



Helicity-changing Decays of Relic Neutrinos and Detections in PTOLEMY-like Experiments

Jihong Huang 黄吉鸿

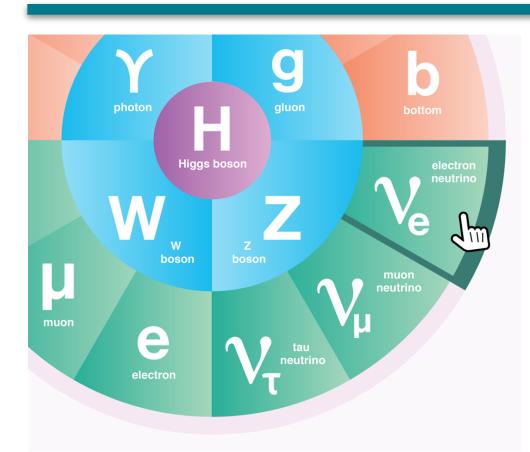
Institute of High Energy Physics (IHEP)

Based on: JH & Shun Zhou, JCAP 09 (2024) 067

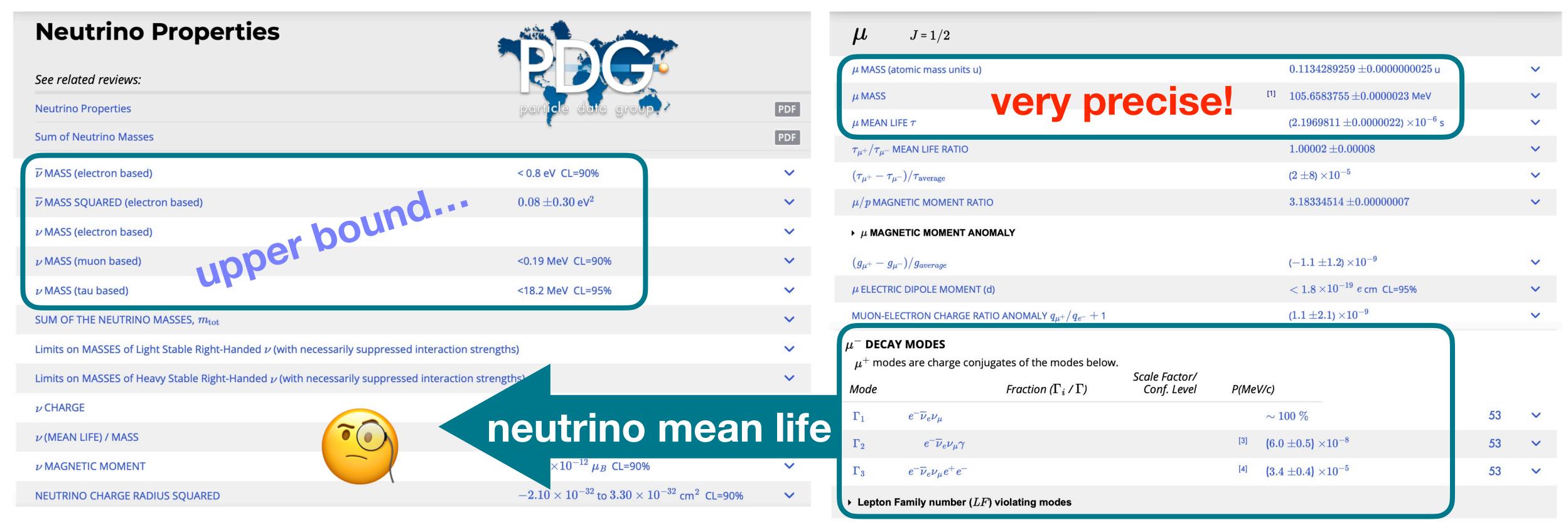
The XIX International Conference on Topics in Astroparticle and Underground Physics (TAUP2025)

Xichang, 2025/08/24-30

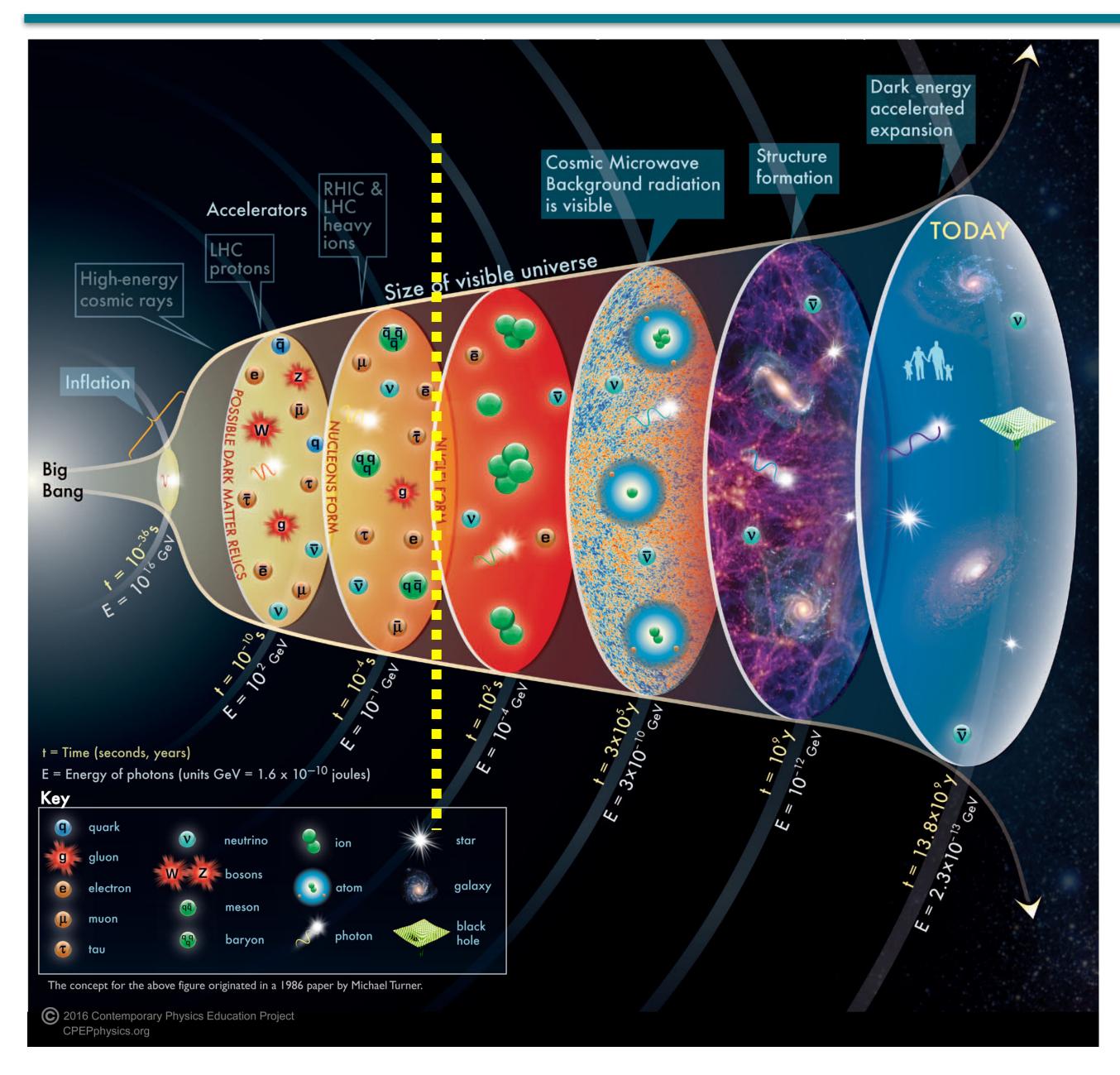
Neutrino Properties



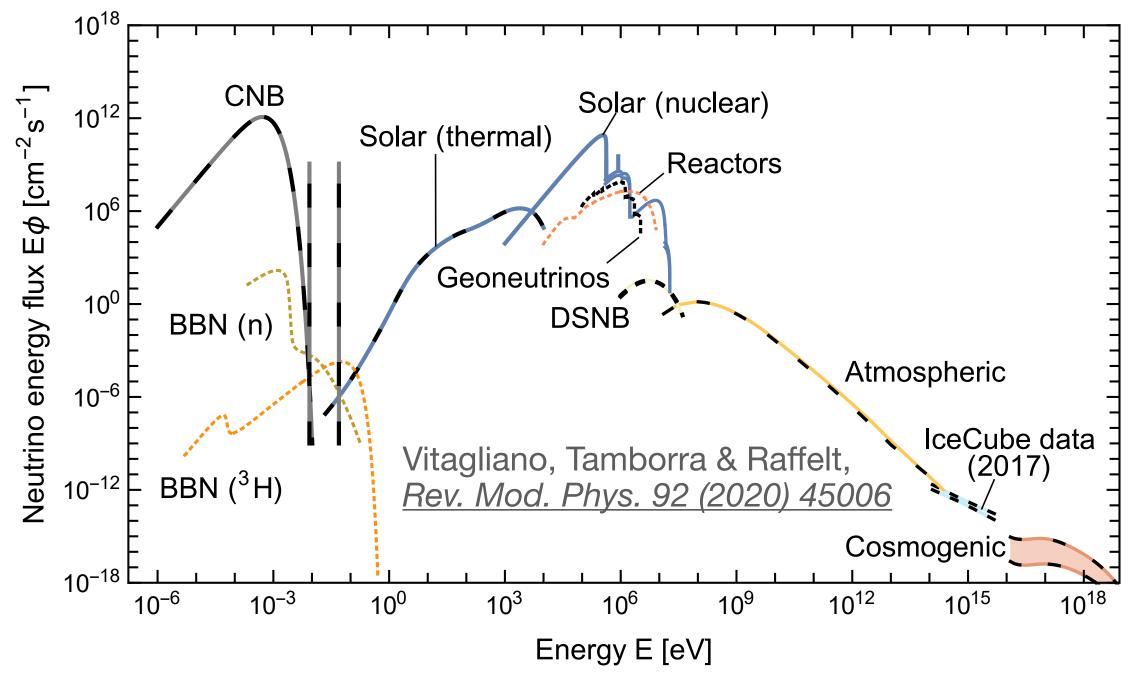
- √ Neutrino oscillation: massive neutrinos & significant leptonic flavor mixing
- * Neutrino mass ordering: normal mass ordering (NO, $m_1 < m_2 < m_3$) or inverted mass ordering (IO, $m_3 < m_1 < m_2$)?
- * The absolute neutrino mass scale is still unknown. Neutrino mass origin?
- * Dirac or Majorana particles? Lepton number/flavor violation? (New physics!)



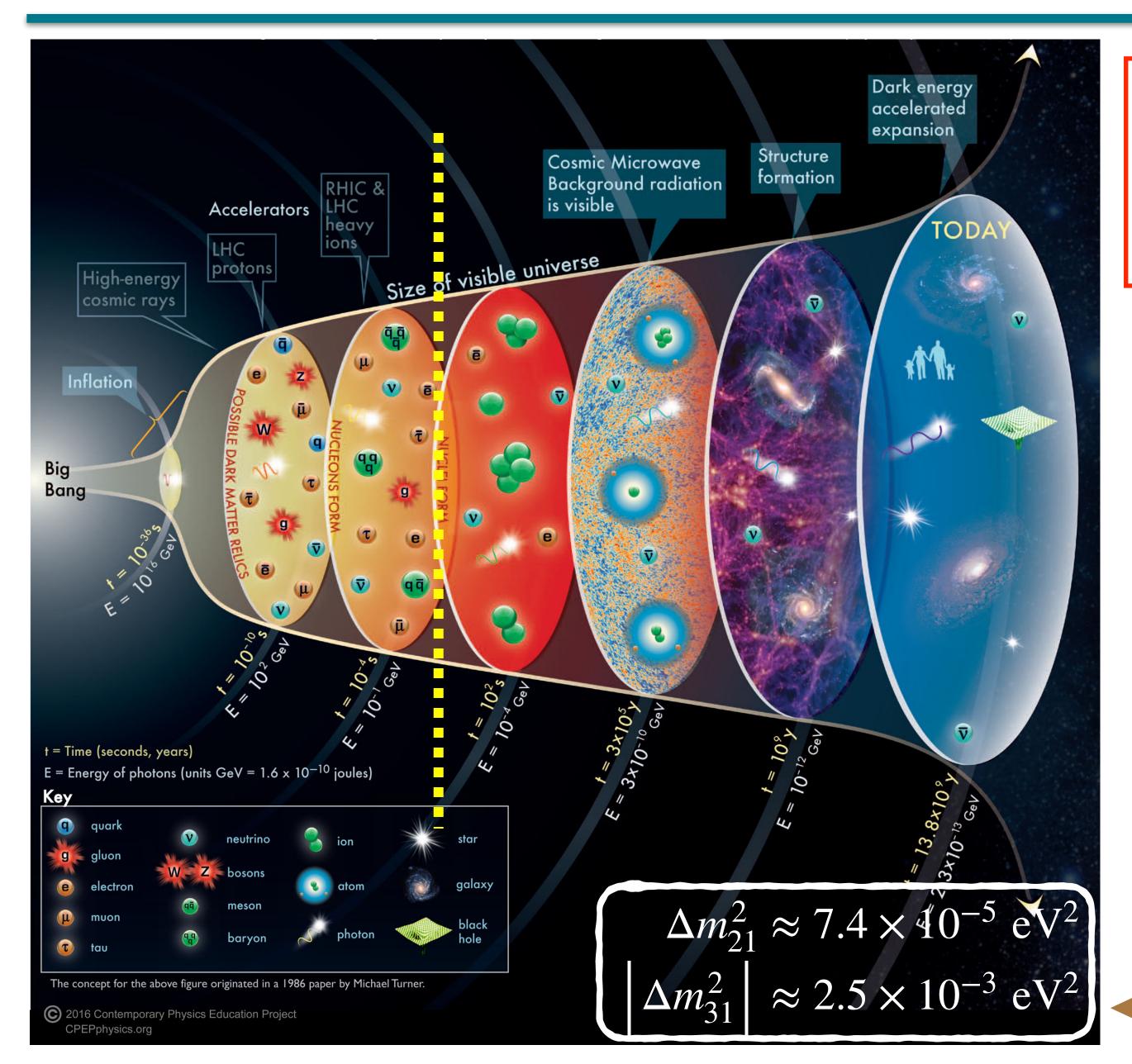
Relic Neutrinos



- √ Neutrinos decoupled @ the 1st sec.
- √ A non-relativistic neutrino source
- √ Reveal neutrinos' intrinsic properties



Relic Neutrinos



- √ Neutrinos decoupled @ the 1st sec.
- √ A non-relativistic neutrino source
- √ Reveal neutrinos' intrinsic properties
- Number density per flavor per helicity

$$n_{\text{C}\nu\text{B}} \approx 56 \left(\frac{T_{\gamma}}{2.726 \text{ K}}\right) \text{ cm}^{-3}$$

• Temperature today

$$T_{\text{C}\nu\text{B}} \approx 1.95 \text{ K} \sim 0.168 \text{ meV}$$

• Average momentum $\langle p_{\nu} \rangle \approx 0.5 \text{ meV}$

At least two mass states are non-relativistic!

