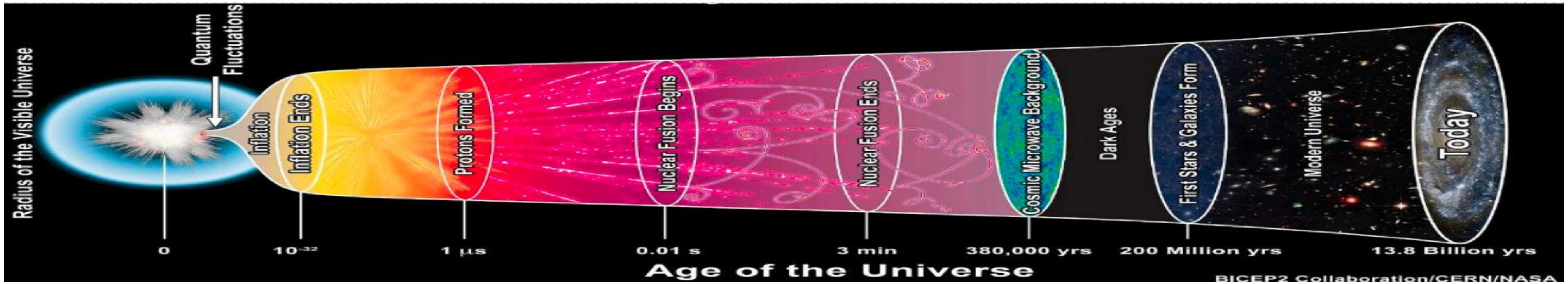
1 Introduction

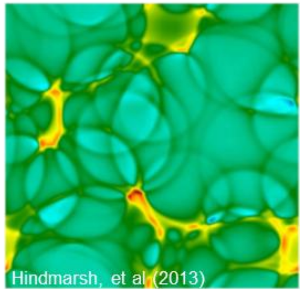
SWs dominated GWs production： Nucleation, Expansion, Percolation



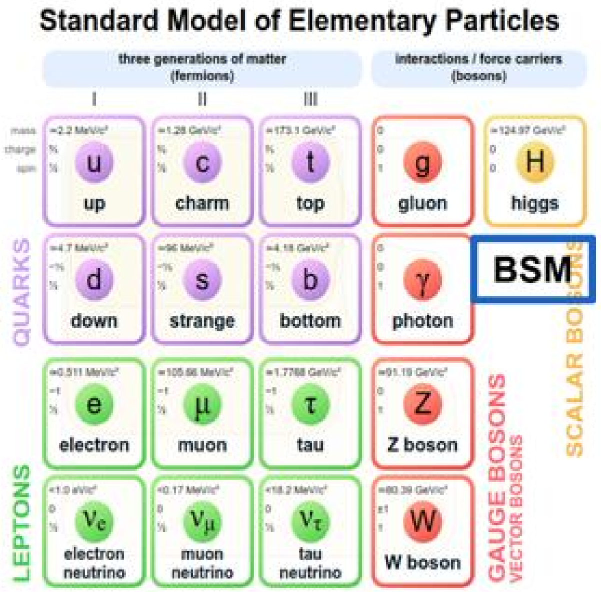
Energy density Spectrum

$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$

sound waves



$$\Omega_{\rm sw}(f)h^2 = 2.65\times10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$



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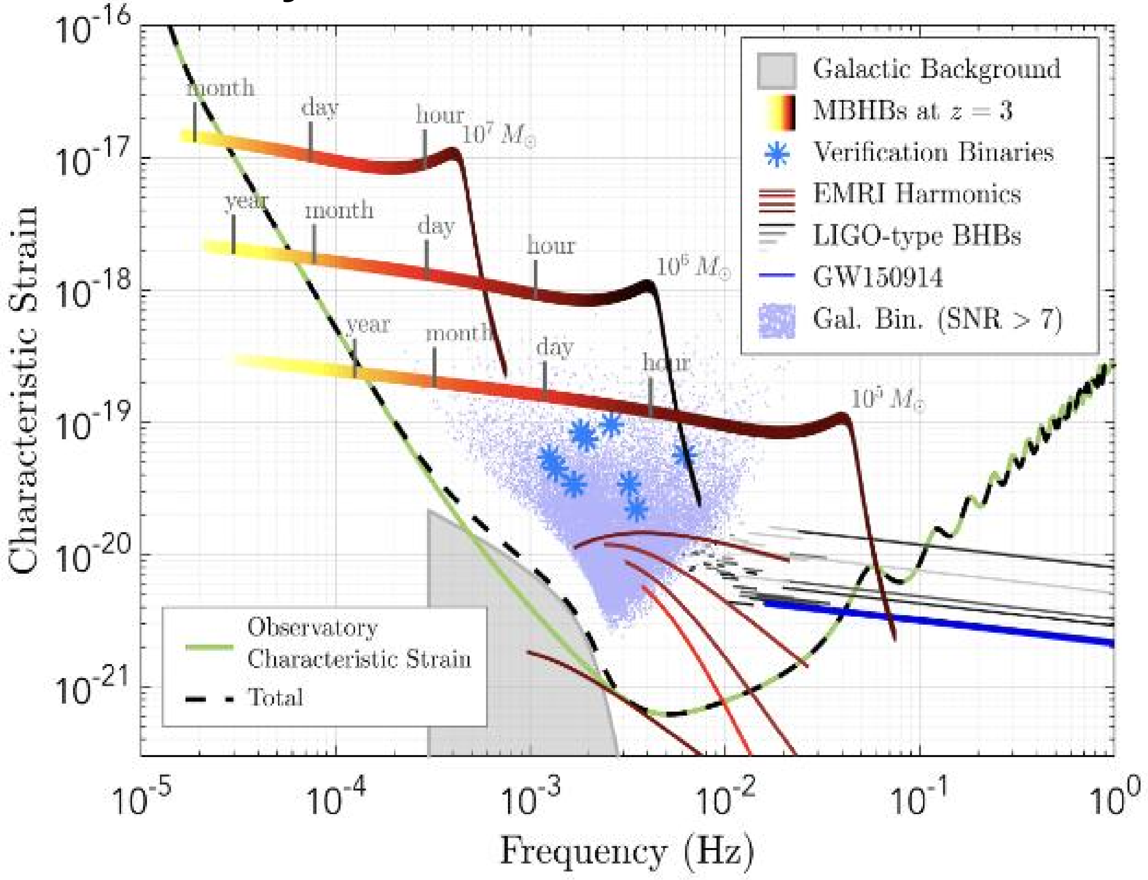
1 Introduction

