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Gravitational Waves of GUT Phase Transition during Inflation

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Based on **XHH** and Y.L. Zhou, 2501.01491

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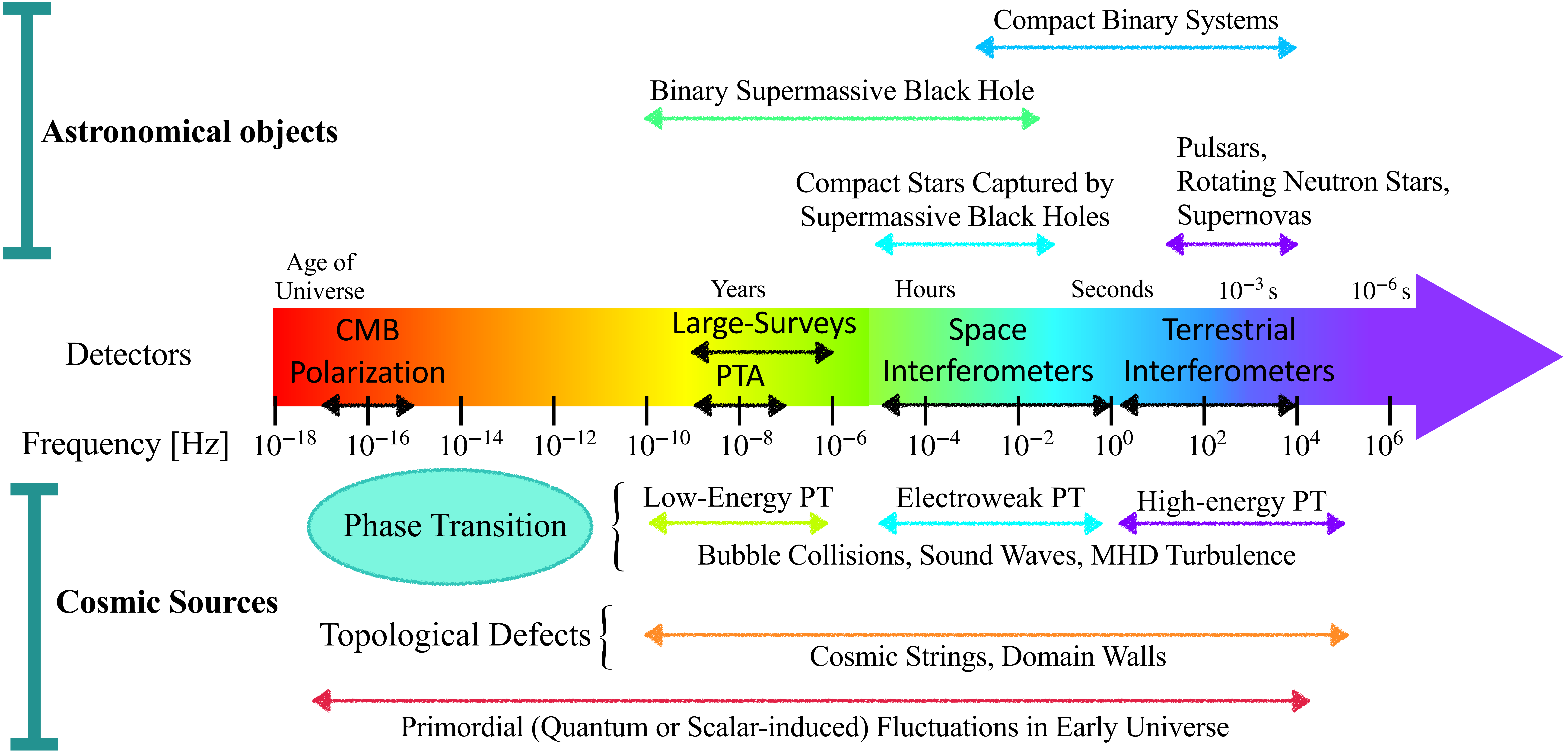
Motivation—Gravitational Waves



Motivation—Grand Unified Theory

Unification of symmetries $G_{\rm GUT} \xrightarrow{\text{broken}} G_{\rm SM}$ → Cosmic Phase Transition → topological defects

Unification of gauge couplings → GUT Scale

Unification of matters → Proton Decay

Unwanted topological defects: monopoles and domain walls. The both defects must be diluted by inflation or other mechanisms.

Case I: GUT breaking before inflation

GUT

→ GUT breaking

Inflation

→ Diluting monopoles and domain walls

$G_1 \subset G_{\rm SM} \times U(1)$

→
- Seesaw Mechanism
- Matter-antimatter asymmetry

→ Cosmic strings from $U(1)$ breaking

GWs from cosmic string:

- Test Seesaw Mechanism, 1908.03227
- Test GUT, 1912.03695, 2005.13549

Standard Model

Case II: GUT breaking during inflation

GUT

Inflation

→ GUT Phase Transition

GWs from GUT PT

→ Super-high scale PT

GWs from SHSPT

$G_1 \subset G_{\rm SM} \times U(1)$

Standard Model

Are GWs from GUT PT during inflation observable today and used to test GUT?

Motivation—Phase Transition during Inflation

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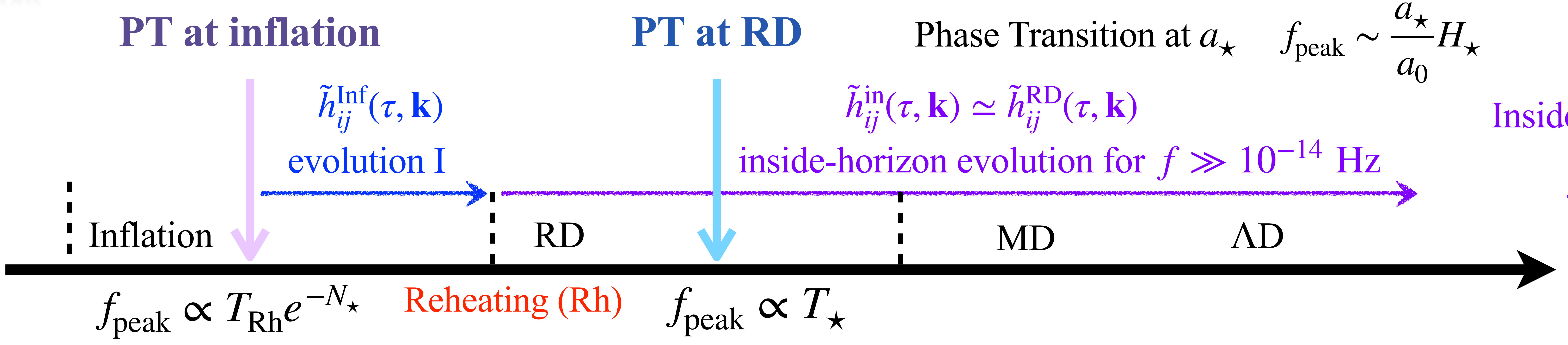

Inside Horizon: $\lambda \ll 1/\mathscr{H}$

$$\frac{d\rho_{\rm GW}^{\rm in}}{d\log k} \propto \frac{1}{a^4(\tau)}$$

A unique gravitational wave signal from phase transition during inflation

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We study the properties of the gravitational wave (GW) signals produced by first order phase transitions during the inflation era. We show that the power spectrum of the GW oscillates with its wave number. This signal can be observed directly by future terrestrial and spatial gravitational wave detectors and through the B-mode spectrum in CMB. This oscillatory feature of GW is generic for any approximately instantaneous sources occurring during inflation, and is distinct from the GW from phase transitions after the inflation. The details of the GW spectrum contain information about the scale of the phase transition and the later evolution of the universe.