Detection CvB with Tritium

PHYSICAL REVIEW

VOLUME 128, NUMBER 3

NOVEMBER 1, 1962

Universal Neutrino Degeneracy

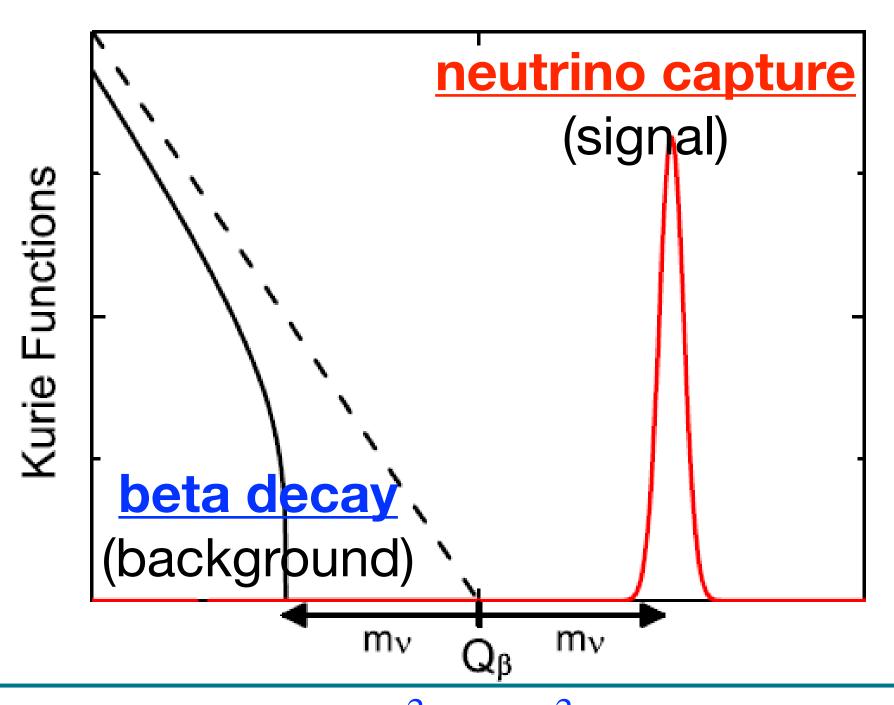
Steven Weinberg*

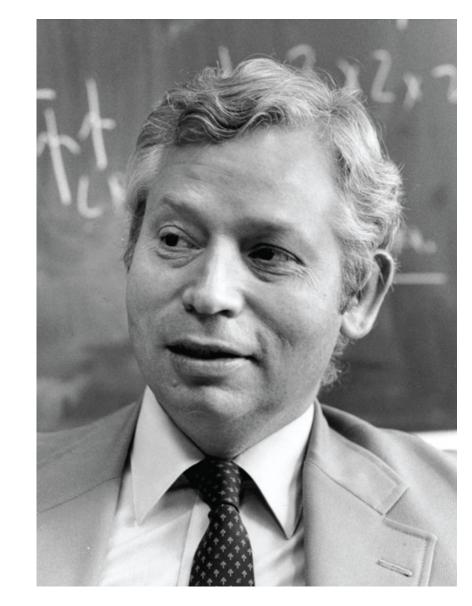
Imperial College of Science and Technology, London, England

(Received March 22, 1962)

200+ citations

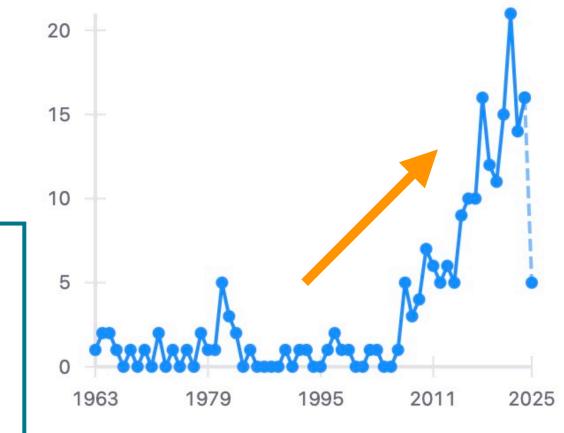
It would evidently be very worthwhile to do a counter experiment specifically designed to look for electrons with energies just above the end point in a β^- decay. Tritium might be preferable because of background problems and because it has an accurately known end point; a decay process with a higher Q value would give more counts above the end point, though a smaller proportion. For tritium Q=17.95 keV and the half-life is 12.5 yr, so (using 144) the number of events above the end point per gram of tritium is $76/\sec$ if $E_F=1$ eV. It varies as E_F^3 (and for other decays roughly as Q^2). The limiting factor on such an experiment is energy resolution and our imperfect knowledge of β^- end points rather than the rarity of absorption events. Probably it





Steven Weinberg (1933~2021)

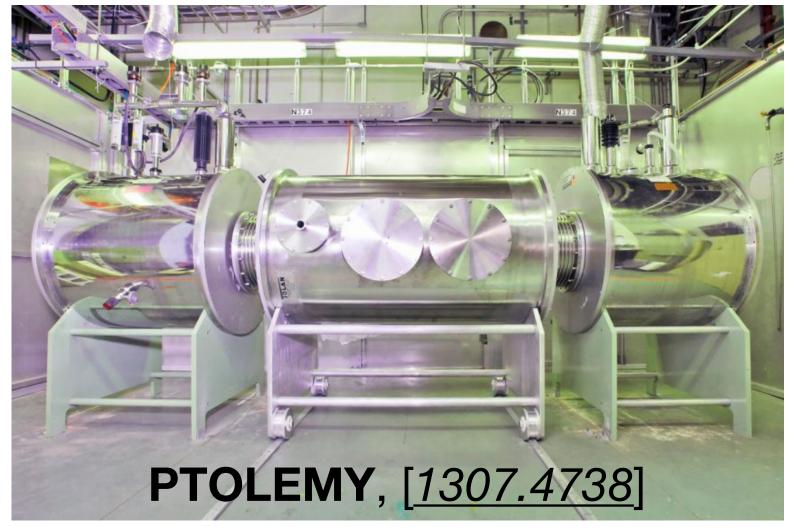
Citations per year



- \P Measuring <u>neutrino absolute mass scale</u> from tritium decay $^3{
 m H}
 ightarrow ^3{
 m He} + e^- + \overline{
 u}_e$
- \bigstar **Detecting cosmic relic neutrinos** through the tritium capture $\nu_e + {}^3{
 m H} \to {}^3{
 m He} + e^-$
- √ No energy threshold (vs 1.8 MeV for IBD), suitable for non-relativistic neutrino case

Capture Rates of CvB

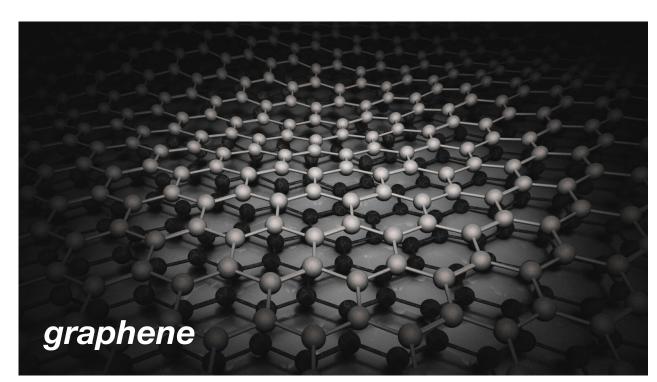




Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield (PTOLEMY)

• Capture Rates: Long, Lunardini & Sabancilar, JCAP 08 (2014) 038

$$\left[\Gamma_{\text{C}\nu\text{B}} = N_{\text{T}}\overline{\sigma} \sum_{s_i = \pm 1/2}^{3} \left| U_{ei} \right|^2 n_i(s_i) \mathcal{A}(s_i)\right]$$

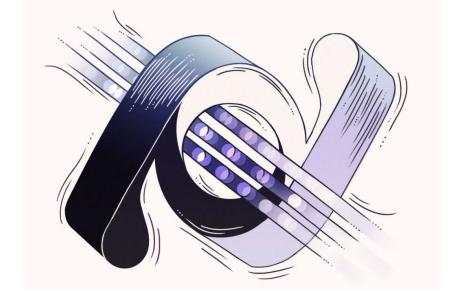


- Number of tritium nuclei in the target (100 g tritium) $N_{
 m T} pprox 2 imes 10^{25}$
- Cross section $\bar{\sigma} \approx 3.8 \times 10^{-45} \text{ cm}^2$

•
$$\mathscr{A}(s_i) \equiv 1 - 2s_i\beta_i = \begin{cases} 1 - \beta_i & s_i = +1/2 \\ 1 + \beta_i & s_i = -1/2 \end{cases}$$



√ Velocity ?!



 $\Gamma_{\text{C}\nu\text{B}}^{\text{D}} \approx 4 \text{ yr}^{-1}$ $\Gamma_{\text{C}\nu\text{B}}^{\text{M}} \approx 8 \text{ yr}^{-1}$

A factor of 2! (distinguish D or M)

 $\Gamma_{\text{C}\nu\text{B}}^{\text{D}} \approx 7 \text{ yr}^{-1} \text{ for NO}$

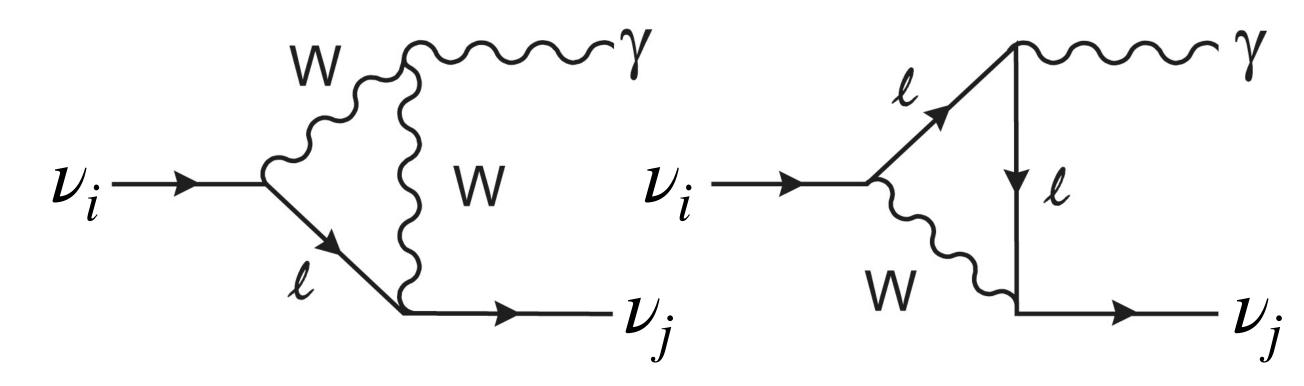
considering velocity distribution Roulet & Vissani, <u>JCAP 10 (2018) 049</u>

Neutrino Decays

It is interesting to investigate whether a heavier neutrino can decay into a lighter one and other elementary particles within or beyond the SM.