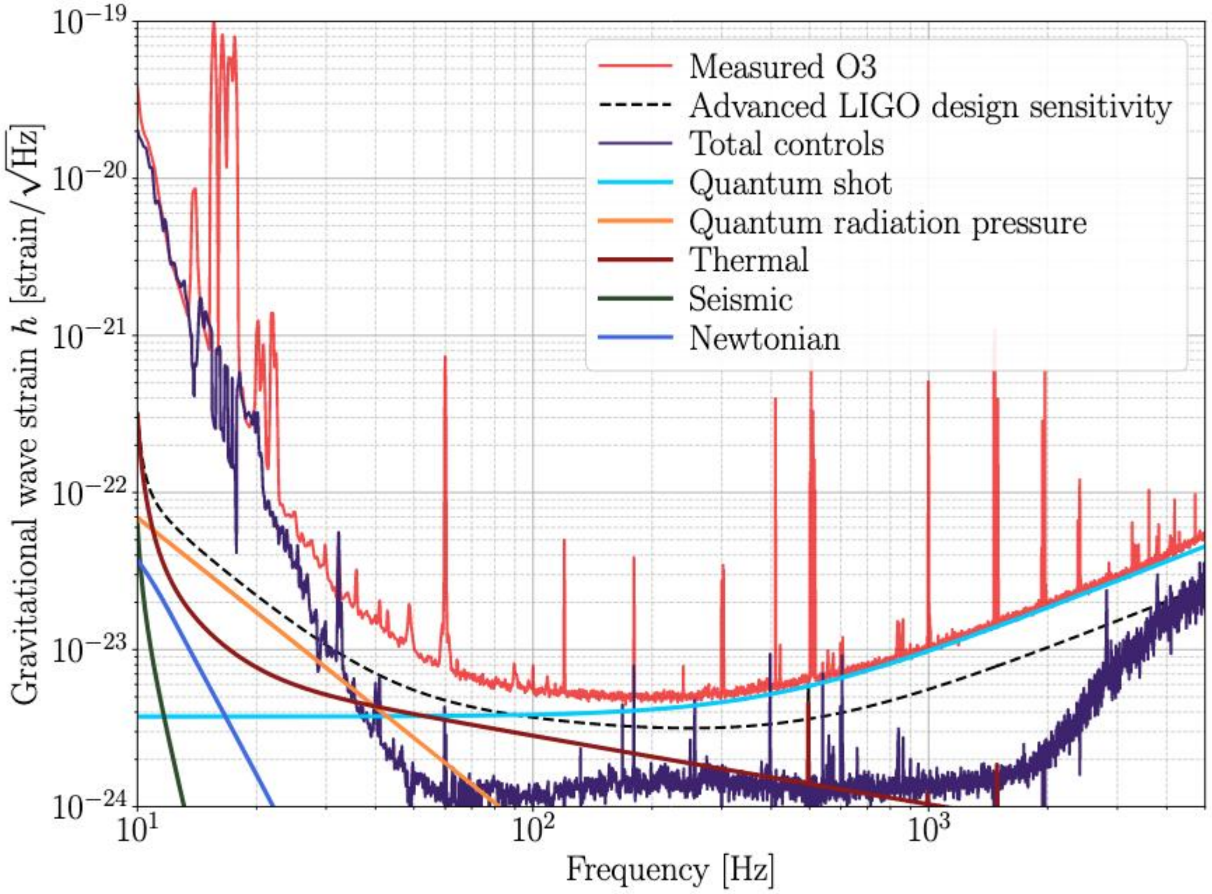
Space-based GWs detectors
Miliherze range