Haipeng An, et al., 2009.12381, 2201.05171

Phase transition during inflation: $\Lambda_{\rm PT} \gtrsim \Lambda_{\rm Inf}$

Match at reheating: $\tilde{h}_{ij}^{\rm Inf}(\tau, \mathbf{k})\big|_{\rm Rh} = \tilde{h}_{ij}^{\rm RD}(\tau, \mathbf{k})\big|_{\rm Rh}$

- Frequency is redshifted by inflation
- GWs aren't extremely depressed by inflation
- GWs spectrum has special features

Super-High Scale Phase Transition (SHSPT), $\Lambda > 10^9$ GeV

- SHSPT during RD: $f_{\rm peak} > 10^4$ Hz, Unobservable today
- SHSPT during inflation: GWs are possibly observable

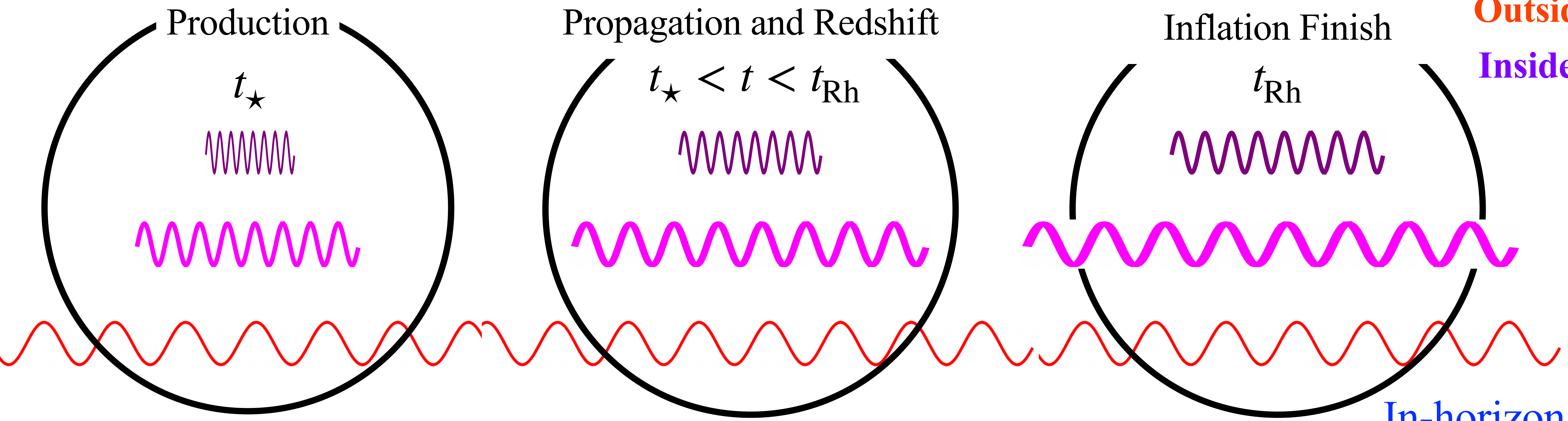
New Physics: GUT, Intermediate Symmetries of GUT, Seesaw Model, New Gauge Symmetries, Flavor Symmetries, Extra Dimensions, SUSY

Inflated GWs from instant sources

GWs' Evolution in inflation

$H_\star = H_{\rm Inf} \simeq \text{constant}$

$$\tilde{h}_{ij}^{\rm Inf}(\tau,\mathbf{k}) = 16\pi G_{\rm N}\tilde{\sigma}_{ij}(\mathbf{k}) \times \frac{a_\star}{k}\left\{\frac{a_\star H_\star}{k}\left(\frac{a_\star}{a(\tau)}-1\right)\cos[k(\tau-\tau_\star)] + \left(\frac{a_\star^2 H_\star^2}{k^2}+\frac{a_\star}{a(\tau)}\right)\sin[k(\tau-\tau_\star)]\right\}$$



Outside Horizon $a(t)\,\lambda \gg 1/H$

Inside Horizon $a(t)\,\lambda \ll 1/H$

$\dfrac{k}{a_{\rm Rh}H_\star} \gg 1$, Far UV

$\dfrac{k}{a_{\rm Rh}\,H_\star} \ll 1 \ll \dfrac{k}{a_\star\,H_\star}$, UV

$\dfrac{k}{a_\star\,H_\star} \ll 1$, IR

In-horizon evolution depresses GW, but out-horizon evolution doesn't.

GWs from instant sources during inflation evolving in RD, MD, ΛD

$$\tilde{h}_{ij}(\tau,\mathbf{k}) = 16\pi G_{\rm N}\tilde{\sigma}_{ij}(\mathbf{k}) \times \left[h_0(\tau,\mathbf{k}) + h_1(\tau,\mathbf{k})\right]$$

h_0 is Haipeng's result

$$h_0(\tau,\mathbf{k}) = \frac{-a_{\rm Rh}^2}{a(\tau)}\,\frac{1}{a_\star H_\star} \times \frac{\sin[k\tau - y\epsilon]}{y^3} \times \left\{\cos[y(1-\epsilon)] - \frac{\sin[y(1-\epsilon)]}{y}\right\}$$

$$h_1(\tau,\mathbf{k}) = \frac{-a_{\rm Rh}^2}{a(\tau)}\,\frac{y\epsilon}{a_\star H_\star} \times \left\{\frac{1-\epsilon}{y^3}\cos[k\tau + y(1-2\epsilon)] - \left(\frac{1+y\epsilon}{y^4}\right)\sin[k\tau + y(1-2\epsilon)]\right\}$$

h_0 dominates GWs in UV and IR, h_1 dominates GWs in far UV.

Matching Conditions

$$\tilde{h}_{ij}^{\rm Inf}(\tau,\mathbf{k})\Big|_{\rm Rh} = \tilde{h}_{ij}^{\rm RD}(\tau,\mathbf{k})\Big|_{\rm Rh}$$

$$\partial_t\tilde{h}_{ij}^{\rm Inf}(\tau,\mathbf{k})\Big|_{\rm Rh} = \partial_t\tilde{h}_{ij}^{\rm RD}(\tau,\mathbf{k})\Big|_{\rm Rh}$$

Conventions $y = \dfrac{k}{a_\star H_\star}$, $\epsilon = \dfrac{a_\star}{a_{\rm Rh}} = e^{-N_\star} \le 1$

Inflated GWs from instant sources

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Inflated GWs Spectrum — GWs produced during inflation

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log k} = \frac{\mathrm{d}\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\mathrm{d}\log k} \times \frac{a_{\mathrm{Rh}}^4}{a^4(t)} \times \mathcal{S}(t,k)$$

correlation

instant source

GWs in flat spacetime $\displaystyle \frac{\mathrm{d}\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\mathrm{d}\log k} = \frac{2G_{\mathrm{N}}}{\pi}\frac{k^3}{V}\left|\tilde{\sigma}_{ij}(\mathbf{k})\right|^2$