Radiative decays: $\nu_i \rightarrow \nu_i + \gamma$

$$\left[\Gamma\left(\nu_i \to \nu_j + \gamma\right) = 5.3 \text{ s}^{-1} \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1 \text{ eV}}\right)^3 \left(\frac{\mu_{\text{eff}}}{\mu_{\text{B}}}\right)^2\right]$$



Effective magnetic dipole moment $\mu_{\rm eff} \approx 10^{-23} \mu_{\rm B}$

$$\mu_{\rm eff} \approx 10^{-23} \mu_{\rm B}$$

⇒ Neutrino lifetime $\tau_{\nu} \approx 10^{49}$ s, *MUCH LONGER* than the age of the Universe $t_0 \approx 4 \times 10^{17} \text{ s}$

Direct decays: $\nu_i \rightarrow \nu_i + \phi$

Neutrinos can interact with the Nambu-Goldstone boson, the Majoron, to explain the neutrino mass origin (Majoron model).

$$\mathcal{L}_{\mathbf{M}} = \frac{1}{2} \sum_{i} \left(\overline{\nu_{i}} i \partial \!\!\!/ \nu_{i} - m_{i} \overline{\nu_{i}} \nu_{i} \right) + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \left[i \phi \sum_{i,j} g_{ij} \overline{\nu_{i}} \gamma^{5} \nu_{j} + \text{h.c.} \right]$$

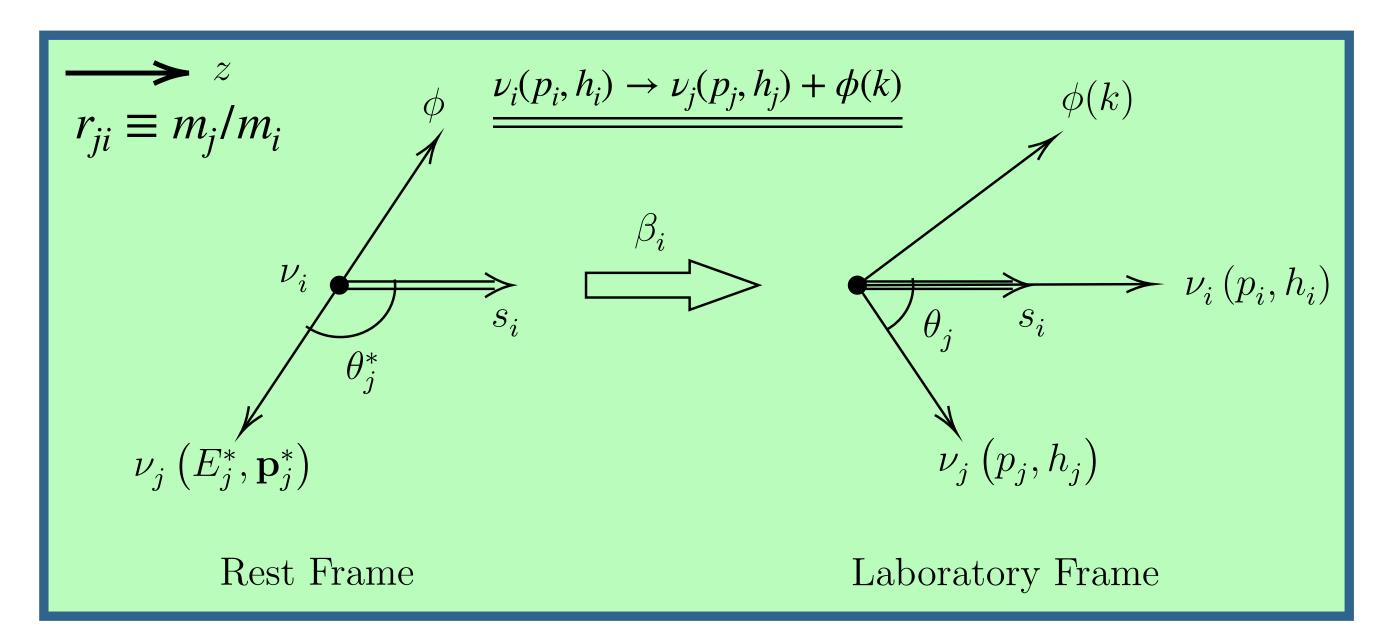
- Dirac case: $\nu_i \to \nu_j + \phi$ and $\overline{\nu}_i \to \overline{\nu}_j + \phi$
- The decay <u>amplitudes</u> for Majorana neutrinos are twice that for Dirac ones.

Neutrino Invisible Decays

Direct decays: $\nu_i \rightarrow \nu_j + \phi$

Neutrinos can interact with the Nambu-Goldstone boson, the Majoron, to explain the neutrino mass origin (Majoron model).

$$\mathcal{L}_{\mathbf{M}} = \frac{1}{2} \sum_{i} \left(\overline{\nu_{i}} i \partial \!\!\!/ \nu_{i} - m_{i} \overline{\nu_{i}} \nu_{i} \right) + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \left[i \phi \sum_{i,j} g_{ij} \overline{\nu_{i}} \gamma^{5} \nu_{j} + \text{h.c.} \right]$$



Modern Physics Letters A Vol. 5, No. 5 (1990) 297-299 © World Scientific Publishing Company

SOME REMARKS ON NEUTRINO DECAY VIA A NAMBU-GOLDSTONE BOSON

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Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA

Received 30 August 1989

$$\Gamma(\nu_2 \to \nu_1) = \frac{m_1 m_2}{16\pi E_2} \left[g_1^2 \left(\frac{x}{2} + 2 + \frac{2}{x} \log x - \frac{2}{x^2} - \frac{1}{2x^3} \right) \right]$$

relativistic limit $E \gg m$

$$+ g_2^2 \left(\frac{x}{2} - 2 + \frac{2}{x} \log x + \frac{2}{x^2} - \frac{1}{2x^3} \right) \right]$$

$$\Gamma(\nu_2 \to \overline{\nu}_1) = \frac{m_1 m_2}{16\pi E_2} \left[(g_1^2 + g_2^2) \left(\frac{x}{2} - \frac{2}{x} \log x - \frac{1}{2x^3} \right) \right] \qquad \left(x = \frac{m_2}{m_1} \right).$$

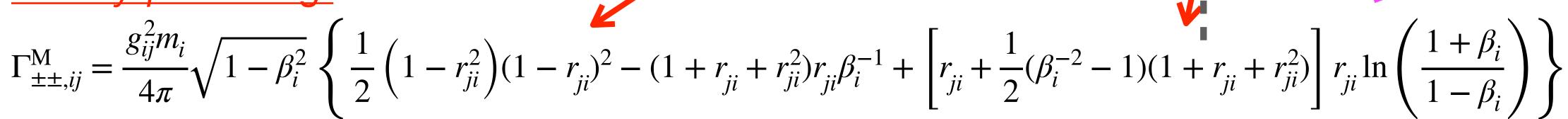
also see: Funcke, Raffelt & Vitagliano, PRD 101 (2020) 1, 015025

<u>Decay amplitudes</u> (with specific helicities) → Amplitude squared → Decay rates

Total Decay Rates

Total decay rates (the most general for the first time)

- ① For $\beta_i \leq \beta_{ji}^* \equiv (1 r_{ji}^2)/(1 + r_{ji}^2)$
 - Helicity-preserving:



Helicity-changing:

$$\Gamma_{\pm\mp,ij}^{\mathrm{M}} = \frac{g_{ij}^2 m_i}{4\pi} \sqrt{1 - \beta_i^2} \left\{ \frac{1}{2} \left(1 - r_{ji}^2 \right) (1 - r_{ji})^2 + (1 + r_{ji} + r_{ji}^2) r_{ji} \beta_i^{-1} - \left[r_{ji} + \frac{1}{2} (\beta_i^{-2} - 1) (1 + r_{ji} + r_{ji}^2) \right] r_{ji} \ln \left(\frac{1 + \beta_i}{1 - \beta_i} \right) \right\}$$

- ② For $\beta_i > \beta_{ji}^* \equiv (1 r_{ji}^2)/(1 + r_{ji}^2)$
 - Helicity-preserving:

$$\Gamma^{\mathrm{M}}_{\pm\pm,ij} = \frac{g_{ij}^2 m_i}{4\pi} \sqrt{1 - \beta_i^2} \left\{ \frac{1}{2} \left(1 - r_{ji}^2 \right) (1 - r_{ji})^2 - r_{ji} (1 - r_{ji}^2) - 2r_{ji}^2 \ln r_{ji} - (\beta_i^{-2} - 1) \left[(1 + r_{ji} + r_{ji}^2) r_{ji} \ln r_{ji} + \frac{1}{4} (1 - r_{ji}^2) (1 + 4r_{ji} + r_{ji}^2) \right] \right\}$$

• Helicity-changing:

Consistent with previous results in the relativistic limit, i.e., $\beta_i \rightarrow 1$

$$\Gamma^{\mathrm{M}}_{\pm\mp,ij} = \frac{g_{ij}^2 m_i}{4\pi} \sqrt{1 - \beta_i^2} \left\{ \frac{1}{2} \left(1 - r_{ji}^2 \right) (1 - r_{ji})^2 + r_{ji} (1 - r_{ji}^2) + 2r_{ji}^2 \ln r_{ji} + (\beta_i^{-2} - 1) \left[(1 + r_{ji} + r_{ji}^2) r_{ji} \ln r_{ji} + \frac{1}{4} (1 - r_{ji}^2) (1 + 4r_{ji} + r_{ji}^2) \right] \right\}$$

Decay-rate Asymmetries

\checkmark For $\beta_i \le \beta_{ii}^* \equiv (1 - r_{ji}^2)/(1 + r_{ji}^2)$

$$\Gamma_{\pm\pm,ij}^{\mathcal{M}} = \frac{g_{ij}^{2} m_{i}}{4\pi} \sqrt{1 - \beta_{i}^{2}} \left\{ \frac{1}{2} \left(1 - r_{ji}^{2} \right) (1 - r_{ji})^{2} - (1 + r_{ji} + r_{ji}^{2}) r_{ji} \beta_{i}^{-1} \right.$$

$$\left. + \left[r_{ji} + \frac{1}{2} (\beta_{i}^{-2} - 1) (1 + r_{ji} + r_{ji}^{2}) \right] r_{ji} \ln \left(\frac{1 + \beta_{i}}{1 - \beta_{i}} \right) \right\},$$

$$\Gamma_{\pm\mp,ij}^{\mathcal{M}} = \frac{g_{ij}^{2} m_{i}}{4\pi} \sqrt{1 - \beta_{i}^{2}} \left\{ \frac{1}{2} \left(1 - r_{ji}^{2} \right) (1 - r_{ji})^{2} + (1 + r_{ji} + r_{ji}^{2}) r_{ji} \beta_{i}^{-1} \right.$$

$$\left. - \left[r_{ji} + \frac{1}{2} (\beta_{i}^{-2} - 1) (1 + r_{ji} + r_{ji}^{2}) \right] r_{ji} \ln \left(\frac{1 + \beta_{i}}{1 - \beta_{i}} \right) \right\},$$

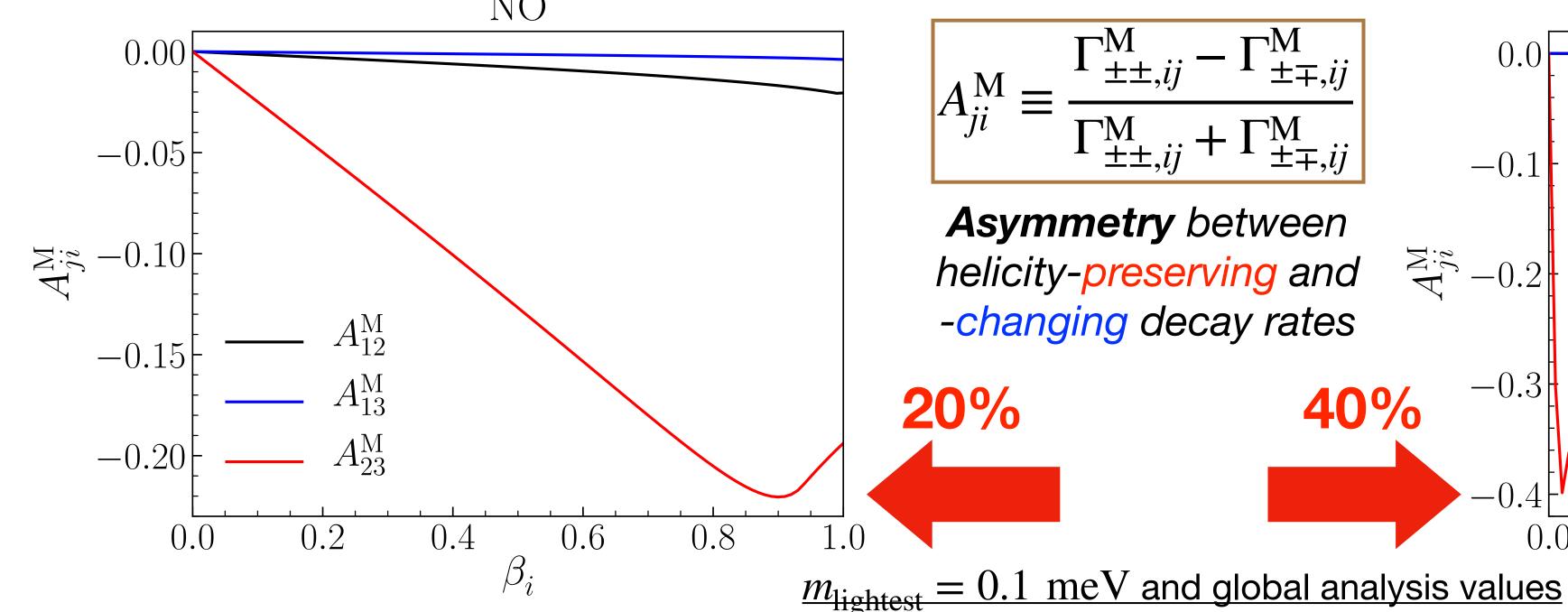
$$\checkmark$$
 For $\beta_i > \beta_{ii}^* \equiv (1 - r_{ji}^2)/(1 + r_{ji}^2)$

