A research of proto-neutron star evolution and the structure near the surface

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Introduction

After a supernova explosion (SN), the new-born neutron star left in the central region is called proto-neutron star (PNS). The general understanding is that the emission of neutrinos leads to the cooling of the PNS. In the discussion of PNS evolution, neutrinos play a crucial role. With upcoming neutrino detectors such as upgraded water-Cherenkov observatories and the liquid scintillator detector JUNO, the observable duration is expected to extend to several tens of seconds during a Galactic supernova event. Thus, conducting long-term simulations of PNS cooling is necessary to accurately estimate the emitted neutrinos.

In the previous studies (Nakazato et al., 2018), the impact of neutrino reactions and evolutionary calculations due to the distribution of heavy atomic nuclei near the surface of PNS, using different EOS for nuclear matter, was discussed. One of our focus has been on the late thermal evolution of the outer layers of PNS, revealing the formation of temperature distribution through evolutionary calculations. Moreover, the code we are using is based on spherical symmetry. Since convective motions within the PNS are expected to significantly affect not only thermal quantities' distribution, but also neutrinos emitted, we implemented a diffusion-based treatment of convection for comparing the early stage of evolution.

Method

To investigate the long-term evolution of PNS, we employed a quasi-static evolution code that solves neutrino transport using a multi-group flux-limited diffusion scheme under spherical symmetry with general relativity. The flux limiter we adopted can be found in Mayle & Wilson (1987). We computed our evolution model without convection up to 50 seconds for two different nuclear equations of state (Shen 1998, Togashi 2017), and we only calculate a convection model with Togashi EOS up to about 4 seconds. The neutrino reactions considered in this code are the following.

Absorption and emission (AE)	$e^- + p \leftrightarrow \nu_e + n \quad e^+ + n \leftrightarrow \bar{\nu}_e + p$ $e^- + A \leftrightarrow \nu_e + A'$
Electron scattering (ES)	$\nu + e^{\pm} \leftrightarrow \nu + e^{\pm}$
Thermal pair (TP)	$e^- + e^+ \leftrightarrow \nu + \bar{\nu} \quad \tilde{\gamma} \leftrightarrow \nu + \bar{\nu}$ $N + N \leftrightarrow N + N + \nu + \bar{\nu}$
Isoenergetic scattering off nucleon (IS)	$\nu + n \leftrightarrow \nu + n \quad \nu + p \leftrightarrow \nu + p$ $\nu + A \leftrightarrow \nu + A$

For our convective model, we study the effect of convection on PNS evolution within the mixing length theory. An unstable stratification can be found by using Ledoux criterion $C_{\rm L}$. Thus, the Brunt-Väisälä frequency $f_{\rm BV}$ are obtained as

$$f_{\rm BV}^2 = -\frac{1}{\rho}\frac{\partial \Phi}{\partial r}C_{\rm L} \qquad C_{\rm L} = \left.\frac{\partial \rho}{\partial s}\right|_{P,Y_{\rm e}}\frac{ds}{dr} + \left.\frac{\partial \rho}{\partial Y_{\rm e}}\right|_{P,s}\frac{dY_{\rm e}}{dr}$$

and ρ, s, $Y_{\rm e}$, r, P, Φ are baryon density, entropy per baryon, electron fraction, radius, pressure and the gravitational potential. The areas are convectively unstable with $f_{\rm BV}^2 < 0$, and convective motions tend to bring these areas towards $f_{\rm BV} = 0$. To model the convection, we solved diffusion equations of s, $Y_{\rm e}$ in $f_{\rm BV}^2 < 0$ areas,

$$\rho\frac{ds}{dt} = \nabla \cdot (\rho D \nabla s) \qquad \rho\frac{dY_{\rm e}}{dt} = \nabla \cdot (\rho D \nabla Y_{\rm e})$$

where diffusion coefficient D is determined by mixing length λ_P and convective velocity $v_{\rm c}$. λ_P is the distance a blob moves before mixing, it's assumed to be equal to pressure scale height in our research. $v_{\rm c}$ is estimated using energy conservation(Mayle & Wilson 1988).

$$v_{\rm c} = \lambda_P\sqrt{\frac{2gC_{\rm L}}{\rho}} \qquad \lambda_P = -\frac{dr}{d\ln P} \qquad D = \frac{1}{3}v_{\rm c}\lambda_P$$

All the PNS evolution equations are solved with implicit scheme, in the split operator approximation. The convection equations are solved individually, after a new state variables are found from other equations (neutrino transport, TOV, etc.).