Uninflated GWs for $f \gg 10^{-14}$ Hz — in-horizon evolution

$$h^2\Omega_{\mathrm{GW}}(f) = h^2\widetilde{\Omega}_{\mathrm{GW}}(\tilde{f})\Big|_{\tilde{f}=f\frac{a_{\mathrm{Rh}}}{a_\star}} \times S(f)$$

redshift from inflation

deformation function

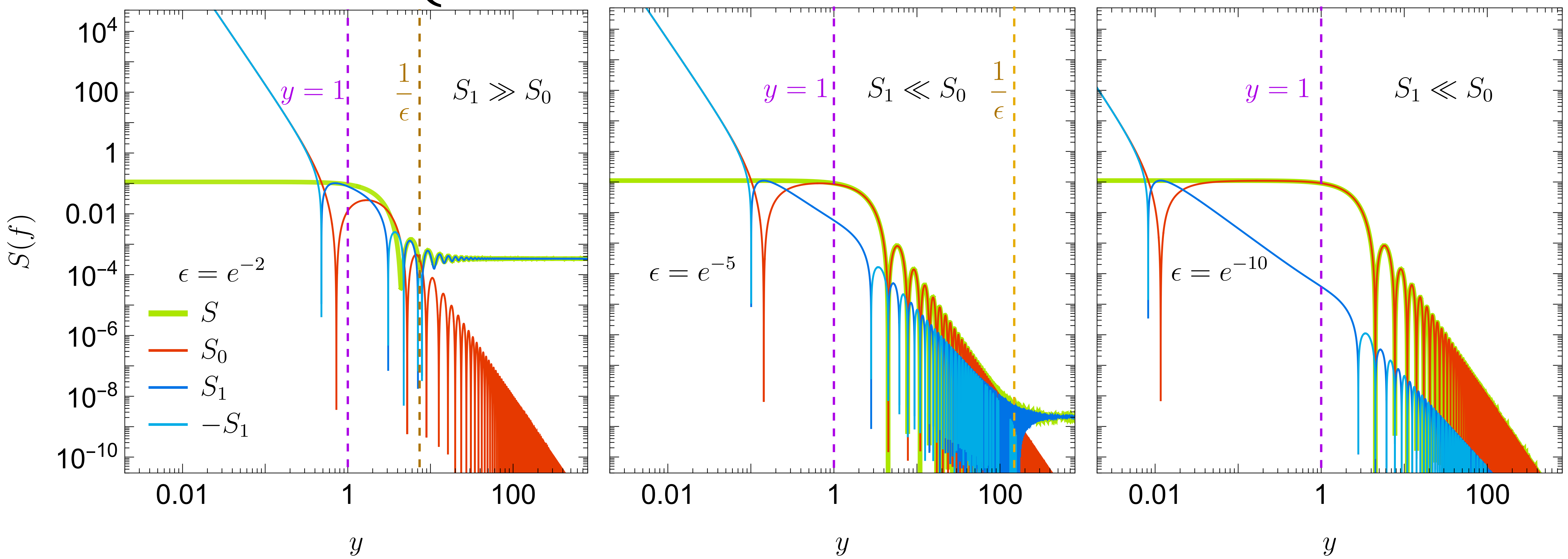
$$h^2\widetilde{\Omega}_{\mathrm{GW}}(\tilde{f}) = \frac{h^2}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\mathrm{d}\log k} \times \frac{a_{\mathrm{Rh}}^4}{a_0^4}$$ decreasing by a^4

$$S(f) = \mathcal{S}(t_0, 2\pi a_0 f) = S_0(f) + S_1(f) \qquad S_0(f) = \left\{\frac{\cos[y(1-\epsilon)]}{y^2} - \frac{\sin[y(1-\epsilon)]}{y^3}\right\}^2$$

S_0 from h_0 is Haipeng's result

$$S_1(f) = y\epsilon \times \left\{\left[\frac{1}{y^2} + 2\epsilon - 1\right]\frac{\sin[2y(1-\epsilon)]}{y^4} - \left[\frac{2-\epsilon}{y^2} + \epsilon\right]\frac{\cos[2y(1-\epsilon)]}{y^3} + \frac{\epsilon^3}{y}\left(\frac{1}{y^2}+1\right)\right\}$$

S_1 is correction in our work for adapting $\epsilon \gtrsim e^{-10}$



- Specific feature — $S(f)$ Oscillates!
- GWs aren't extremely depressed by inflation!
- $N_\star \gg 10$: $S_0(f)$ dominates GWs spectrum in IR ($y \ll 1$) and UV ($y \gg 1 \wedge y\epsilon \ll 1$).
- $N_\star \ll 10$: $S_1(f)$ is important.

Hence, Our result is applicable for any $N_\star$.

Inflated GWs from short-time sources

For more realistic, **Short-Time Sources** at $t_\star$

$$\bar{S}(f) = \frac{1}{\Delta_t}\int_{\bar{t}-\Delta_t/2}^{\bar{t}+\Delta_t/2} \mathrm{d}t_\star\, S(f) = \frac{1}{\Delta_y}\int_{\bar{y}-\Delta_y/2}^{\bar{y}+\Delta_y/2} \mathrm{d}y\, S(f)\Big|_{\bar{y}=\frac{2\pi a_0 f}{a_\star H_\star}}$$

$$h^2\Omega_{\rm GW}(f) = h^2\widetilde{\Omega}_{\rm GW}(\tilde{f}) \times \bar{S}(f)$$

$\bar{S}$ is complicated but analytic

Δ_τ conformal duration of sources $\Delta_y = \frac{a_\star \Delta_\tau}{1/H_\star}\bar{y}$

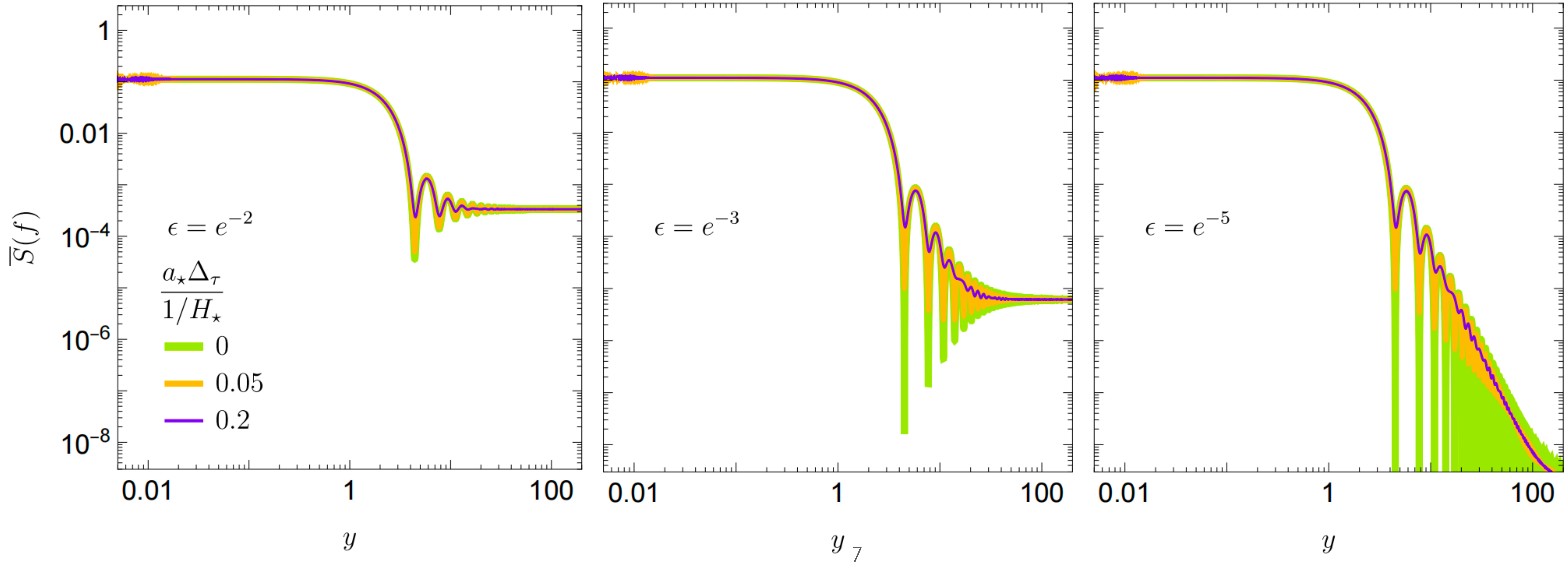
FUV: $y\epsilon \gg 1,\ S^{\rm FUV}(f) \simeq \epsilon^4 = a_\star^4/a_{\rm Rh}^4$

UV: $y\epsilon \ll 1 \ll y,\ S^{\rm UV}(f) \sim 1/\bar{y}^4$

$$S^{\rm UV}(f) \simeq \frac{(1-\epsilon) + \cos[2\bar{y}(1-\epsilon)]\sin[\Delta_y(1-\epsilon)]/\Delta_y}{2\bar{y}^4}$$

IR: $y \ll 1,\ S^{\rm IR}(f) \simeq 1/9$

The duration Δ_τ damps oscillation but don't affect envelope.