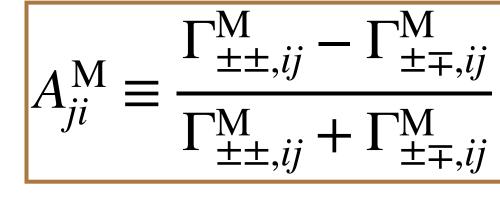
$$\Gamma_{\pm\pm,ij}^{\mathcal{M}} = \frac{g_{ij}^{2} m_{i}}{4\pi} \sqrt{1 - \beta_{i}^{2}} \left\{ \frac{1}{2} \left(1 - r_{ji}^{2} \right) (1 - r_{ji})^{2} - r_{ji} (1 - r_{ji}^{2}) - 2r_{ji}^{2} \ln r_{ji} \right.$$

$$\left. - (\beta_{i}^{-2} - 1) \left[(1 + r_{ji} + r_{ji}^{2}) r_{ji} \ln r_{ji} + \frac{1}{4} (1 - r_{ji}^{2}) (1 + 4r_{ji} + r_{ji}^{2}) \right] \right\},$$

$$\Gamma_{\pm\mp,ij}^{\mathcal{M}} = \frac{g_{ij}^{2} m_{i}}{4\pi} \sqrt{1 - \beta_{i}^{2}} \left\{ \frac{1}{2} \left(1 - r_{ji}^{2} \right) (1 - r_{ji})^{2} + r_{ji} (1 - r_{ji}^{2}) + 2r_{ji}^{2} \ln r_{ji} \right.$$

$$\left. + (\beta_{i}^{-2} - 1) \left[(1 + r_{ji} + r_{ji}^{2}) r_{ji} \ln r_{ji} + \frac{1}{4} (1 - r_{ji}^{2}) (1 + 4r_{ji} + r_{ji}^{2}) \right] \right\},$$

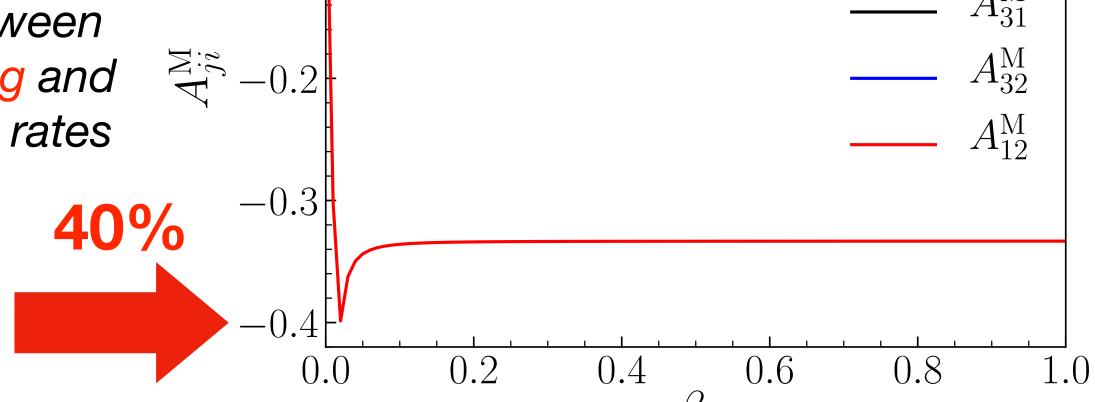




Asymmetry between helicity-preserving and -changing decay rates

20%





Experimental Constraints on g

- ✓ Set all couplings to be equal $g_{ii} = g$
- \checkmark The massless ϕ serves as an extra radiation affecting the BBN and the CMB power spectrum through decays $\nu_i \to \nu_i + \phi$ and scatterings $\nu_i + \nu_j \to \phi + \phi$
 - * For $g \leq 10^{-7}$ the scalar ϕ will never be in thermal equilibrium via scatterings before the CMB formation see, Hannestad & Raffelt, Phys. Rev. D 72 (2005) 103514
- ✓ The lower bound on τ_i (ultra-relativistic): Barenboim et al., <u>JCAP 03 (2021) 087</u>

$$au_i \gtrsim 4 \times 10^{(5\cdots 6)} \; {
m s} \left(\frac{m_{\nu}}{50 \; {
m meV}} \right)^5$$

 $\frac{Daughter neutrino masses may}{{
m weaken the lifetime constraint up to a factor of 50}}$

up to a factor of 50

i.e. (for NO),
$$g \lesssim (1.6 \cdots 5.0) \times 10^{-10}$$
 (for ν_3), $g \lesssim (0.4 \cdots 1.3) \times 10^{-7}$ (for ν_2)

We choose $10^{-16} \lesssim g \lesssim 10^{-10}$ in this work (a benchmark value: $g = 10^{-12}$)

Experimental Constraints on g

- ✓ Set all couplings to be equal $g_{ij} = g$ $\mathscr{L} \supset -\frac{1}{2}\overline{N_R^c}\mathbf{y}_N N_R S + \text{h.c.} \Rightarrow \frac{\mathrm{i}\phi}{2f}\sum_{i=1}^3 m_i \overline{\nu}_i \gamma^5 \nu_i \ [S \equiv (f + \rho + \mathrm{i}\phi)/\sqrt{2}]$
- \checkmark The massless ϕ serves as an extra radiation affecting the BBN and the CMB power spectrum through decays $\nu_i o \nu_j + \phi$ and scatterings $\nu_i + \nu_i o \phi + \phi$
 - * For $g \leq 10^{-7}$ the scalar ϕ will never be in thermal equilibrium via scatterings before the CMB formation see, Hannestad & Raffelt, Phys. Rev. D 72 (2005) 103514
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$$au_i \gtrsim 4 \times 10^{(5\cdots 6)} \text{ s} \left(\frac{m_{\nu}}{50 \text{ meV}}\right)^5$$

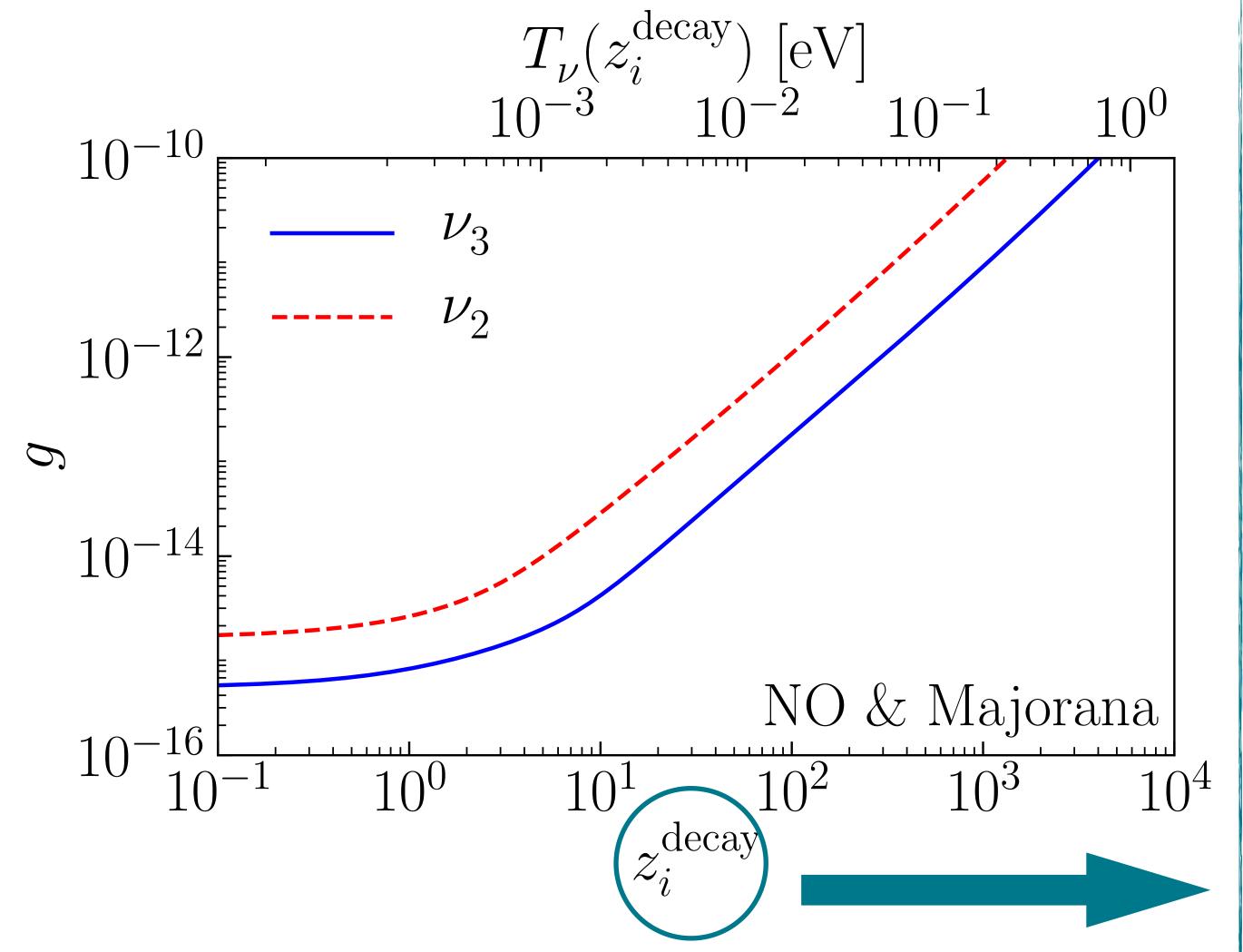
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i.e. (for NO), $g \lesssim (1.6 \cdots 5.0) \times 10^{-10}$ (for ν_3), $g \lesssim (0.4 \cdots 1.3) \times 10^{-7}$ (for ν_2)

We choose $10^{-16} \lesssim g \lesssim 10^{-10}$ in this work (a benchmark value: $g = 10^{-12}$)

• Modified number density $n_i(z) = \overline{n}_i(z)a(z)^3$ • Estimate z_i^{decay} from



$$\frac{T_{\nu}(z_{i}^{\text{decay}}) [\text{eV}]}{10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^{0}} = e^{-\widetilde{\lambda}_{i}(z_{i}^{\text{decay}})} f_{\text{FD}} \left[|\mathbf{p}|, T_{\nu}(z_{i}^{\text{decay}}) \right] |\mathbf{p}|^{2} d|\mathbf{p}|$$

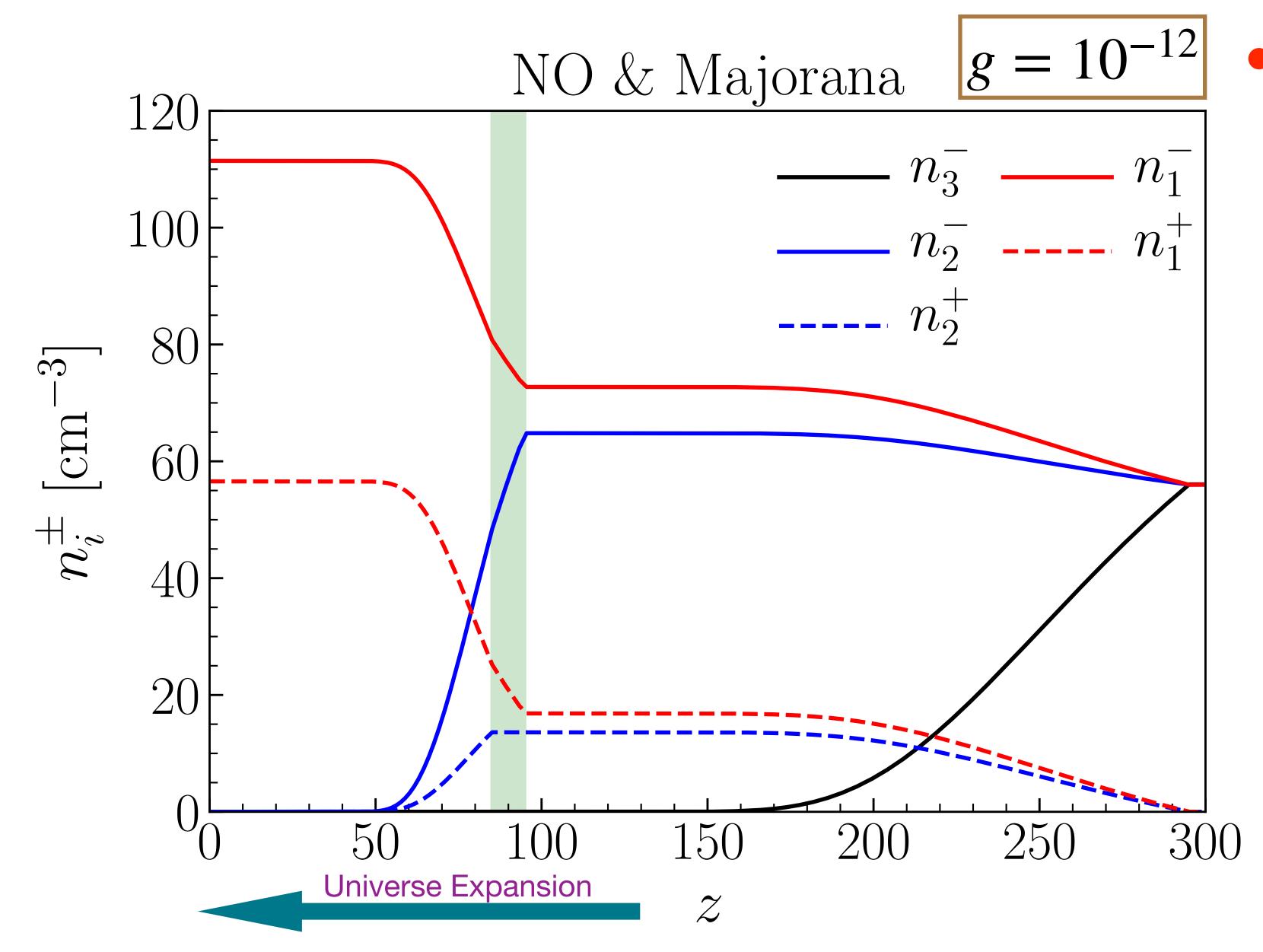
$$= e^{-1}$$

$$\int_{0}^{\infty} f_{\text{FD}} \left[|\mathbf{p}|, T_{\nu}(z_{i}^{\text{decay}}) \right] |\mathbf{p}|^{2} d|\mathbf{p}|$$

where the <u>suppression factor</u>

$$\widetilde{\lambda}_{i}(z_{i}^{\text{decay}}) \equiv \int_{z_{i}^{\text{decay}}}^{\infty} \frac{\mathrm{d}z \; \Gamma_{\pm,i}^{\mathrm{M}^{*}}}{(1+z)H(z)\gamma_{i}(z)}$$