Moreover, we only solved the diffusion equations at areas $f_{\rm BV}^2 < 0$. When the convective boundary is treated with such a simple on/off threshold criterion, the solution can exhibit numerical oscillations. To address this issue, we adopted a smoothing scheme for the diffusion coefficient D using a Gaussian function. We assume a gauss function with a maximum $D_{\rm i} = \frac{1}{3}v_{\rm c,i}\lambda_{P,\rm i}$ at each mesh point $\rm i \in [1, n]$. Then, at arbitrary mesh point k, we get several new diffusion coefficient $D'_{\rm i\to k}$ from each mesh point as below. Then, we adopt a new diffusion coefficient $D'_{\rm k}$ as the maximum of $D'_{\rm i\to k}$.

$$D'_{\rm i\to k} = D_{\rm i}\exp\left(-\frac{(r_{\rm k} - r_{\rm i})^2}{\lambda_P^2}\right), \ (\rm i = 1, \ldots, n)$$

Results

We calculated the PNSC process up to 50 seconds based on 2 initial models.

1. M162: PNS mass = 1.62 $M_{\odot}$, post bounce 400 ms, provided by Mayle & Wilson
2. M147: PNS mass = 1.475 $M_{\odot}$, post bounce 300 ms, Woosley15

The simulations reveal the presence of a local maximum temperature (T-peak) near the surface of PNS. In the discussion of non-convection models, we only explain results using M162 here for simplicity. And convection is discussed by models using M147.

(1) non-convection models

At the late thermal evolution of the outer layers of PNS, the density of the PNS increases with evolution, while the temperature continues to decrease. The neutrino reactions exchanging the energy at a specific position induce entropy changes there. Figure 1 depict the radial distribution of physical quantities in the outer layers of the PNS, such as mass ratio and specific heat. The mass ratio and the temperature derivative of entropy (ds/dT) are determined based on the EOS, and the specific heat is derived from the following formula.

$$ds = \frac{dQ}{T}, \quad C = \frac{dQ}{dT}, \quad C = T\frac{ds}{dt}\frac{dt}{dT}$$

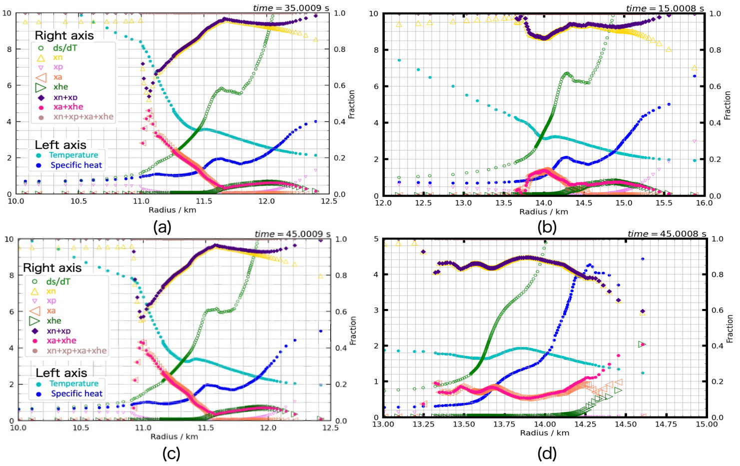
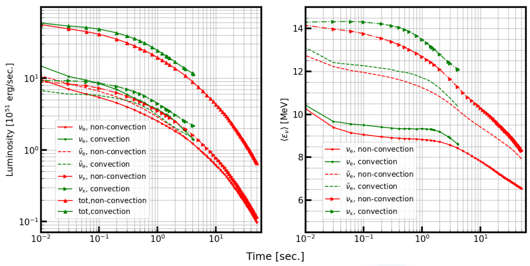


Figure 1. the radial distribution of mass ratios for neutrons(n), protons(p), heavy nuclei(a), and helium(he) per nucleon, as well as the temperature and specific heat of the PNS as 35 s and 45 s. (b), (d) illustrate results based on the Shen EOS calculation, the others use the Togashi EOS. The unit of Temperature is MeV.