Uninflated GWs from Phase Transition

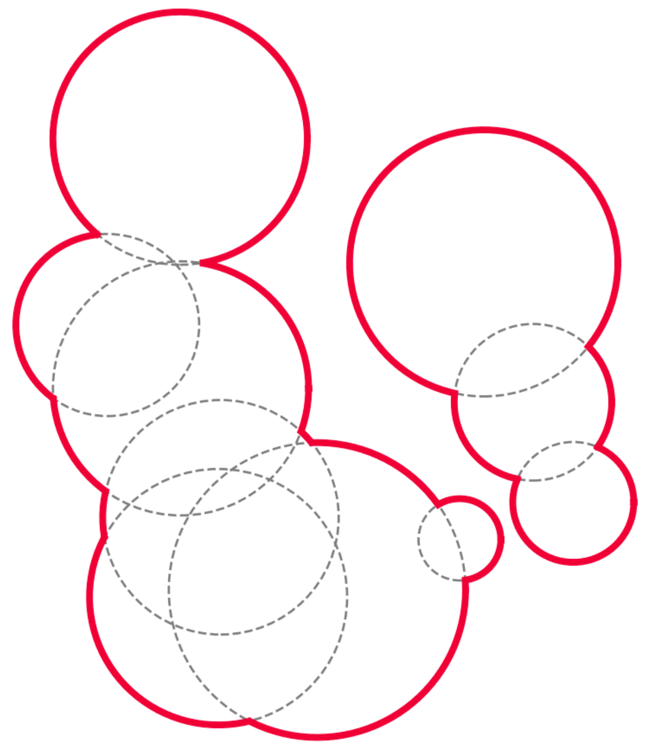
$h^2\Omega_{\rm GW}(f) = h^2\widetilde{\Omega}_{\rm GW}(\tilde{f}) \times \bar{S}(f)$ After having $\bar{S}(f)$, we will study the uninflated GWs spectrum.

~~plasma: sound waves, MHD turbulence~~
phase transition field: bubble collisions;

Plasma is unnecessary during inflation,
so sound waves and MHD turbulence are negligible,
and we only consider bubble collisions during inflation.

GWs from Phase Transition in flat spacetime

I. Envelope Approximation



The energy-momentum is mainly contained in envelope of bubbles and vanishes in intersection.

PRL 69 (1992) 2026-2029
PRD 47 (1993) 4372-4391
PRD 49 (1994) 2837-2851

II. Broken Power Law

$$\Omega_{\rm GW\star}(f_\star) = \Omega^{\rm peak}_{\rm GW\star}\frac{(a+b)(f_\star/f^{\rm peak}_\star)^a}{a(f_\star/f^{\rm peak}_\star)^{a+b}+b}$$

III. Simulation JCAP 09 (2008) 022

$a = 2.8, b = 1$

bubble wall velocity: $v_{\rm w}$
effective factor about fluid: κ

$$f^{\rm peak}_\star = \frac{0.62\,\beta}{1.8 - 0.1 v_{\rm w} + v_{\rm w}^2}$$

$\alpha = \rho_{\rm PT}/\rho_{\rm rad}$ $\quad \kappa(\alpha) = \dfrac{0.715\alpha + 0.181\sqrt{\alpha}}{1+0.715\alpha}$

$$\Omega^{\rm peak}_{\rm GW\star} = \frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^2}\times\kappa^2\left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2\times\left(\frac{H_\star}{\beta}\right)^2$$

$n_\beta = \dfrac{\beta}{H} \in (10, 10^4)$

IV. phase transition during inflation — few radiation

$v_w = 1, \alpha \to \infty, \kappa \to 1 \longrightarrow$ strong phase transition

phase transition parameters:
duration of phase transition: $1/\beta$
vacuum energy density: $\rho_{\rm PT} = \rho_{\rm F} - \rho_{\rm T}$

Uninfalted GWs Spectrum from Phase Transition

$$\tilde{f}^{\rm peak} = 37.8\ {\rm MHz}\left(\frac{\beta}{H_\star}\right)\left(\frac{T_\star}{10^{15}\ {\rm GeV}}\right)\left(\frac{g_\star}{100}\right)^{1/6}$$

$$h^2\widetilde{\Omega}_{\rm GW}(\tilde{f}) = 1.27\times10^{-6}\left(\frac{H_\star}{\beta}\right)^2\left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2\left(\frac{100}{g_\star}\right)^{1/3}\frac{(a+b)(\tilde{f}/\tilde{f}^{\rm peak})^a}{a(\tilde{f}/\tilde{f}^{\rm peak})^{a+b}+b}$$

Inflated GWs from GUT Phase Transition

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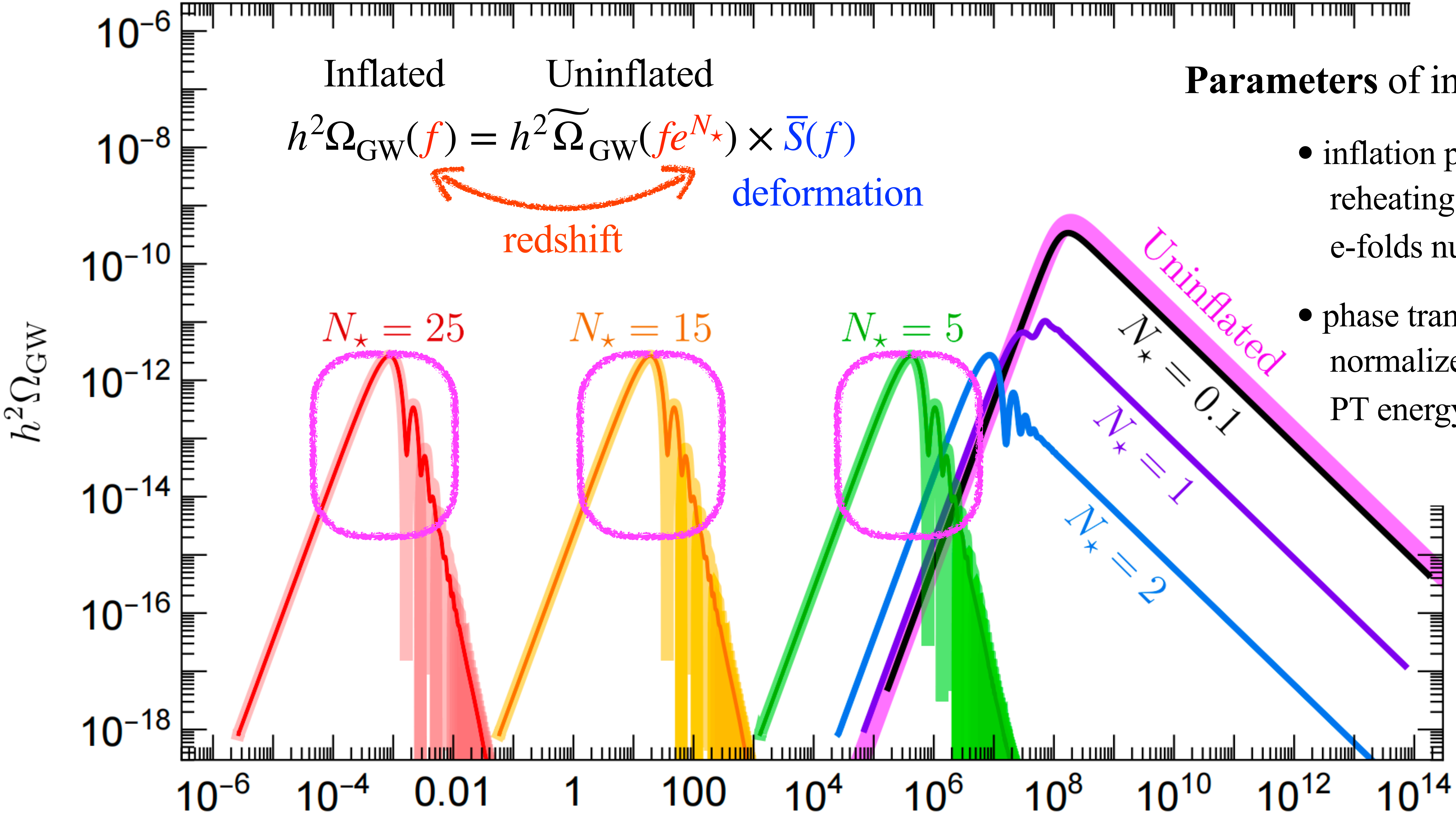