• z_i^{decay} characterizes the redshift when <u>a substantial fraction</u> of ν_i starts to decay for a given g after taking account of its *momentum distribution*

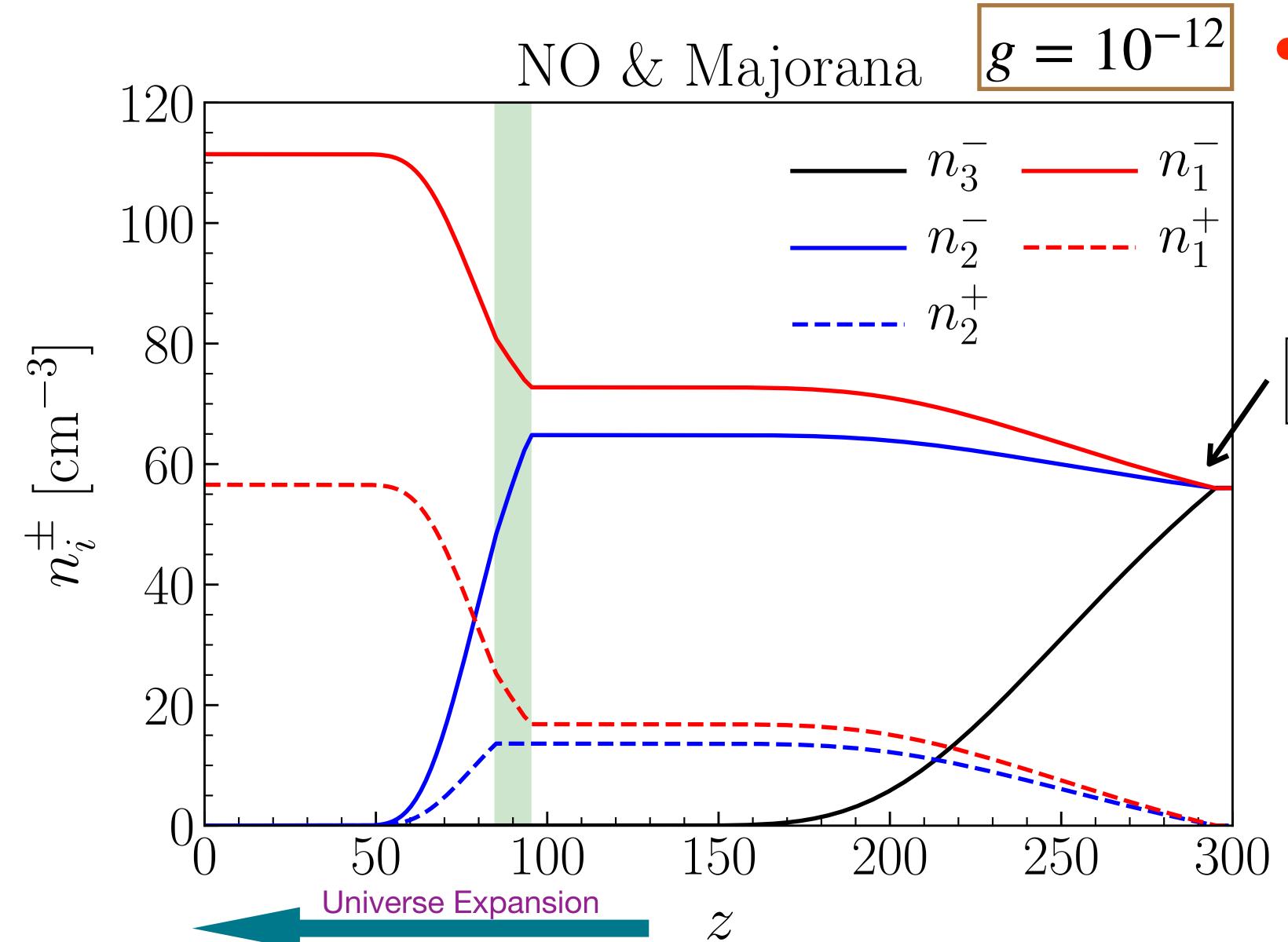


Number densities

$$n_i^{\pm}(z) = n_0 e^{-\lambda_i(z)}$$

$$n_j^{\pm}(z) = \left[n_0 - n_i^{\pm}(z) \right] \mathcal{B}_{ij}^{M+}$$

branching ratio of helicitypreserving decays

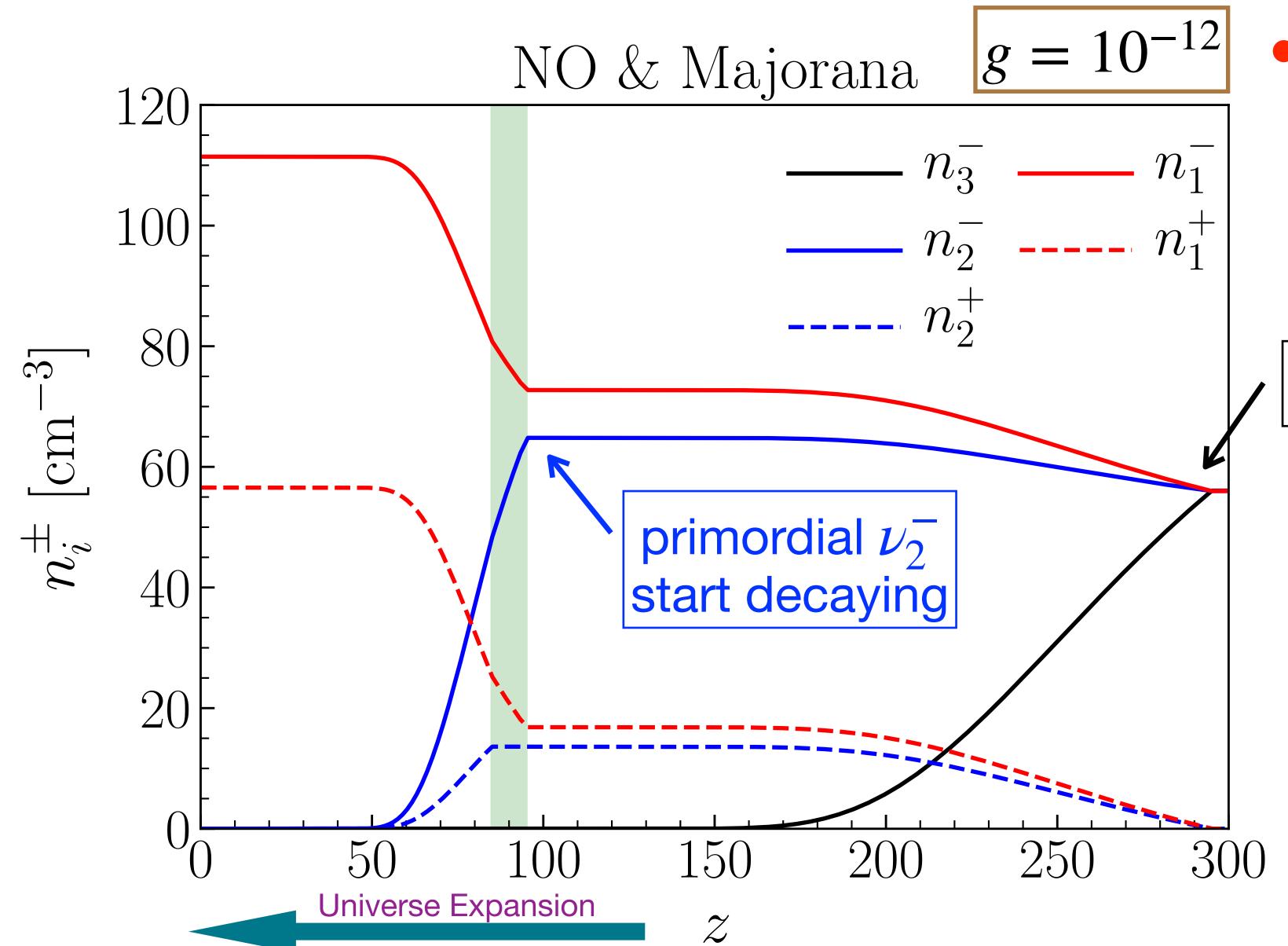


Number densities

$$n_i^{\pm}(z) = n_0 e^{-\lambda_i(z)}$$

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 ν_3^- start decaying

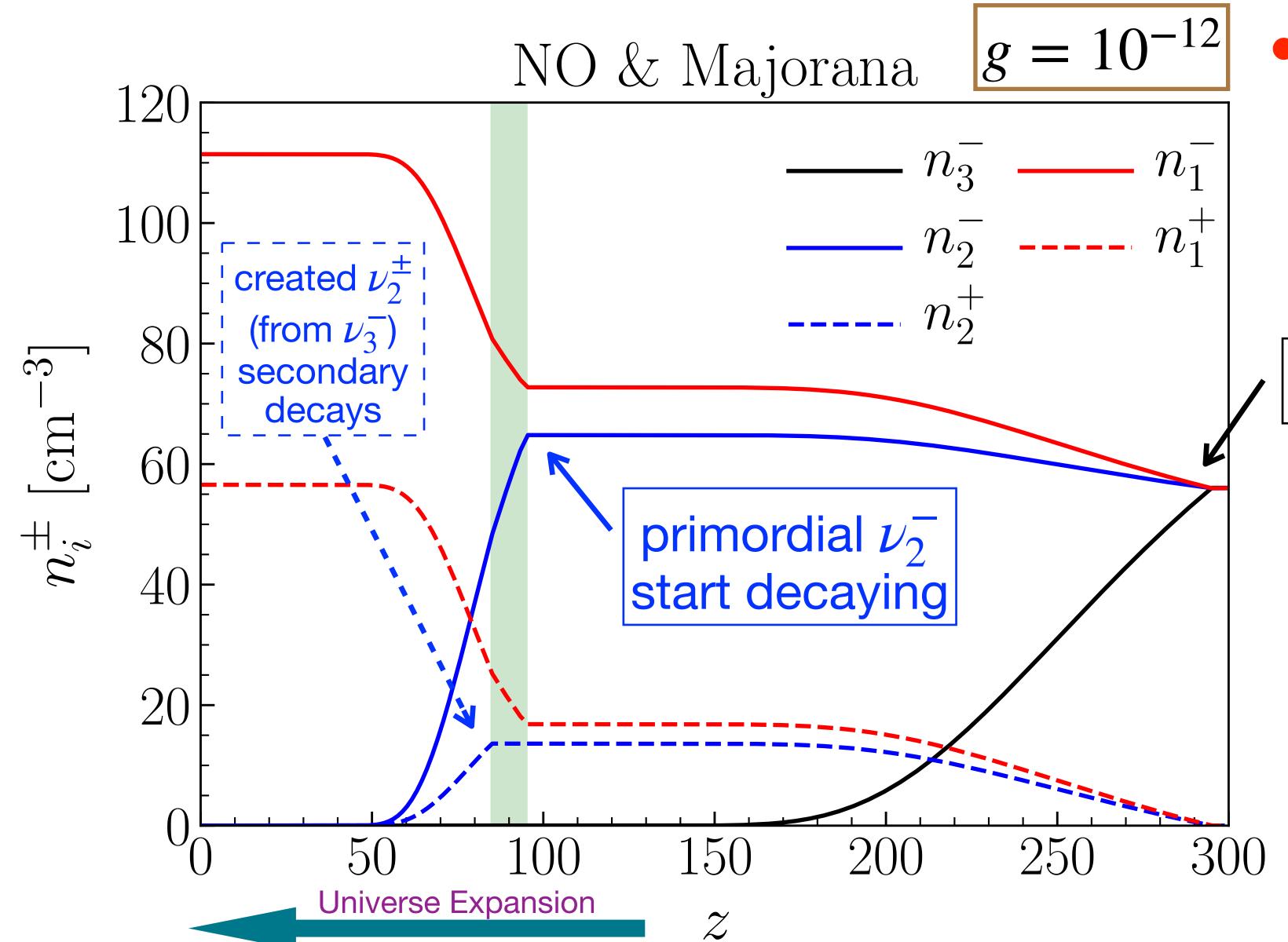


Number densities

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 $|\nu_3^-|$ start decaying

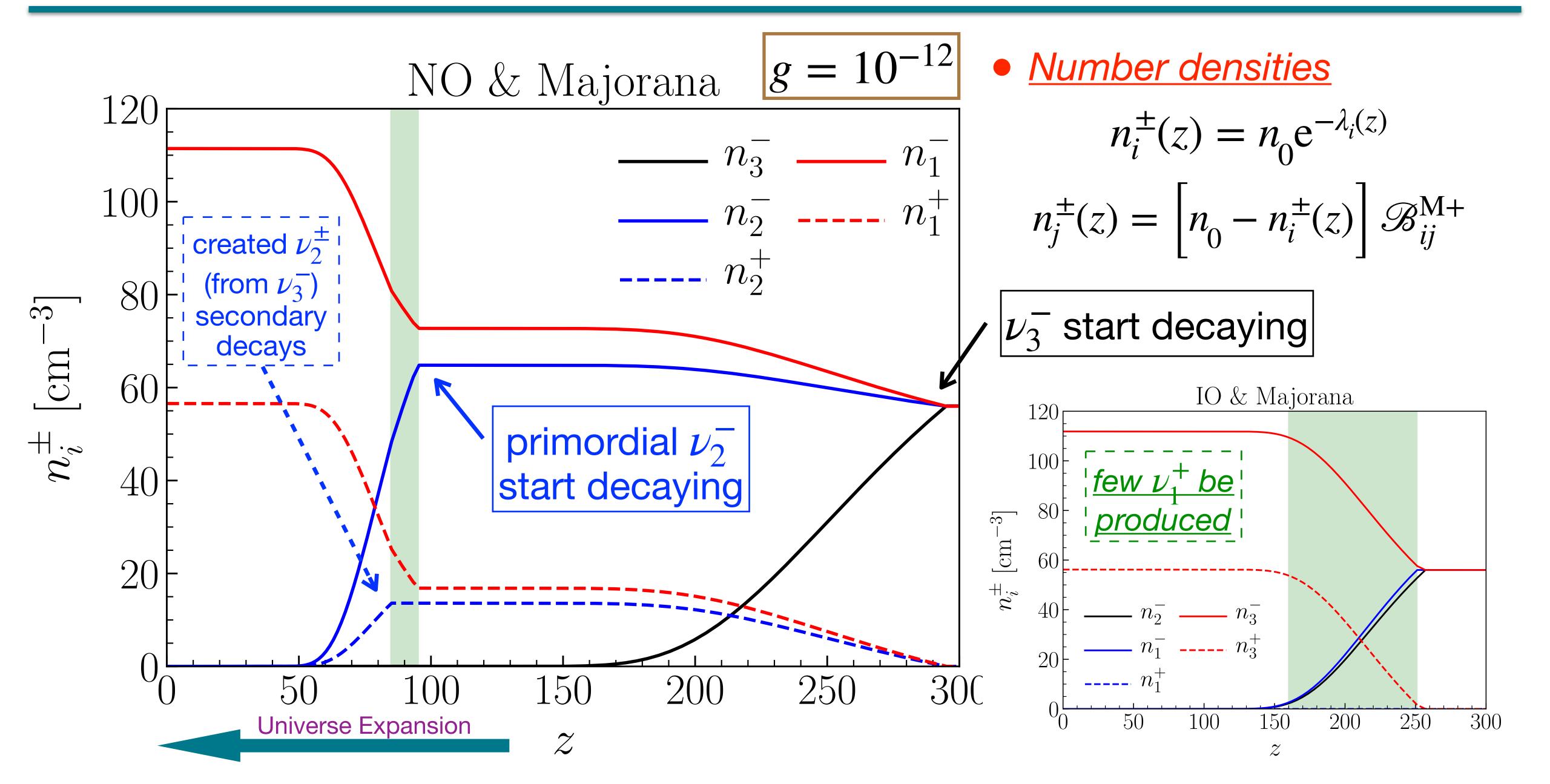


Number densities

$$n_i^{\pm}(z) = n_0 e^{-\lambda_i(z)}$$

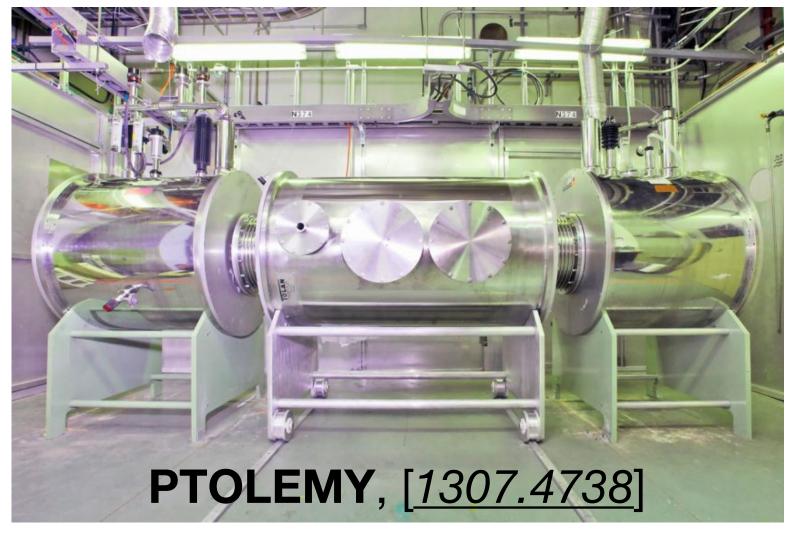
$$n_j^{\pm}(z) = \left[n_0 - n_i^{\pm}(z) \right] \mathcal{B}_{ij}^{M+}$$

 ν_3^- start decaying



CvB Capture Rates





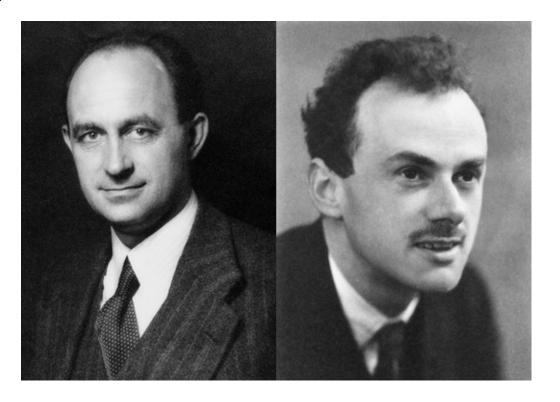
Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield (PTOLEMY)