John Baker arXiv:1907.06482



Ground-based GWs detectors
Handred Hertz range

Craig Cahillane arXiv:2202.00847



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02

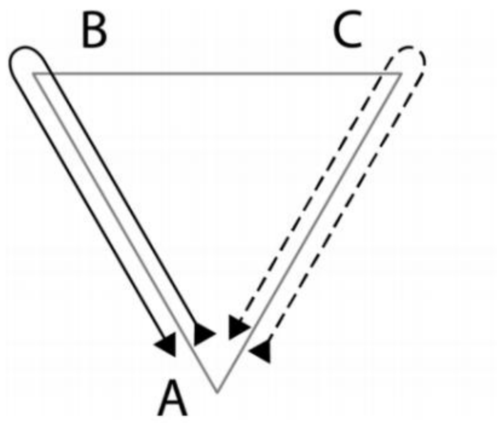
Data generation

◎2.1 Time delay interferometry

◎2.2 AET channels

◎2.3 Frequency domain data

2.1 Time delay interferometry (TDI)



phase difference at vertex A

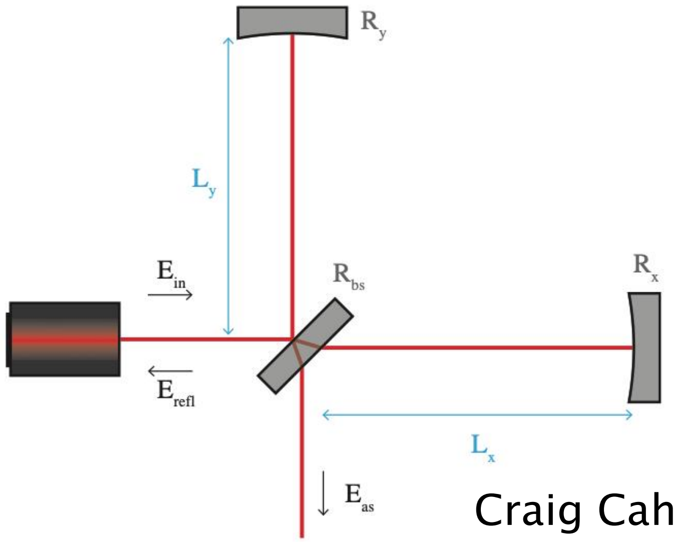
$$\Phi_{A_{BC}}(t) = \Delta\varphi_{A_{BC}}(t) + n_{A_{BC}}(t)$$

noise at vertex A

Arm Length $L = 2.5 \times 10^9\ m$

TDI (cancel laser frequency noise)