Parameters of inflated GWs from PT

- inflation parameters:
  reheating temperature $T_{\rm Rh}$,
  e-folds number of sources $N_\star$,
- phase transition parameters:
  normalized PT velocity $\beta/H_\star$,
  PT energy density ratio $\rho_{\rm PT}/\rho_{\rm tot}$

$T_{\rm Rh} = 10^{15}$ GeV

$\rho_{\rm PT}/\rho_{\rm tot} = 0.1$

$\beta/H_\star = 5$

- $N_\star \geq 5$: results are early obtained by Haipeng's group.
- $N_\star \leq 5$: our new results are shown and consistent with uninflated.
- Uninflated frequency from GUT PT is beyond observation.
- Inflated GWs might be observable and have oscillations.

Inflated GWs from GUT Phase Transition: $T_{\rm Rh} = 10^{15}$ GeV

Large $N_\star$ only affects frequency, $\rho_{\rm PT}/\rho_{\rm tot}$ only affects amplitude, and $\beta/H_\star$ affects shape and amplitude.

Inflated GWs from Phase Transition of New Physics below GUT Scale

Our method and results are applicable for these phase transitions.

$T_{\rm Rh}$ only affects GWs' frequency and doesn't change shape and amplitude of spectrum.

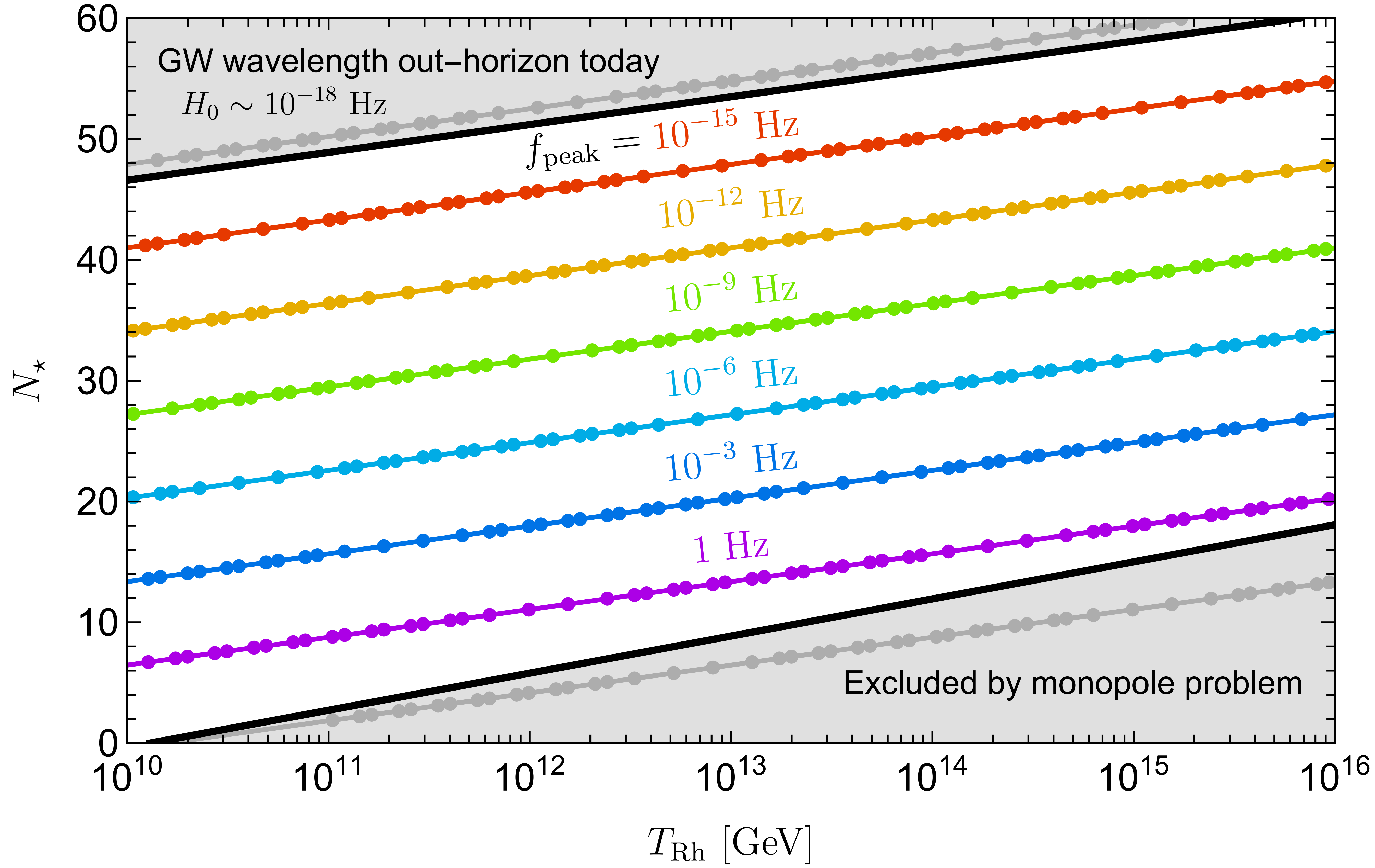
Super-High Scale Phase Transition, $\Lambda \sim T_{\rm Rh} > 10^9$ GeV

- Intermediate Symmetries of GUT
- Seesaw Model/$U(1)_{B-L}$
- New Gauge Symmetries
- Flavor Symmetries