Over time, the PNS emits neutrinos and cools down. To understand the formation of T-peak, we analyzed locations in the outer layers of the PNS that are less prone to cooling than inner side. From the distribution of $\dot{s}$, it was confirmed that the outer layers of the PNS release neutrinos and experience cooling. However, the $\dot{s}$ does not exhibit a distribution structure like the T-peak around the location where the T-peak exists. When examining the time evolution of temperature, it is observed that the temperature changes in the outer regions where heavy nuclei exist are smaller than in the inner regions. Furthermore, beyond the location of the T-peak(at 11.6 km and 11.5 km at 35 s and 45 s, respectively), there exists a local maximum value of specific heat. This location corresponds to the boundary region where heavy nuclei and helium are generated. Places with high specific heat capacity are less susceptible to temperature changes. Additionally, the results based on the Shen EOS calculation have a difference in T-peak formation time. This is attributed to the difference in the transition temperatures of nuclei in different EOS.

(2) convection model



- $\langle \varepsilon_{\nu_{\rm e,con.}} \rangle > \langle \varepsilon_{\nu_{\rm e,non-con.}} \rangle$
- $\langle \varepsilon_{\bar{\nu}_{\rm e,con.}} \rangle > \langle \varepsilon_{\bar{\nu}_{\rm e,non-con.}} \rangle$
- $\langle \varepsilon_{\nu_{\rm x,con.}} \rangle > \langle \varepsilon_{\nu_{\rm x,non-con.}} \rangle$

$L_{\rm tot,con.} > L_{\rm tot,non-con.}$

Figure 2. Mean energy of emitted neutrinos (right) and neutrino energy luminosity (left) as a function of time, for each neutrinos flavor, with and without convection.

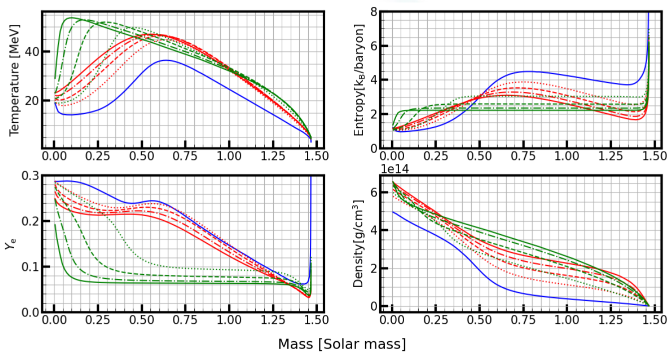


Figure 3. T, s, $Y_{\rm e}$, ρ on mass. Dotted, dashed, dash dot, solid line represent time = 1s, 2s, 3s, 4s in the simulation. Red, green, blue lines are non-convection model, convection model and initial distributions.

Figure 3 shows a comparation between convection model with non-convection model. A convection region exist at the early phase during PNS evolution. The range of the region changed in time, which make the entropy and $Y_{\rm e}$ profiles in the convective layer become flatter. These distributions show qualitative agreement with earlier work (Pascal 2022)

Conclusion

In the late thermal evolution of non-convection models, the appearance of T-peak at the outer layers of the PNS is attributed to the occurrence of heavy nuclei and helium. In those locations, it acts to impede cooling, making them less prone to cooling than the inner regions. Convective effects lead to an increase in the average neutrino energy and luminosity emitted from the PNS during the first 4 seconds of its early evolution, while the temperature inside the PNS rises. However, we mention that a diffusion with neutrino should be considered during the convective process. We intend to incorporate the aforementioned improvements and subsequently perform long-term numerical simulations with a range of progenitor stars and EOS models, with the aim of analyzing supernova neutrino emission.