Tristan L. Smith arXiv: 1908.00546



Arm Length $L = 4\ km$

Cross correlation method for LIGO

Craig Cahillane arXiv: 2202.00847

2.2 AET channels

XYZ channels

Vertex A : X

Vertex B : Y

Vertex C : Z

AET channels

$$A = \frac{1}{\sqrt{2}}(Z - X)$$

$$E = \frac{1}{\sqrt{6}}(X - 2Y + Z)$$

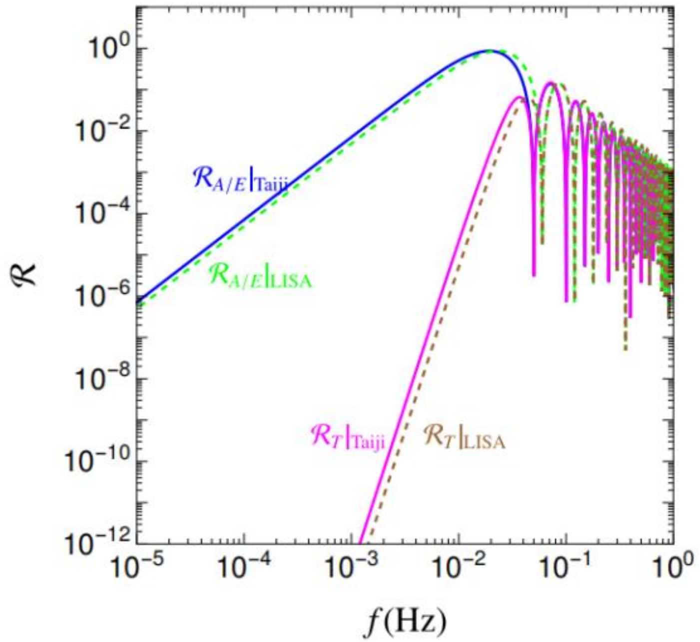
$$T = \frac{1}{\sqrt{3}}(X + Y + Z)$$

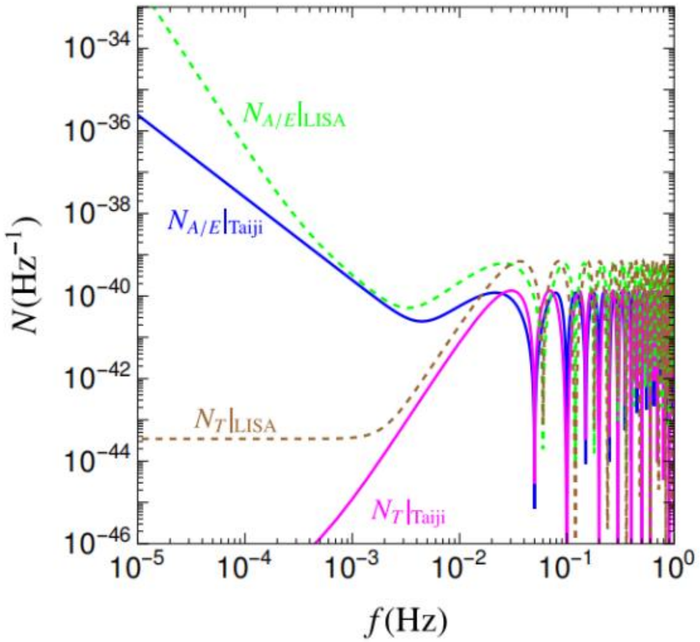
Power spectral densities

$$\mathrm{PSD}_A(f) = S_h(f)\mathcal{R}_A(f) + N_A(f)$$

$$\mathrm{PSD}_E(f) = S_h(f)\mathcal{R}_E(f) + N_E(f)$$

$$\mathrm{PSD}_T(f) = S_h(f)\mathcal{R}_T(f) + N_T(f)$$





Null channel method

$$\mathrm{PSD}_T(f) = N_T(f)$$

Tristan L. Smith arXiv: 1908.00546

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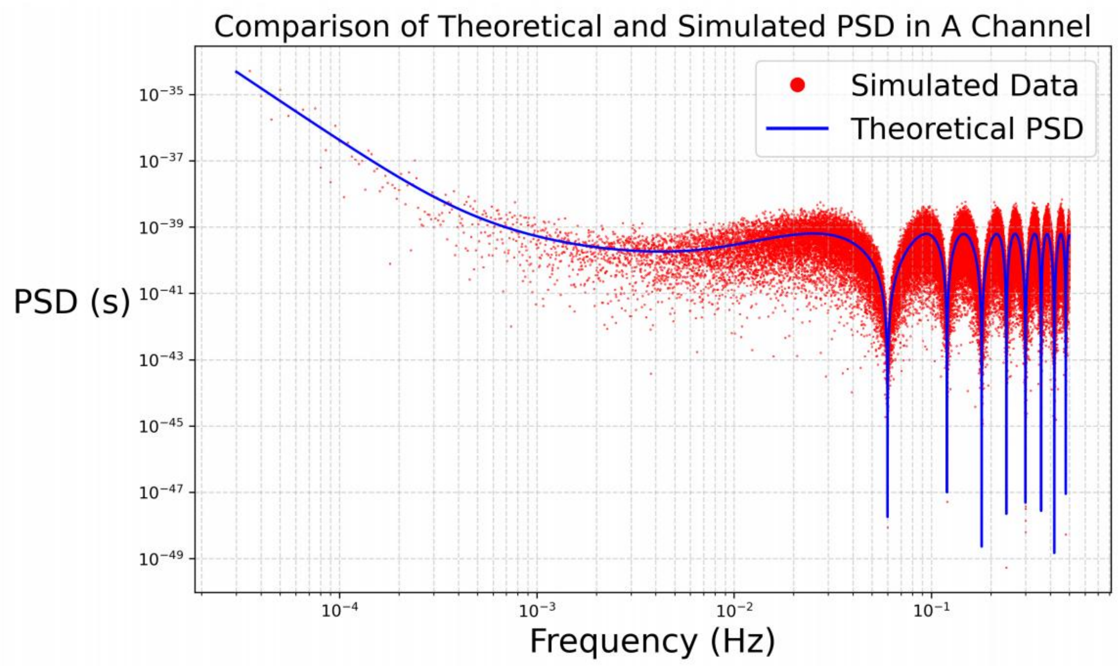
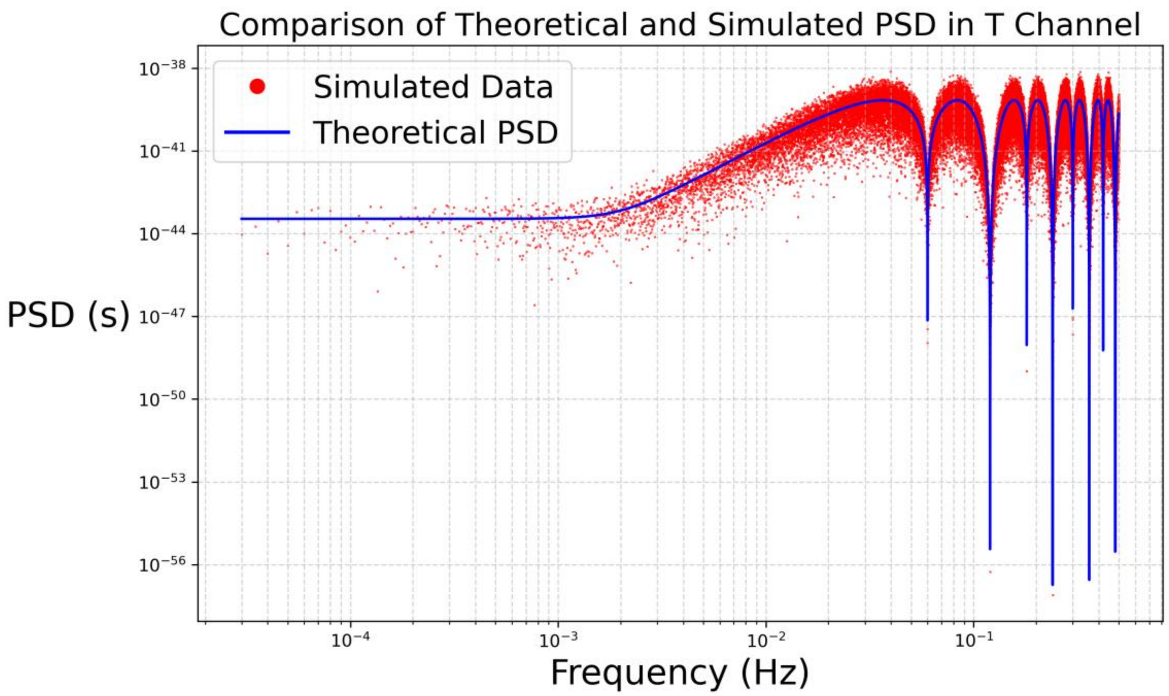
2.3 Frequency domain data