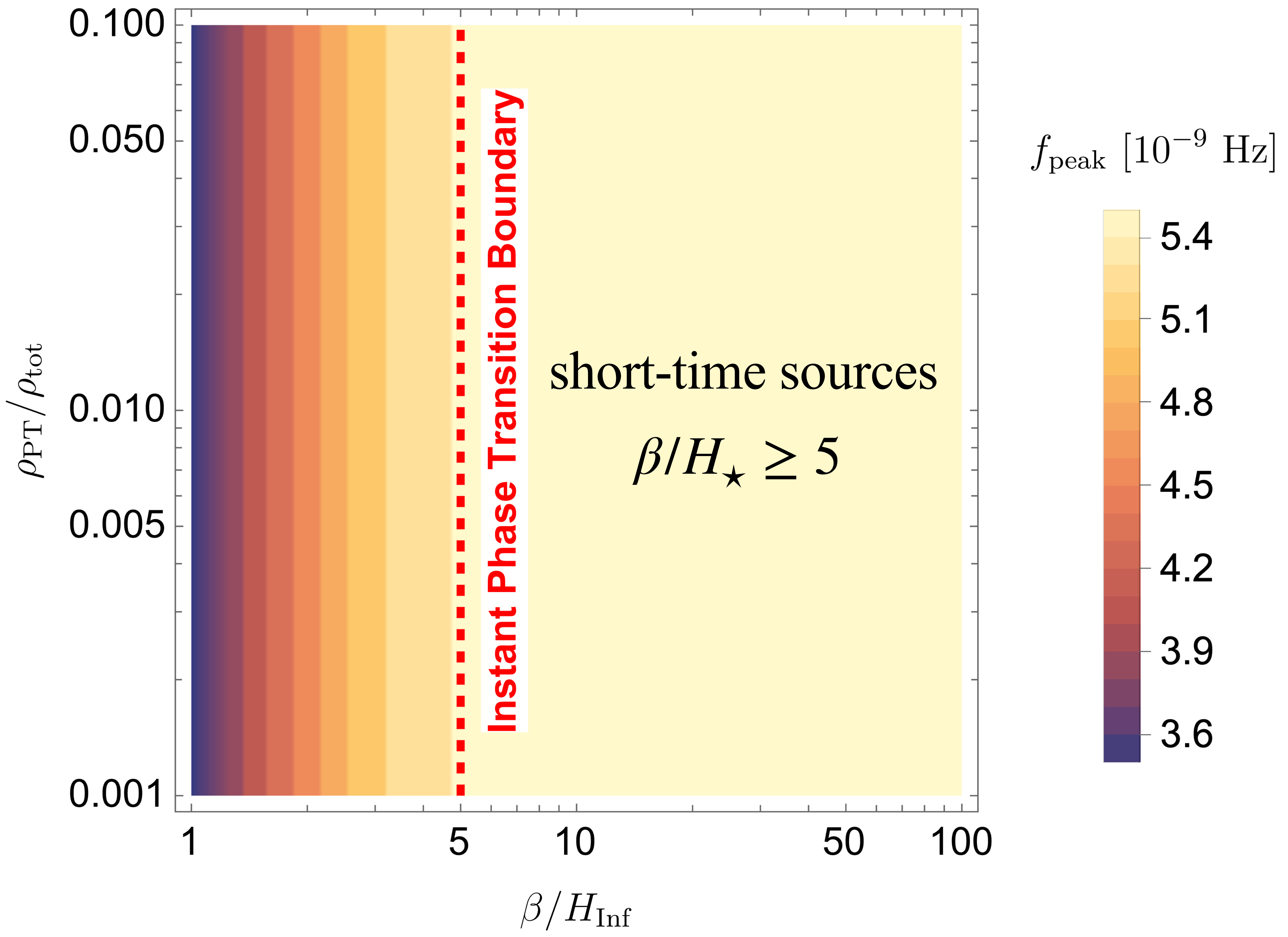
Inflated GWs from Phase Transition

Scanning peak frequency of inflated GW from PT in parameters space, T_{Rh} and $N_\star$, $\beta/H_\star$ and $\rho_{\mathrm{PT}}/\rho_{\mathrm{tot}}$

Inflation Parameters



Phase Transition Parameters



Monopoles Problem Limit $n_\star \sim L_{\mathrm{mono}}^{-3} \sim H_\star^{-3}, M_{\mathrm{mono}} \sim 10 T_{\mathrm{Rh}}$

short-time sources $\longrightarrow$ $f_{\mathrm{peak}} \sim H_\star a_\star / a_0$

$$\frac{\Omega_{\mathrm{mono}}}{10^{-5}} \sim \left(\frac{T_{\mathrm{Rh}}}{10^{15}\ \mathrm{GeV}}\right)^4 e^{-3(N_\star - 15)} \lesssim 1$$

- T_{Rh} and $N_\star$ decide the peak frequency, and $\beta/H_\star$ and $\rho_{\mathrm{PT}}/\rho_{\mathrm{tot}}$ don't affect.
- Monopoles problem just excludes unobservable high frequency region.

Inflated GWs from Phase Transition

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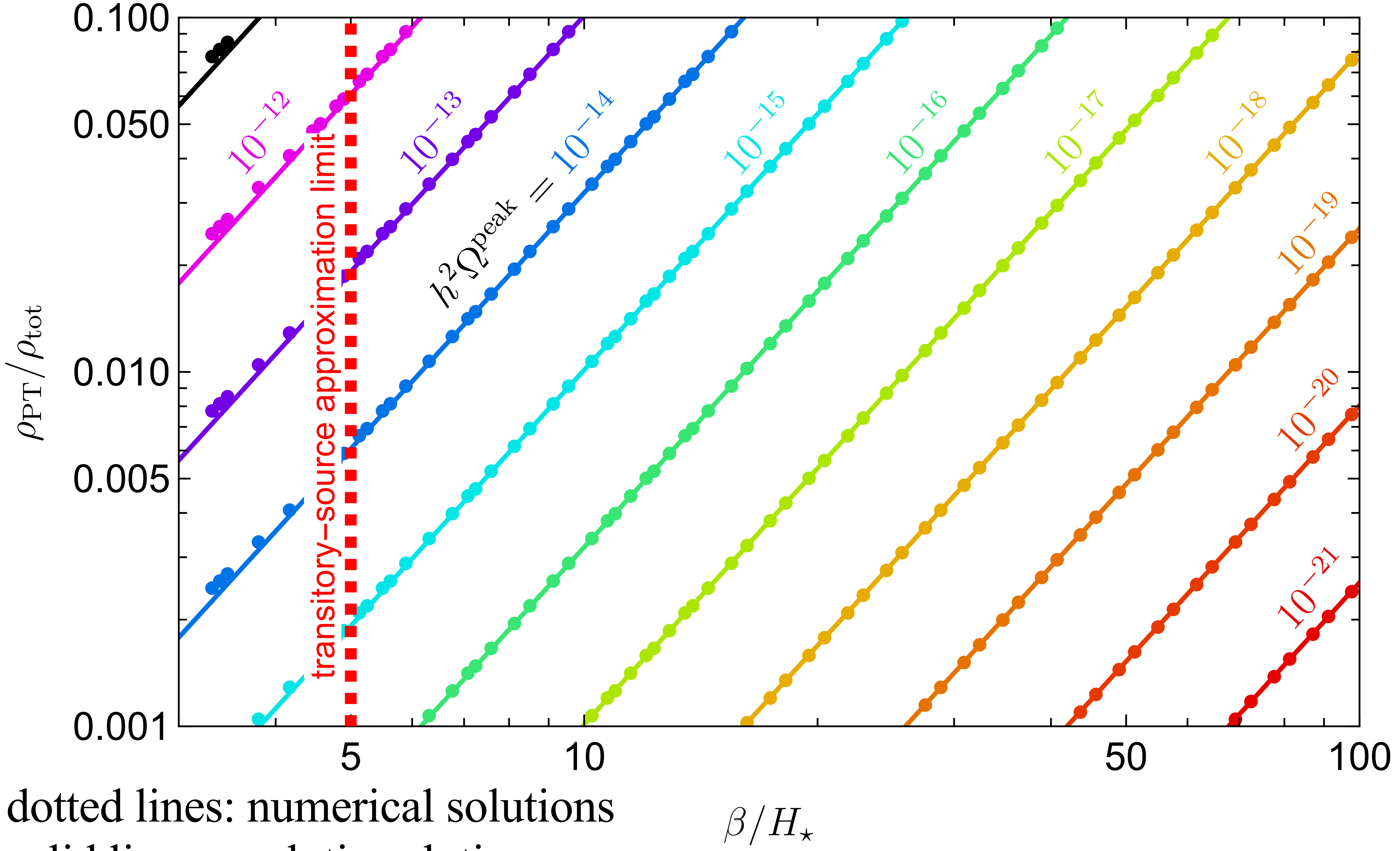
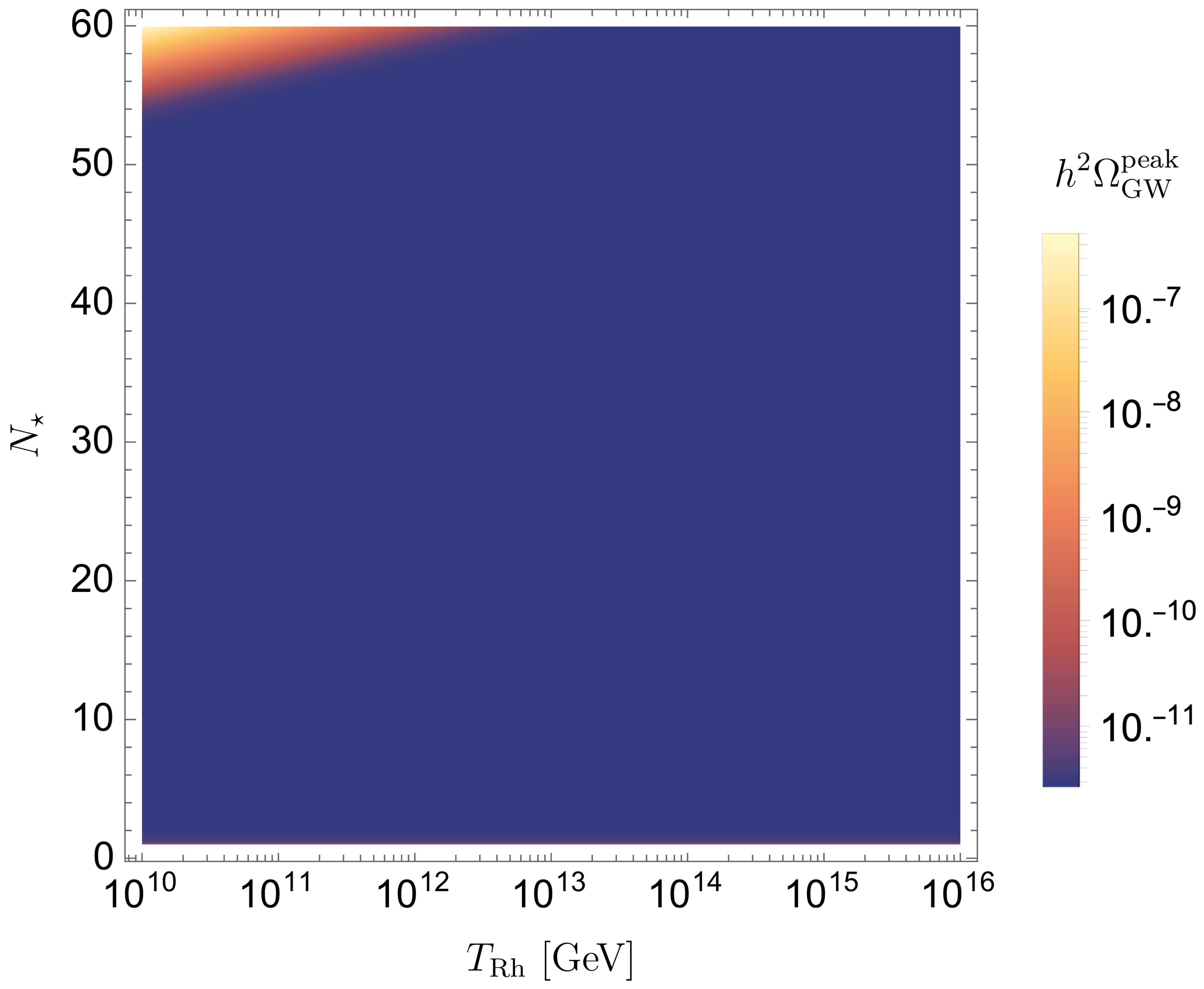
Scanning peak spectrum of inflated GWs from PT in parameters space, $T_{\rm Rh}$ and $N_\star$, $\beta/H_\star$ and $\rho_{\rm PT}/\rho_{\rm tot}$