• Capture Rates: Long, Lunardini & Sabancilar, JCAP 08 (2014) 038

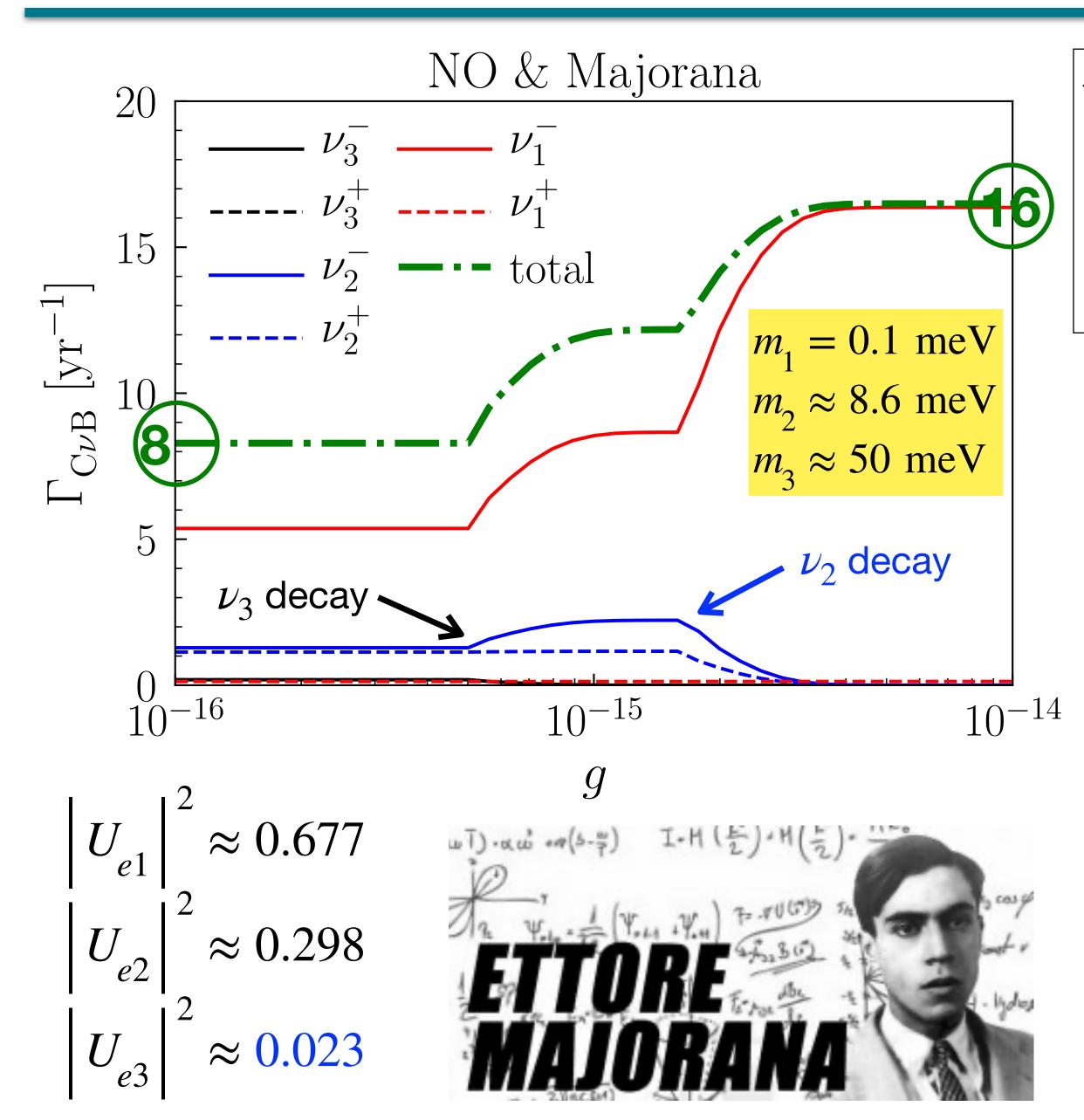
$$\left[\Gamma_{\text{C}\nu\text{B}} = N_{\text{T}}\overline{\sigma} \sum_{s_i = \pm 1/2} \sum_{i=1}^{3} \left| U_{ei} \right|^2 n_i(s_i) \mathcal{A}(s_i)\right]$$

- Number of tritium nuclei in the target (100 g tritium) $N_{
 m T} pprox 2 imes 10^{25}$
- Cross section $\bar{\sigma} \approx 3.8 \times 10^{-45} \text{ cm}^2$
- Leptonic flavor mixing matrix $\left|U_{e1}\right|^2 \approx 0.677$, $\left|U_{e2}\right|^2 \approx 0.298$, $\left|U_{e3}\right|^2 \approx 0.023$
- $\mathscr{A}(s_i) \equiv 1 2s_i \left< \beta_i \right>$ with the momentum Fermi-Dirac distribution

$$\left\langle \beta_{i} \right\rangle = \frac{\int_{0}^{\infty} \beta_{i} f_{FD} \left(\left| \mathbf{p}_{i} \right|, T_{\nu_{i}}^{0} \right) |\mathbf{p}_{i}|^{2} d|\mathbf{p}_{i}|}{\int_{0}^{\infty} f_{FD} \left(\left| \mathbf{p}_{i} \right|, T_{\nu_{i}}^{0} \right) |\mathbf{p}_{i}|^{2} d|\mathbf{p}_{i}|}$$

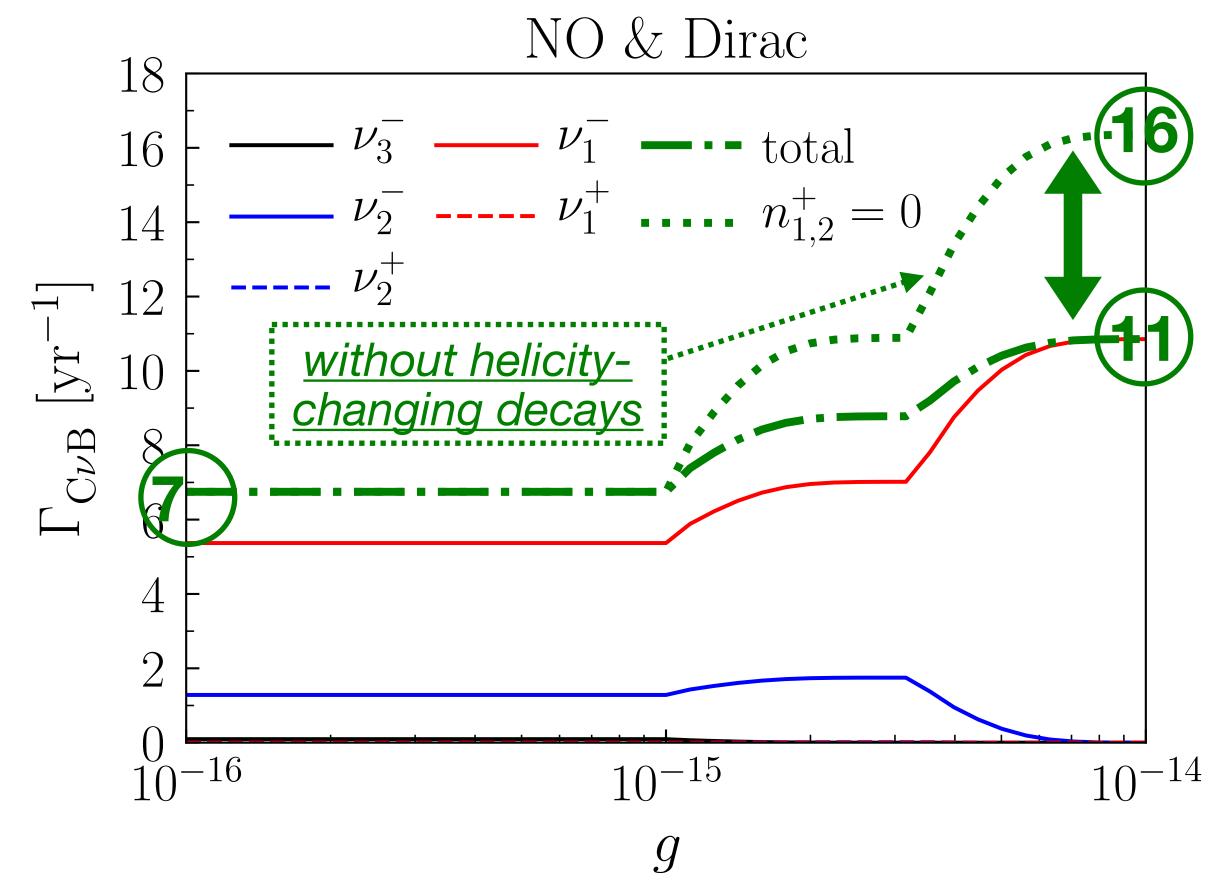


Capture Rates (NO)