Distribution $P\left(\tilde{d}_a(f_k)\right) = \frac{1}{2\pi\sigma_a^2}\exp\left[-\frac{\left|\tilde{d}_a(f_k)\right|^2}{2\sigma_a^2}\right]$ $\sigma_a^2 = \frac{Tf_s^2}{4}P_a(f_k)$





Generate in frequency domain

03 Data analysis

◎3.1 Likelihood

◎3.2 Comparison between FIM and MCMC

◎3.3 Constraints on xSM parameters

◎3.4 Constraints on Higgs couping parameters

3.1 Likelihood

Logarithmic Likelihood function

$$\ln \mathcal{L} = -\sum_{\kappa=1}^{N_0}\sum_{k=1}^{N/2}\left\{ \ln \frac{\pi^3 T^3 f_s^6 \left[S_A(f_k)+N_A(f_k)\right]\left[S_E(f_k)+N_E(f_k)\right] N_T(f_k)}{8} + \frac{2}{T f_s^2}\left[\frac{\left|\tilde{d}_A^\kappa(f_k)\right|^2}{S_A(f_k)+N_A(f_k)} + \frac{\left|\tilde{d}_E^\kappa(f_k)\right|^2}{S_E(f_k)+N_E(f_k)} + \frac{\left|\tilde{d}_T^\kappa(f_k)\right|^2}{N_T(f_k)}\right]\right\}$$

k : index of discrete frequency

$N = 10^6$: data number in one segment

κ : segments index

$N_0 = 126$: number of segments

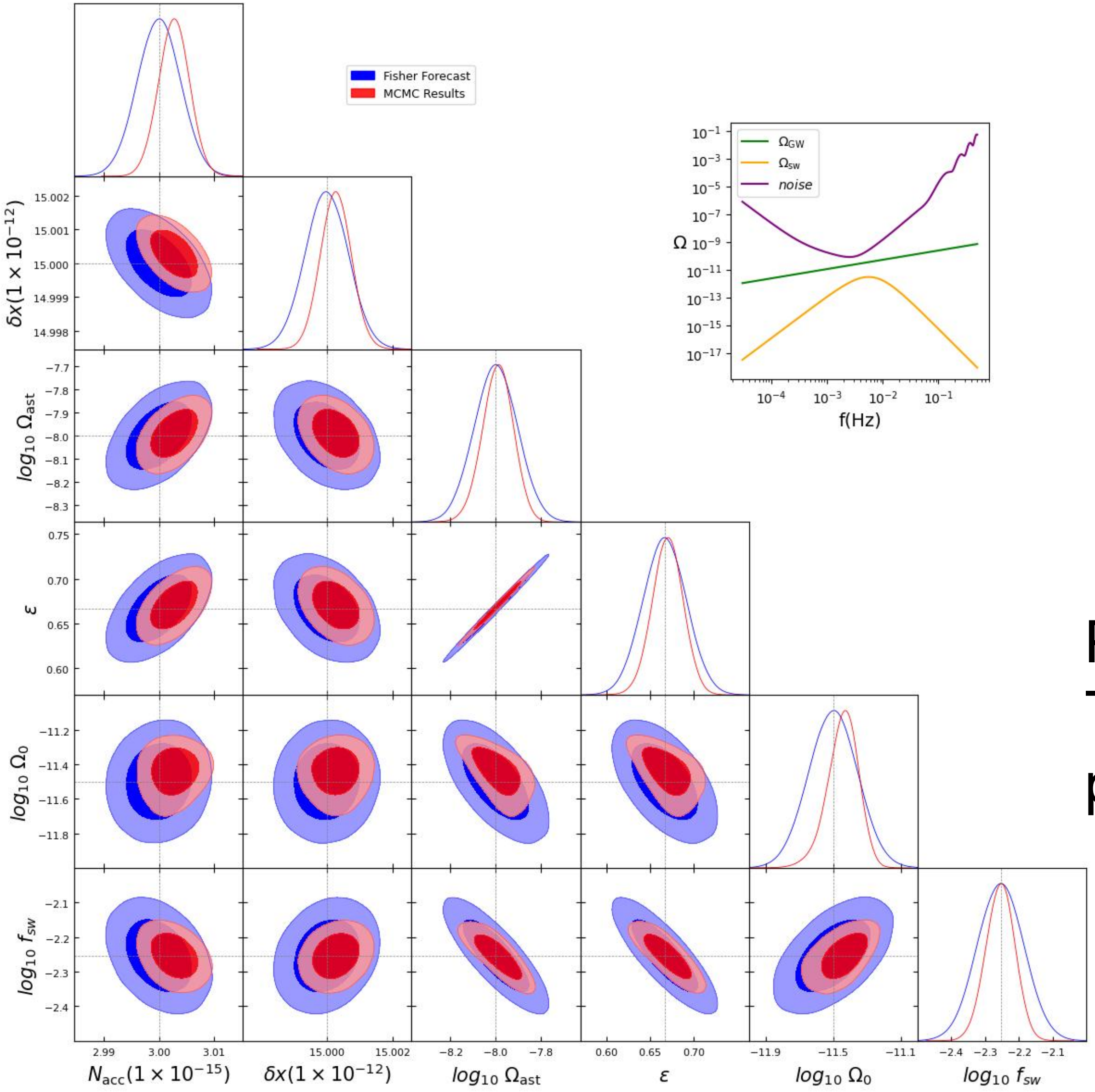
$T = 4$ years : observation time

Fisher information matrix

$$F_{ij} = -E\left(\frac{\partial^2 \ln p(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{\theta})}{\partial\theta_i \partial\theta_j}\right)$$