Phase Transition Parameters



Inflation Parameters



dotted lines: numerical solutions
solid lines: analytic solutions

$$h^2\Omega_{\rm GW}(f) \propto f^a S_0(f) \Rightarrow f^{\rm peak} \simeq 62.7\ {\rm MHz} \times \left(\frac{g_\star}{100}\right)^{1/6}\left(\frac{T_\star}{10^{15}\ {\rm GeV}}\right) \times e^{-N_\star}$$

$$h^2\Omega_{\rm GW}^{\rm peak} \simeq 6.27 \times 10^{-7} \times \left(\frac{H_\star}{\beta}\right)^{2+a}\left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2\left(\frac{100}{g_\star}\right)^{1/3}$$

- Phase transition parameters decide peak spectrum.
- GWs spectrum in some parameters space are observable.

Results

- **After solving monopole problem, GWs from GUT phase transition during inflation**, if it is first-order, can be redshifted and deformed to oscillate, and thus **might be observable today and in foreseeable future.**

- **The general correlation between inflated GWs and uninflated GWs** is established **for short-time sources**, which is **applicable for any e-folds number of sources**.

- There are three region — IR, UV and far UV for inflated GWs, where IR and UV usually aren't extremely depressed, and FUV is depressed by $a_\star^4/a_{\rm Rh}^4$.

- The inflated GWs from phase transition have

$$f^{\rm peak} \simeq 62.7\ \mathrm{MHz} \times \left(\frac{g_\star}{100}\right)^{1/6} \left(\frac{T_\star}{10^{15}\ \mathrm{GeV}}\right) \times e^{-N_\star}, \quad h^2\Omega_{\rm GW}^{\rm peak} \simeq 6.27 \times 10^{-7} \times \left(\frac{H_\star}{\beta}\right)^{2+a} \left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2 \left(\frac{100}{g_\star}\right)^{1/3}$$

  where some parameters' regions can be tested today and in future as shown on the above.

- For inflated GWs from short phase transition, phase transition parameters, $\beta/H_\star$ and $\rho_{\rm PT}/\rho_{\rm tot}$, decide peak spectrum and inflation parameters, $T_{\rm Rh}$ and $N_\star$, decide peak frequency.

Thanks!

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Production and Propagation of GWs

Conformal FLRW Metric $ds^2 = a^2(\tau)\left[\mathrm{d}\tau^2 - (\delta_{ij} + h_{ij}(\tau, \mathbf{x}))\mathrm{d}x^i\mathrm{d}x^j\right]$ Traceless and transverse gauge

E.O.M of CGWs $\tilde{h}''_{ij}(\tau, \mathbf{k}) + 2\mathscr{H}\tilde{h}'_{ij}(\tau, \mathbf{k}) + k^2\tilde{h}_{ij}(\tau, \mathbf{k}) = 16\pi G_{\mathrm{N}} a^2(\tau)\tilde{\sigma}_{ij}(\tau, \mathbf{k})$ $\mathscr{H}$ is conformal Hubble factor

$$\tilde{h}_{ij}(\tau, \mathbf{k}) = C_{ij,1}\, h_1(\tau, \mathbf{k}) + C_{ij,2}\, h_2(\tau, \mathbf{k})$$

$h^{\mathrm{Inf}}(\tau, \mathbf{k}) = \cos k\tau + k\tau \sin k\tau\,,\ \ \sin k\tau - k\tau\cos k\tau$ Inflation, ΛD: $\mathscr{H} = -1/\tau$

$h^{\mathrm{RD}}(\tau, \mathbf{k}) = \dfrac{\cos k\tau}{k\tau}, \dfrac{\sin k\tau}{k\tau}$ RD: $\mathscr{H} = 1/\tau$

$h^{\mathrm{MD}}(\tau, \mathbf{k}) = \dfrac{\cos k\tau + k\tau\sin k\tau}{(k\tau)^3}, \dfrac{\sin k\tau - k\tau\cos k\tau}{(k\tau)^3}$ MD: $\mathscr{H} = 2/\tau$