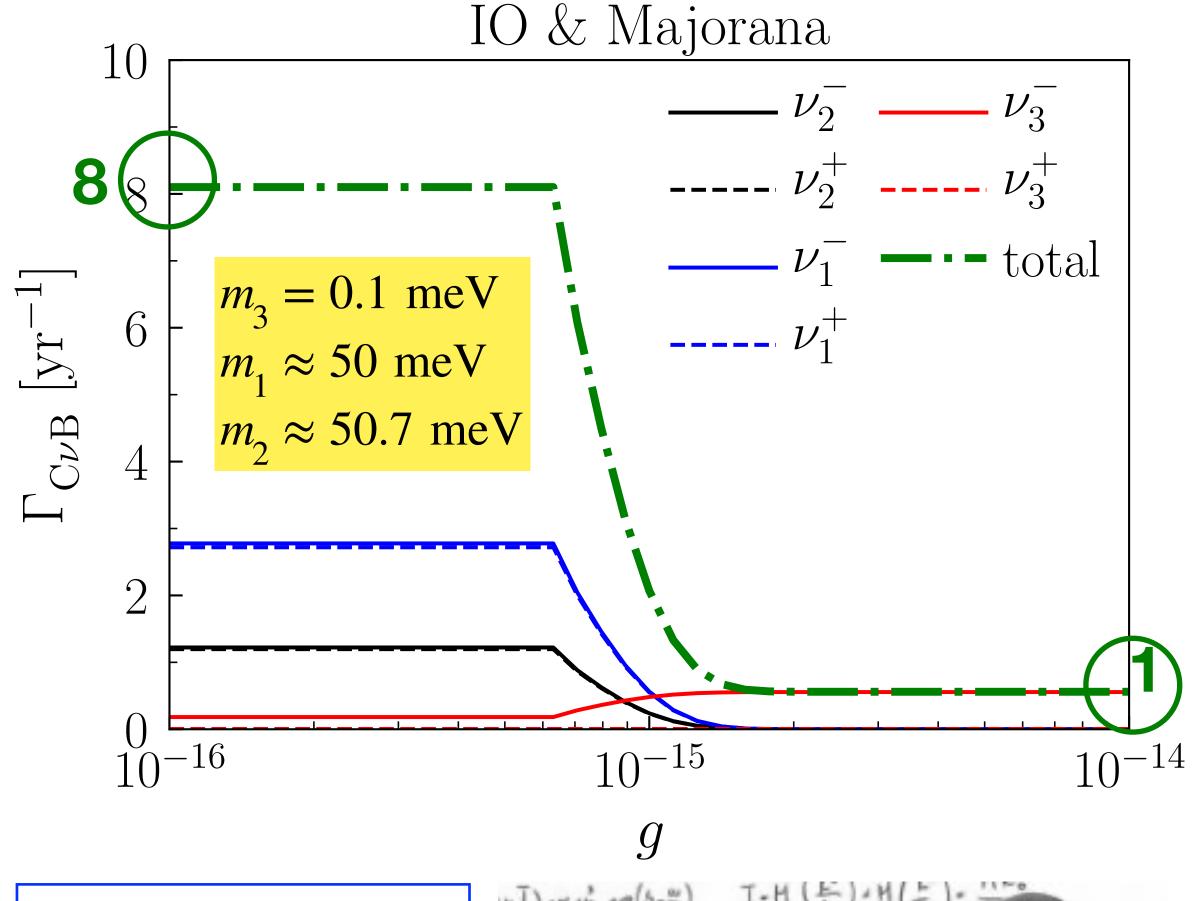


√ The number density of left-helical states will be much larger without helicity-changing decays.

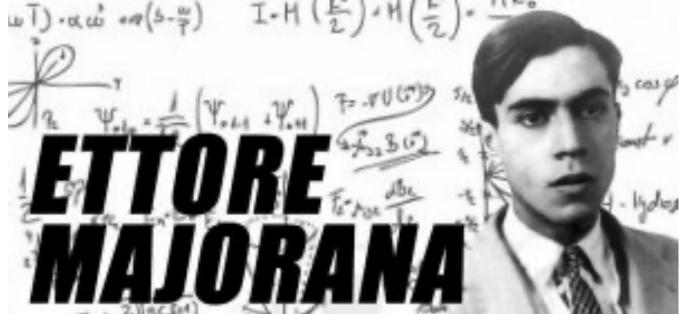




Capture Rates (IO)

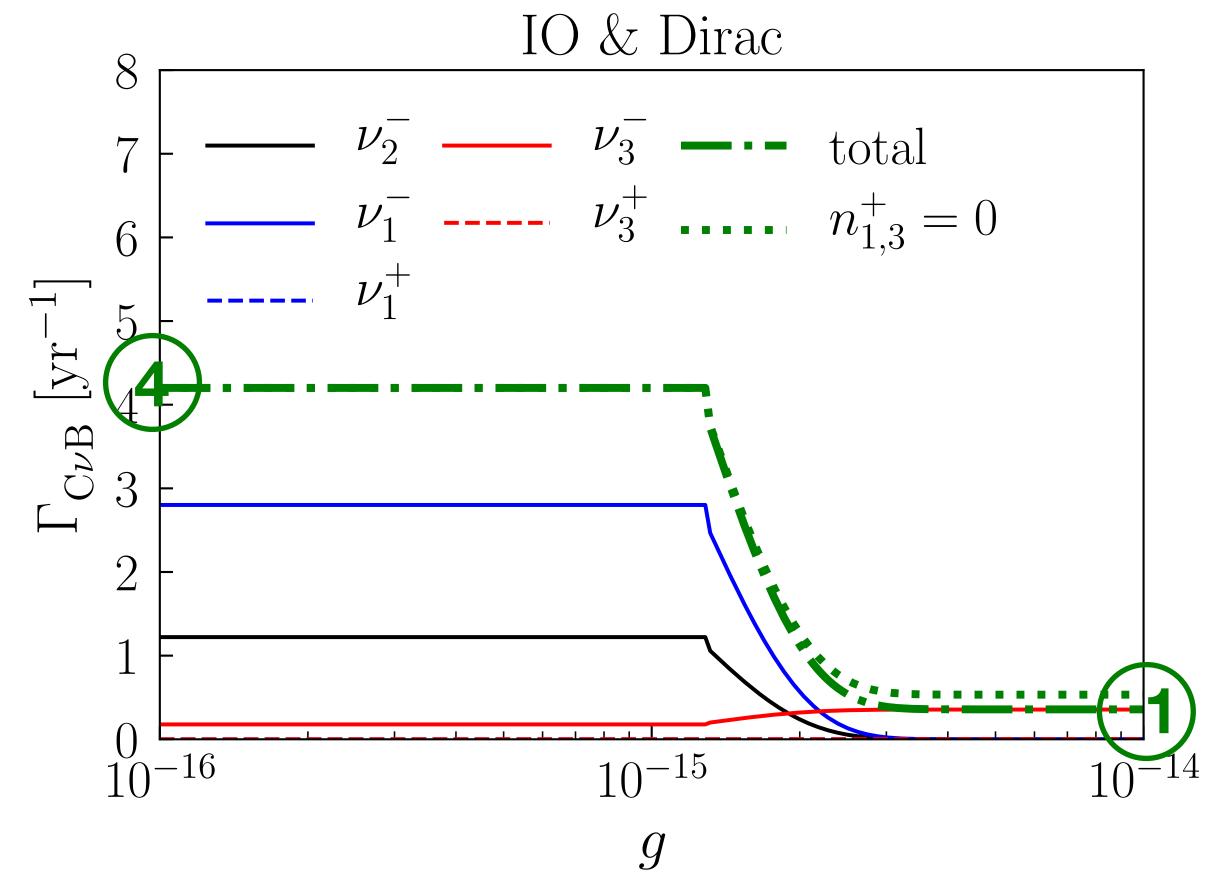


no significant production of ν_1 from ν_2 decays



✓ The difference is negligible for IO ($\nu_2 \rightarrow \nu_1$ decays are suppressed).





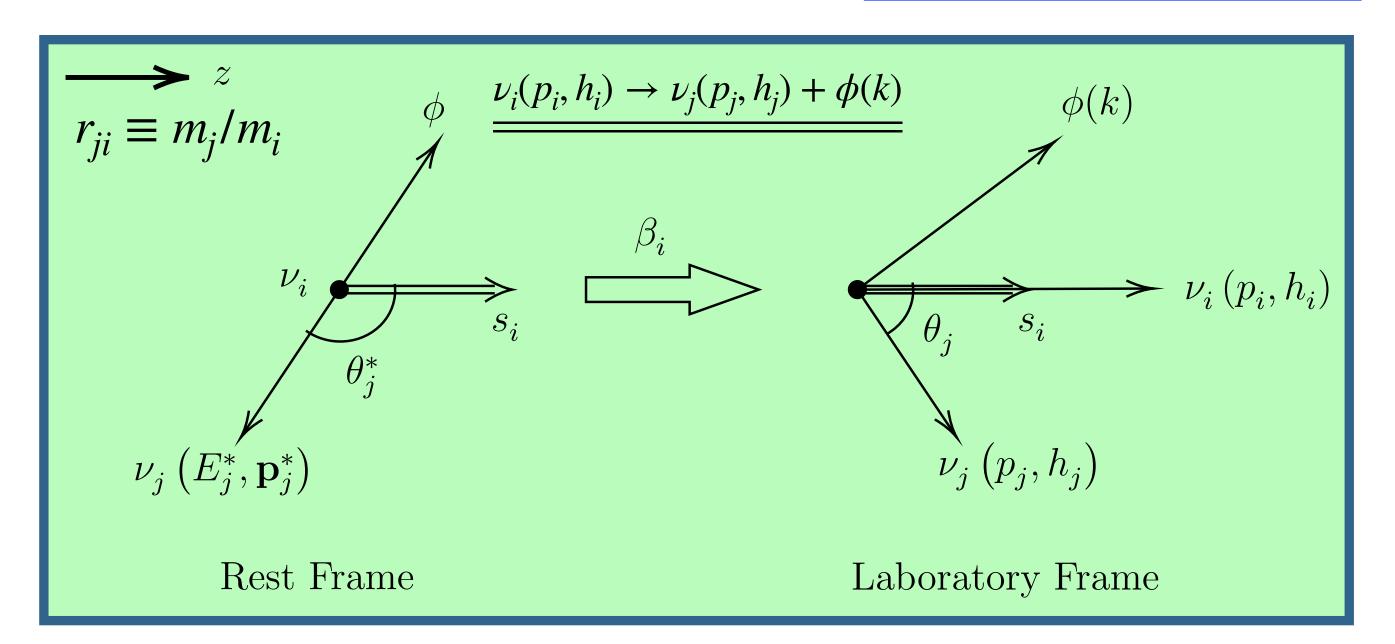
- We study the invisible decays of massive neutrinos and their implications for the CvB detection.
 - 1. The decay rates in the helicity-preserving and -changing decay channels are calculated and discussed in detail.
 - 2. The strategy to evaluate the cosmic neutrino number densities is explained by taking a benchmark value of the coupling between massive neutrinos and the Nambu-Goldstone boson.
 - 3. The capture rates of CvB in the PTOLEMY-like experiment are obtained when considering neutrino decays and the distribution function ($\Gamma^{M}_{C\nu B} \approx 16 \text{ yr}^{-1}$ and $\Gamma^{D}_{C\nu B} \approx 11 \text{ yr}^{-1}$ in the NO case, **1 event per year** in the IO case).
- It is important to probe the intrinsic properties of massive neutrinos (e.g., Dirac or Majorana nature, absolute mass scale, lifetimes, etc.). The PTOLEMY-like experiments have the capability to measure the absolute neutrino mass and detect these-relic neutrinos. It also serve as an instructive platform to test BSM theories (Although the detection of CvB is definitely challenging...).

Thanks for your attention!