$$F_{ij}^{\text{likelihood}} = N_0 \sum_{k=0}^{N/2}\left[\frac{2}{\left[S_A(f_k)+N_A(f_k)\right]^2}\frac{\partial\left[S_A(f_k)+N_A(f_k)\right]}{\partial\theta_i}\frac{\partial\left[S_A(f_k)+N_A(f_k)\right]}{\partial\theta_j} + \frac{1}{N_T^2(f_k)}\frac{\partial N_T(f_k)}{\partial\theta_i}\frac{\partial N_T(f_k)}{\partial\theta_j}\right]$$

3.2 Comparison between FIM and MCMC



$$S_A = S_E = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{\rm ast}\left(\frac{f}{f_{\rm ref}}\right)^{\varepsilon} + \Omega_{\rm sw}(f)}{f^3}\mathcal{R}_A$$

$$\Omega_{\rm sw}(f) = \Omega_0\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

Chiara Caprini arXiv: 2403.03723

Parameters are fully recovered

The deviation arises from the non-Gaussian properties

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3.3 Constraints on xSM parameters

xSM model parameters

$$v_s, \quad m_{h_2}, \quad \theta, \quad b_3, \quad b_4.$$

$$V(H,S) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4,$$

Thermodynamics parameters

$$T_{\rm pt}, \quad \alpha, \quad \beta/H_n$$

$$\Omega_{\rm sw}(f)h^2 = 2.65 \times 10^{-6} \left(\frac{H_{\rm pt}}{\beta}\right) \left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \times v_{\rm w} \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2} \Upsilon(\tau_{\rm sw}) \ ,$$

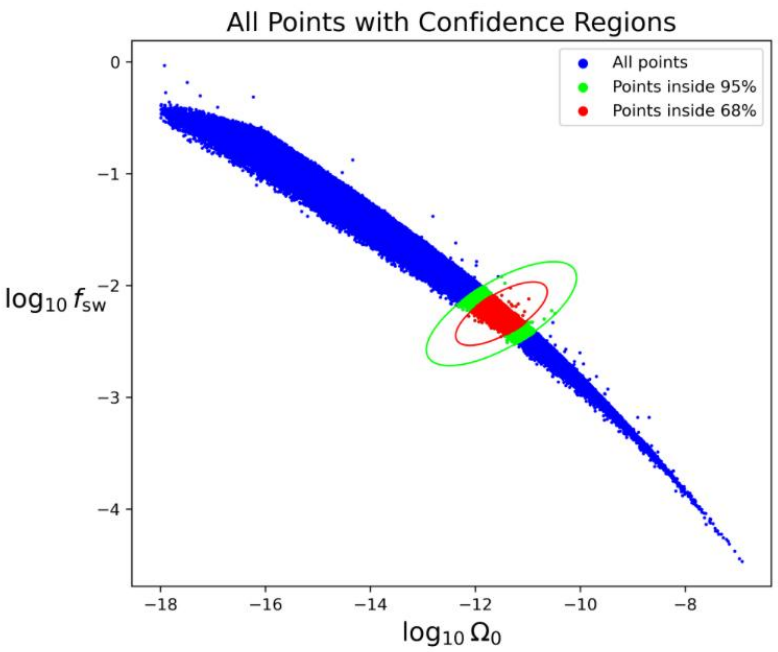
$$f_{\rm sw} = \frac{19}{v_{\rm w}} \left(\frac{\beta}{H_{\rm pt}}\right) \left(\frac{T_{\rm pt}}{100{\rm GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} 10^{-6}\ {\rm Hz}$$

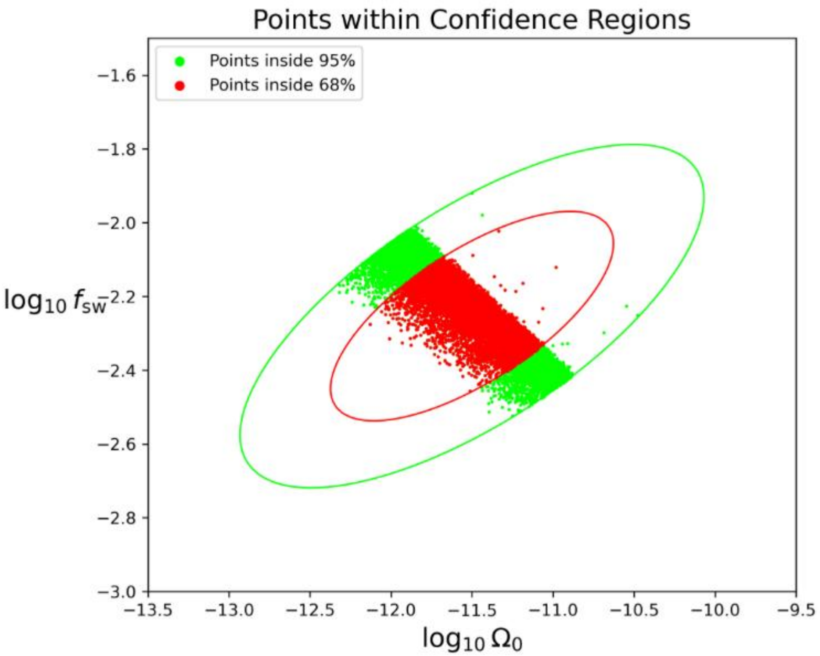
Phenomenological parameters

$$\Omega_{\rm sw}(f) = \Omega_0 \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

Alexandre Alves arXiv: 1812.09333

3.3 Constraints on xSM parameters





physical constraints   Alexandre Alves arXiv: 1812.09333

1.The Higgs potential must be stable and bounded from below.
2.Perturbativity and unitarity must hold at high energies.
3.Higgs couplings must remain close to the Standard Model values.

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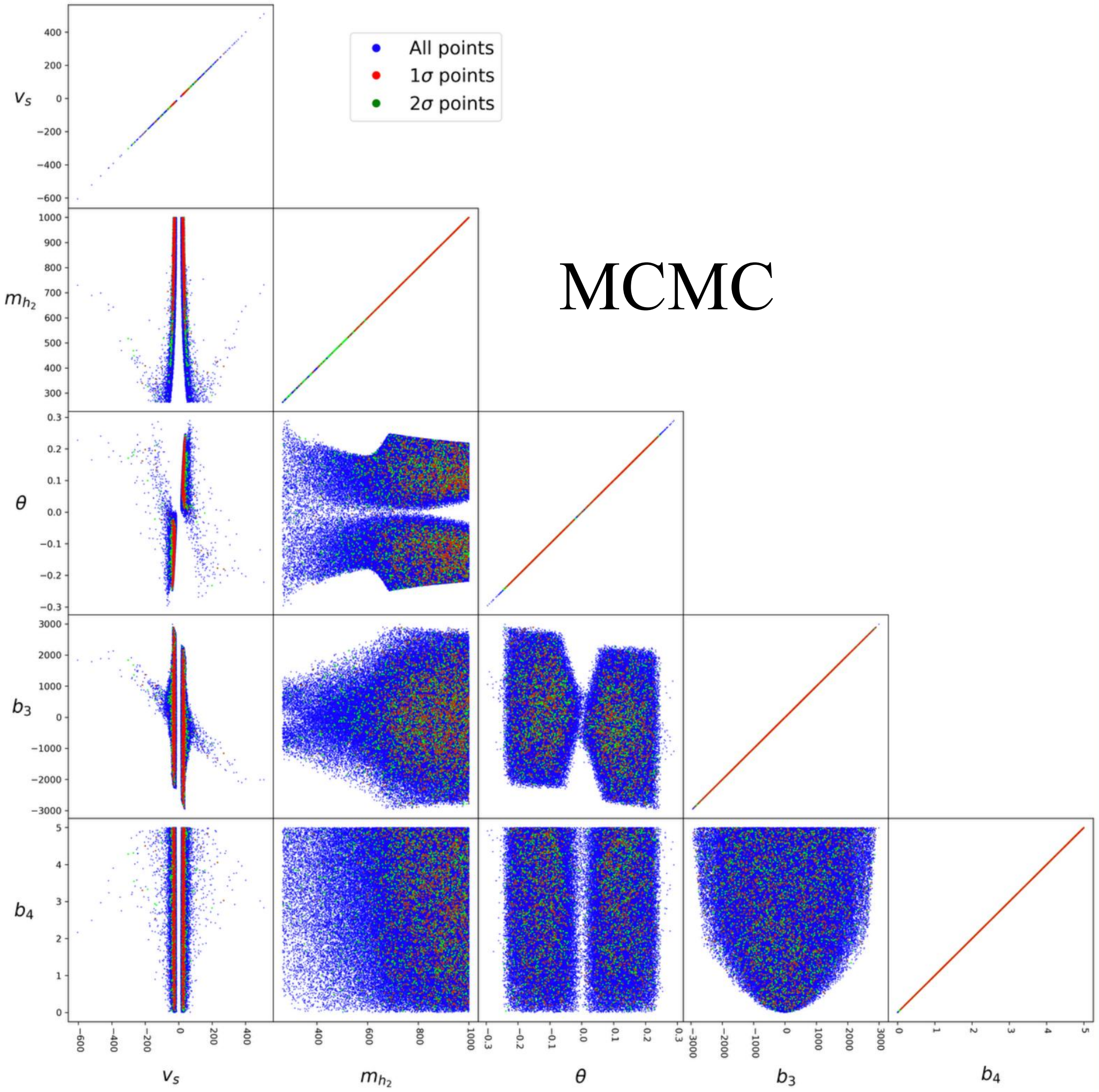
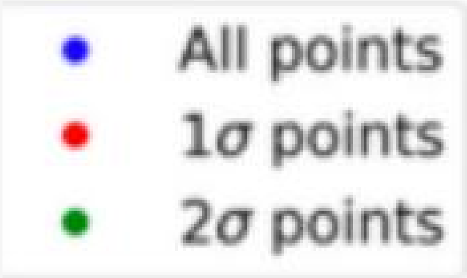
3.3 Constraints on xSM parameters



MCMC

GWs observation significantly restricts the ranges of the xSM parameters and reveal strong parameter correlations, demonstrating it can directly constrain particle physics models beyond the Standard Model.