Backup Slides

Helicity-changing Decays

$$\mathcal{L}_{\mathbf{M}} = \frac{1}{2} \sum_{i} \left(\overline{\nu_{i}} \mathrm{i} \partial \nu_{i} - m_{i} \overline{\nu_{i}} \nu_{i} \right) + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \left[\mathrm{i} \phi \sum_{i,j} g_{ij} \overline{\nu_{i}} \gamma^{5} \nu_{j} + \mathrm{h.c.} \right]$$



$$\underbrace{E.g.}_{u_i(p_i, h_i = +1)} = \begin{pmatrix} \sqrt{E_i - |\mathbf{p}_i|} \\ 0 \\ \sqrt{E_i + |\mathbf{p}_i|} \\ 0 \end{pmatrix} \xrightarrow{helicity-preserving:}_{h_j = +1} \Rightarrow \propto \cos(\theta/2)$$

$$\checkmark \underset{helicity-changing}{helicity-changing}$$

$$h_j = -1 \Rightarrow \propto \sin(\theta/2)$$

√ <u>helicity-preserving:</u>

$$h_j = +1 \Rightarrow \propto \cos(\theta/2)$$

$$h_j = -1 \Rightarrow \propto \sin(\theta/2)$$

• Helicity spinors with $h=\pm 1$

$$\chi^{(+)}(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{+i\phi}\sin\frac{\theta}{2} \end{pmatrix}, \quad \chi^{(-)}(\theta,\phi) = \begin{pmatrix} -\sin\frac{\theta}{2} \\ e^{+i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

► Wave functions with h = +1 & -1

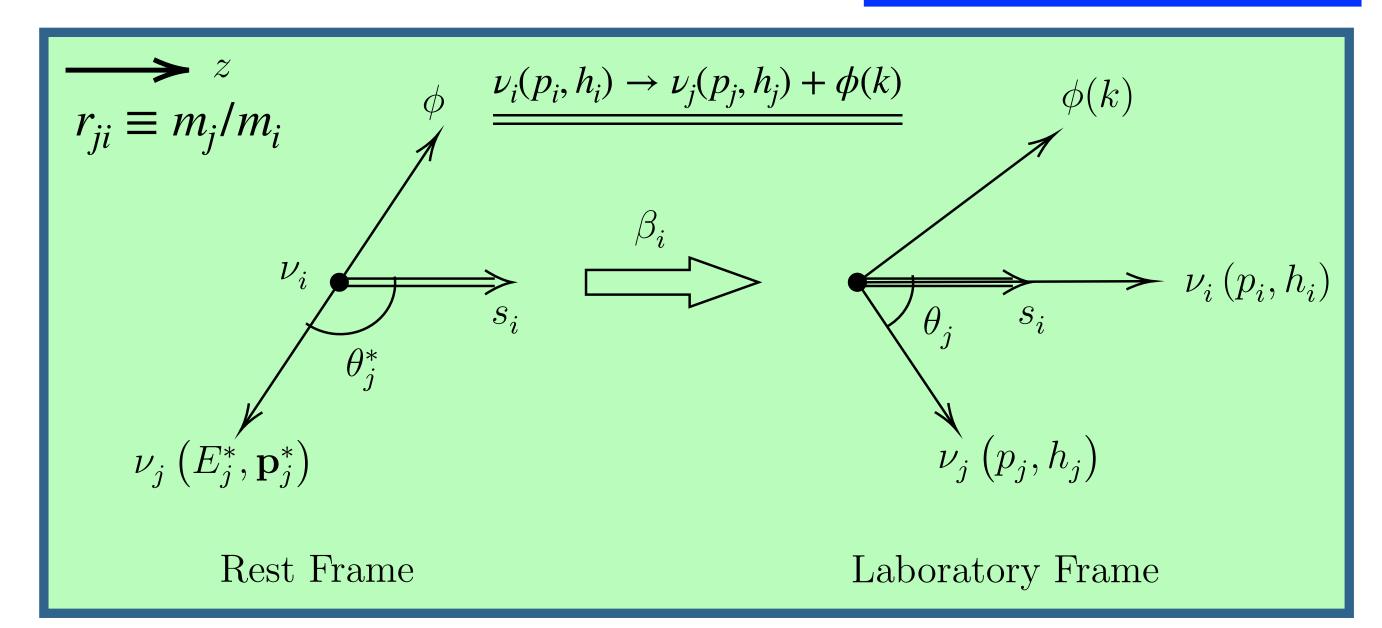
$$u(p, h = +1) = \cos \frac{\theta}{2} \begin{pmatrix} \sqrt{E - |\mathbf{p}|} \\ 0 \\ \sqrt{E + |\mathbf{p}|} \\ 0 \end{pmatrix} + \sin \frac{\theta}{2} e^{i\phi} \begin{pmatrix} 0 \\ \sqrt{E + |\mathbf{p}|} \\ 0 \\ \sqrt{E - |\mathbf{p}|} \end{pmatrix}$$

$$u(p, h = -1) = -\sin\frac{\theta}{2} \begin{pmatrix} \sqrt{E + |\mathbf{p}|} \\ 0 \\ \sqrt{E - |\mathbf{p}|} \\ 0 \end{pmatrix} + \cos\frac{\theta}{2} e^{i\phi} \begin{pmatrix} 0 \\ \sqrt{E - |\mathbf{p}|} \\ 0 \\ \sqrt{E + |\mathbf{p}|} \end{pmatrix}$$

Both positive- and negative-helical states of daughter neutrinos can be produced!

Decay Amplitudes and Decay Rates

$$\mathcal{L}_{\mathbf{M}} = \frac{1}{2} \sum_{i} \left(\overline{\nu_{i}} i \partial \!\!\!/ \nu_{i} - m_{i} \overline{\nu_{i}} \nu_{i} \right) + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \left[i \phi \sum_{i,j} g_{ij} \overline{\nu_{i}} \gamma^{5} \nu_{j} + \text{h.c.} \right]$$



3. Differential decay rates

$$\frac{d\Gamma_{\pm\pm,ij}^{M}}{dE_{j}} = \frac{g_{ij}^{2}m_{i}^{2}}{4\pi E_{i}} \left[+ \frac{E_{i}^{2}r_{ji}^{2} + E_{j}^{2}}{|\mathbf{p}_{i}|^{2}|\mathbf{p}_{j}|} - \frac{(1+r_{ji})^{2}}{2|\mathbf{p}_{i}|} \frac{E_{i}E_{j}}{|\mathbf{p}_{i}||\mathbf{p}_{j}|} + \frac{(1-r_{ji})^{2}}{2|\mathbf{p}_{i}|} \left(1 + \frac{m_{i}^{2}r_{ji}}{|\mathbf{p}_{i}||\mathbf{p}_{j}|} \right) \right] \\
\frac{d\Gamma_{\pm\pm,ij}^{M}}{dE_{j}} = \frac{g_{ij}^{2}m_{i}^{2}}{4\pi E_{i}} \left[-\frac{E_{i}^{2}r_{ji}^{2} + E_{j}^{2}}{|\mathbf{p}_{i}|^{2}|\mathbf{p}_{j}|} + \frac{(1+r_{ji})^{2}}{2|\mathbf{p}_{i}|} + \frac{(1-r_{ji})^{2}}{2|\mathbf{p}_{i}|} \left(1 - \frac{m_{i}^{2}r_{ji}}{|\mathbf{p}_{i}||\mathbf{p}_{j}|} \right) \right] \\
\frac{d\Gamma_{\pm\pm,ij}^{M}}{dE_{j}} = \frac{g_{ij}^{2}m_{i}^{2}}{4\pi E_{i}} \left[-\frac{E_{i}^{2}r_{ji}^{2} + E_{j}^{2}}{|\mathbf{p}_{i}|^{2}|\mathbf{p}_{j}|} + \frac{(1-r_{ji})^{2}}{2|\mathbf{p}_{i}|} \left(1 - \frac{m_{i}^{2}r_{ji}}{|\mathbf{p}_{i}||\mathbf{p}_{j}|} \right) \right] \\
\frac{M_{+-,ij}^{M}}{|\mathbf{p}_{i}|^{2}} = |\mathcal{M}_{--,ij}^{M}|^{2}, \quad |\mathcal{M}_{+-,ij}^{M}|^{2} = |\mathcal{M}_{-+,ij}^{M}|^{2}$$

1. <u>Decay amplitudes</u> (Majorana v)

$$i\mathcal{M}_{h_i h_j, ij}^{\mathbf{M}} = 2g_{ij}\overline{u_j}(p_j, h_j)\gamma^5 u_i(p_i, h_i)$$

2. Amplitude squared

$$|\mathcal{M}_{h_i h_j, ij}^{\mathrm{M}}|^2 = 4g_{ij}^2 \mathrm{Tr} \left[u_i(p_i, h_i) \overline{u_i(p_i, h_i)} \gamma^5 u_j(p_j, h_j) \overline{u_j(p_j, h_j)} \gamma^5 \right]$$

for a specific helicity
$$u(p,h)\overline{u(p,h)} = \frac{1}{2}(\not p+m)(1+h\gamma^5\not s)$$

$$|\mathcal{M}_{++,ij}^{M}|^{2} = 8g_{ij}^{2} \left(E_{i}E_{j} - m_{i}m_{j} - |\mathbf{p}_{i}||\mathbf{p}_{j}| \right) \cos^{2} \frac{\theta_{j}}{2}$$

$$|\mathcal{M}_{+-,ij}^{\mathbf{M}}|^{2} = 8g_{ij}^{2} \left(E_{i}E_{j} - m_{i}m_{j} + |\mathbf{p}_{i}| |\mathbf{p}_{j}| \right) \sin^{2} \frac{\theta_{j}}{2}$$

✓ These identities hold:

$$|\mathcal{M}_{++,ij}^{\mathbf{M}}|^2 = |\mathcal{M}_{--,ij}^{\mathbf{M}}|^2, \quad |\mathcal{M}_{+-,ij}^{\mathbf{M}}|^2 = |\mathcal{M}_{-+,ij}^{\mathbf{M}}|^2$$

Cosmological Constraint

When converting this bound to that on the coupling constant, we have

$$g \lesssim (1.6 \cdots 5.0) \times 10^{-10} \text{ (for } \nu_3), \quad g \lesssim (0.4 \cdots 1.3) \times 10^{-7} \text{ (for } \nu_2), \qquad (2.39)$$

in the NO case. In the IO case, we get $g \lesssim (2 \cdots 6) \times 10^{-10}$ for both ν_1 and ν_2 , since they are almost degenerate in mass. If the masses of daughter neutrinos are taken into account, a phase-space factor comes into play and the constraint on the lifetime will be weakened [43].

The revised bound on the coupling constant reads

$$g \lesssim 3.2 \times 10^{-10} \text{ (for } \nu_3), \quad g \lesssim 5.0 \times 10^{-8} \text{ (for } \nu_2)$$
 (2.40)

in the NO case, while $g \lesssim 4 \times 10^{-10}$ in the IO case.

$$\mathcal{L} = \frac{iJ}{2f} \sum_{i=1}^{3} m_i \overline{\nu}_i \gamma^5 \nu_i \qquad f \sim m_N$$

Chen, Oldengott, Pierobon & Wong, EPJC 82 (2022) 7, 640

Other Constraints on g

- Other constraints on the coupling g come from terrestrial experiments and astrophysical observations.
- The interaction gives rise to the $0\nu\beta\beta$ decays with an extra scalar ϕ . The non-observation of such a signal in the EXO-200 experiment provides a constraint on the coupling constant $g \lesssim (0.4 \cdots 0.9) \times 10^{-5}$ [Kharusi et al., PRD = 104 (2021) 11, 112002]
- Astrophysical constraints on the coupling can also be derived from:
 - **V BBN:** Ahlgren, Ohlsson & Zhou, PRL 111 (2013) 19, 199001; Huang, Ohlsson & Zhou, PRD 97 (2018) 7, 075009; Escudero & Witte, EPJC 80 (2020) 4, 294; Venzor, Pérez-Lorenzana & De-Santiago, PRD 103 (2021) 4, 043534
 - ✓ SN1987A: Kolb & Turner, PRD 36 (1987) 2895; Alekseev, Alekseeva, Krivosheina & Volchenko, PLB 205 (1988) 209; Farzan, PRD 67 (2003) 073015; Zhou, PRD 84 (2011) 038701; Shalgar, Tamborra & Bustamante, PRD 103 (2021) 12, 123008; Fiorillo, Raffelt & Vitagliano, PRL 131 (2023) 2, 021001; Fiorillo, Raffelt & Vitagliano, PRL 132 (2024) 2, 021002; Akita, Im, Masud & Yun, JHEP 07 (2024) 057; Martínez-Miravé, Tamborra & Tórtola, JCAP 05 (2024) 002
 - ✓ **Solar neutrinos**: Bahcall, Cabibbo & Yahil, *PRL 28 (1972) 316*; Berezhiani, Fiorentini, Moretti & Rossi, *ZPC 54 (1992) 581*; Cleveland et al., *Astrophys.J. 496 (1998) 505*; Beacom & Bell, *PRD 65 (2002) 113009*; SAGE collaboration, *PRC 80 (2009) 015807*; Bellini et al., *PRL 107 (2011) 141302*; KamLAND collaboration, *PRC 84 (2011) 035804*; *Phys.Rev.C 92 (2015) 5, 055808*; Super-Kamiokande collaboration, *PRD 94 (2016) 5, 052010*; Borexino collaboration, *PRD 101 (2020) 6, 062001*; SNO collaboration, *PRD 99 (2019) 3, 032013*; Huang & Zhou, *JCAP 02 (2019) 024*
 - ✓ Atmospheric and long-baseline accelerator neutrinos: Lipari & Lusignoli, PRD 60 (1999) 013003; Fogli, Lisi, Marrone & Scioscia, PRD 59 (1999) 117303; Gonzalez-Garcia & Maltoni, PLB 663 (2008) 405; Gomes, Gomes & Peres, PLB 740 (2015) 345; Choubey, Dutta & Pramanik, JHEP 08 (2018) 141
 - ✓ High-energy astrophysical neutrinos: Ng & Beacom, PRD 90 (2014) 6, 065035; Shoemaker & Murase, PRD 93 (2016) 8, 085004; Denton & Tamborra, PRL 121 (2018) 12, 121802; Salas et al., PLB 789 (2019) 472; Song et al., JCAP 04 (2021) 054; Valera, Fiorillo, Esteban & Bustamante, PRD 110 (2024) 4, 043004

Secondary distribution

The distribution function of the final-state ν_i in helicity-changing decay $\nu_i(p_i,\pm) \to \nu_i(p_i,\mp) + \phi$ is

$$f_j(E_j) = \int_{l(E_i)}^{\infty} dE_i \ f_i(E_i) \times \frac{1}{\Gamma_{\pm,i}^M} \frac{d\Gamma_{\pm\mp,ij}^M}{dE_j}$$
 Lipari, Lusignoli & Meloni, PRD 75 (2007) 123005

Here $f_i(E_i)$ is the energy distribution function of parent neutrinos and $l(E_j) = \frac{E_j}{2} \left(1 + \frac{1}{r_{ii}^2} \right) + \frac{|\mathbf{p}_j|}{2} \left(1 - \frac{1}{r_{ji}^2} \right)$