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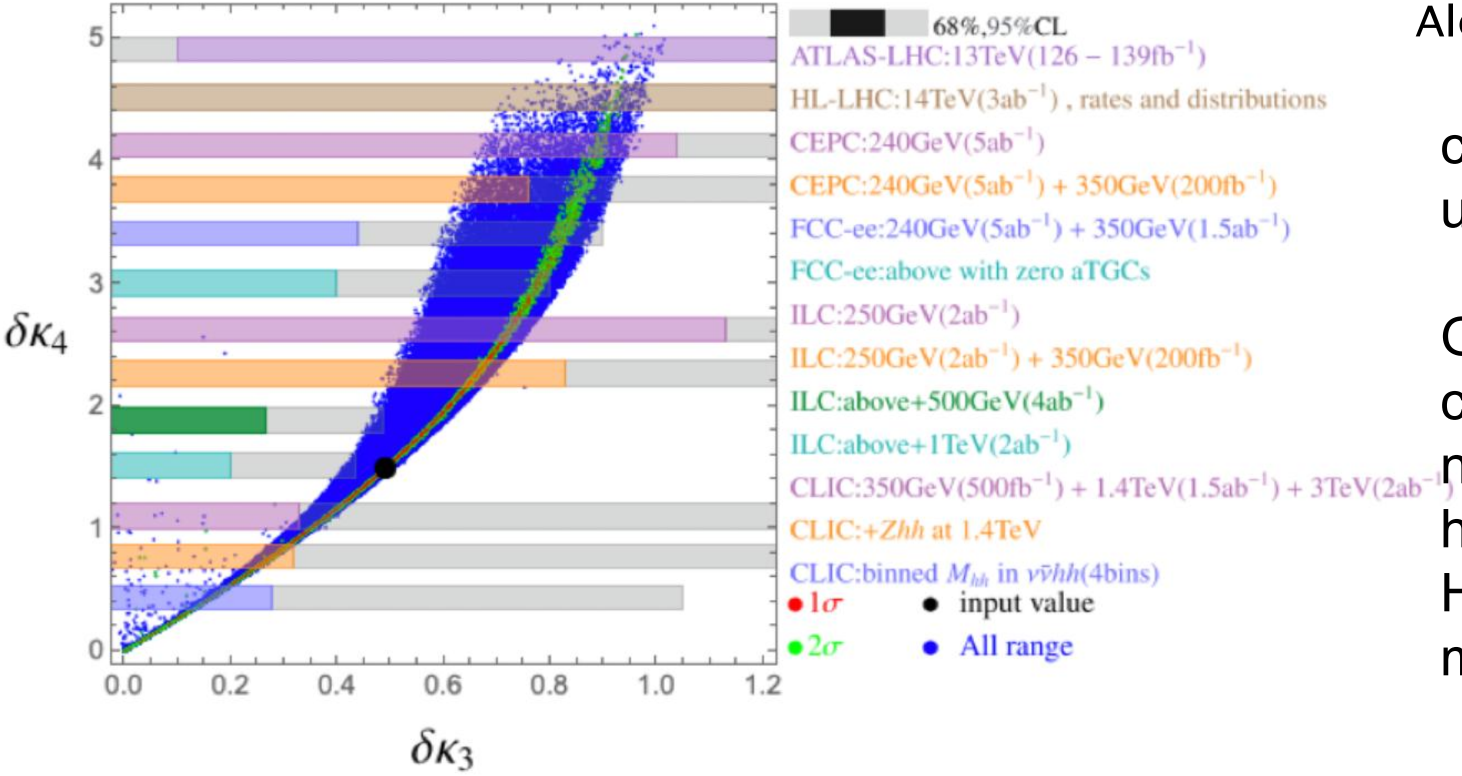
3.4 Constraints on Higgs couping parameters

$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_{h_1}^2}{v}(1+\delta\kappa_3)h_1^3 - \frac{1}{8}\frac{m_{h_1}^2}{v^2}(1+\delta\kappa_4)h_1^4$$

$$\delta\kappa_3 = \theta^2\left[-\frac{3}{2} + \frac{2m_{h_2}^2 - 2b_3 v_s - 4b_4 v_s^2}{m_{h_1}^2}\right] + \mathcal{O}(\theta^3)$$

$$\delta\kappa_4 = \theta^2\left[-3 + \frac{5m_{h_2}^2 - 4b_3 v_s - 8b_4 v_s^2}{m_{h_1}^2}\right] + \mathcal{O}(\theta^3)$$



Alexandre Alves arXiv: 1812.09333

collider-only measurements leave large uncertainties in here

GWs observations from FOPT provide complementary information that helps to narrow the allowed parameter space, highlighting its crucial role in probing the Higgs potential beyond the collider measurements.

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04 Future improvements

4.1 Future improvement

1. Perform simulation in time-domain to capture realistic, non-stationary features.

2. Combine the space-based and ground-based detectors for joint observations.

3. Implement 2nd-generation TDI to improve laser noise cancellation.

4. Apply CLT and CG to reduce computation cost.

5. Explore more particle physics for GW predictions.


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THANKS!