Energy density of SGWB $\dfrac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log k} = \dfrac{1}{64\pi^3 G_{\mathrm{N}}}\dfrac{k^3}{V}\dfrac{1}{a^2(t)}\displaystyle\int_{T_\tau}\dfrac{\mathrm{d}\tau}{T_\tau}\left|\tilde{h}'_{ij}(\tau, \mathbf{k})\right|^2,\quad h^2\Omega_{\mathrm{GW}}(f) = \dfrac{h^2}{\rho_c}\dfrac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log k}\Bigg|_{k=2\pi a_0 f}^{t=t_0}$

Inside Horizon $k^2 \gg \left|\dfrac{a''}{a}\right|,\quad \lambda \ll \dfrac{1}{\mathscr{H}}$ $\longrightarrow$ $\tilde{h}^{\mathrm{in}}_{ij}(\tau, \mathbf{k}) \simeq \tilde{h}^{\mathbf{k}}_{ij}\dfrac{\sin(k\tau+\phi)}{a(\tau)},\quad \dfrac{\mathrm{d}\rho^{\mathrm{in}}_{\mathrm{GW}}}{\mathrm{d}\log k} \propto \dfrac{1}{a^4(\tau)}$ Waves!

depressed by a

Outside Horizon $k^2 \ll \left|\dfrac{a''}{a}\right|,\quad \lambda \gg \dfrac{1}{\mathscr{H}}$ $\longrightarrow$ $\tilde{h}^{\mathrm{out}}_{ij}(\tau, \mathbf{k}) \simeq \tilde{h}_{ij}(\tau_0, \mathbf{k}) + \tilde{h}'_{ij}(\tau_0, \mathbf{k})\displaystyle\int_{\tau_0}^{\tau}\dfrac{\mathrm{d}\tau'}{a^2(\tau')}$

Constant amplitude

Production and Propagation of GWs

$$k^2 \gg \left|\frac{a''}{a}\right|, \quad \lambda \ll \frac{1}{\mathscr{H}}$$

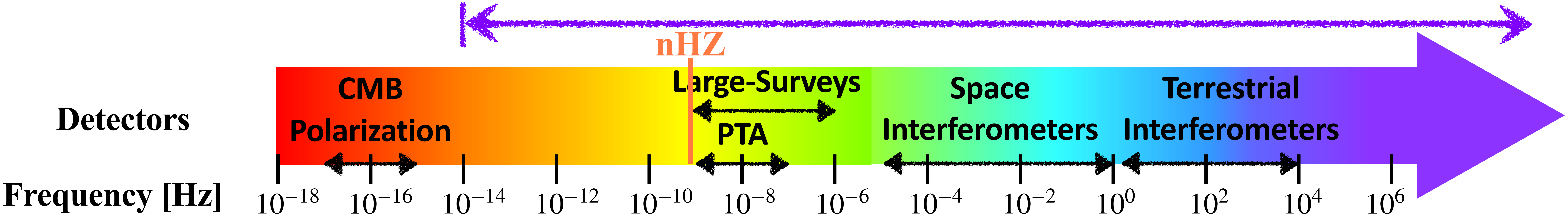
$$k^2 \ll \left|\frac{a''}{a}\right|, \quad \lambda \gg \frac{1}{\mathscr{H}}$$

$$\sqrt{\frac{a''}{a}} \sim \begin{cases} 0\,(\mathrm{HZ}) & \mathrm{RD} \\ 10^{-16} \sim 10^{-18}\,(\mathrm{HZ}) & \mathrm{MD} \\ 10^{-18}\,(\mathrm{HZ}) & \Lambda\mathrm{D} \end{cases}$$

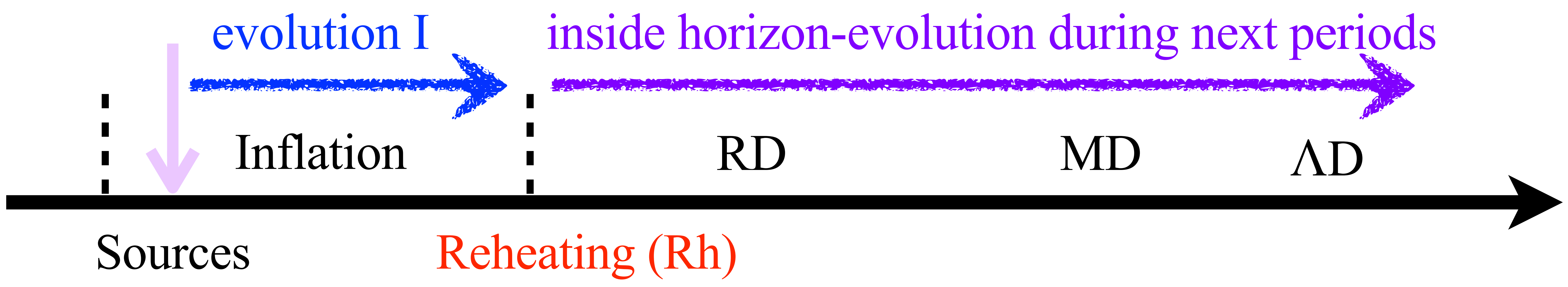
$$\mathscr{H} \sim \begin{cases} 10^{8} \sim 10^{-16}\,(\mathrm{HZ}) & \mathrm{RD} \\ 10^{-16} \sim 10^{-18}\,(\mathrm{HZ}) & \mathrm{MD} \\ 10^{-18}\,(\mathrm{HZ}) & \Lambda\mathrm{D} \end{cases}$$

The GWs of all frequency is inside-horizon evolution during RD

GWs in this frequency range are inside-horizon evolution during RD, MD and ΛD!



$\tilde{h}_{ij}^{\mathrm{Inf}}(\tau, \mathbf{k})$ $\quad$ $\tilde{h}_{ij}^{\mathrm{in}}(\tau, \mathbf{k}) \simeq \tilde{h}_{ij}^{\mathrm{RD}}(\tau, \mathbf{k})$



Matching Conditions of Waves

$$\tilde{h}_{ij}^{\mathrm{Inf}}(\tau, \mathbf{k})\Big|_{\mathrm{Rh}} = \tilde{h}_{ij}^{\mathrm{RD}}(\tau, \mathbf{k})\Big|_{\mathrm{Rh}}$$

$$\partial_t \tilde{h}_{ij}^{\mathrm{Inf}}(\tau, \mathbf{k})\Big|_{\mathrm{Rh}} = \partial_t \tilde{h}_{ij}^{\mathrm{RD}}(\tau, \mathbf{k})\Big|_{\mathrm{Rh}}$$

Phase Transition Potential and Monopoles Problem

Phase transition potential

σ — field of PT, ϕ — inflaton

I. Cubic coupling potential

$$V_{\rm PT}(\phi,\sigma) = D(\phi^2-\phi_0^2)\,\sigma^2 - \mu\,\sigma^3 + \frac{\lambda}{4}\sigma^4$$

possible physical origins

$$SO(10):\ 45\times 45\times 45 = 1 + 45 + \ldots$$

$$SU(5):\ 24\times 24\times 24 = 1 + 24 + \ldots$$

The cubic term must be gauge singlet possibly from adjoint representation of GUT groups.

II. Coleman-Weinberg potential PRD 7 (1973) 1888-1910

$$V_{\rm PT}(\phi,\sigma) = D(\phi^2-\phi_0^2)\,\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{4}\sigma^4\log\frac{\sigma^2}{\Lambda^2}$$

III. High-dimension operator potential

$$V_{\rm PT}(\phi,\sigma) = D(\phi^2-\phi_0^2)\,\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{\Lambda^2}\sigma^6 + \frac{\xi}{\Lambda^4}\sigma^8$$

PRD 71 (2005) 036001, JHEP 02 (2005) 026, JHEP 04 (2008) 029, JHEP 07 (2018) 062

Monopoles Problem

monopoles density $\quad n_\star \simeq L_{\rm mono}^{-3} \sim H_\star^{-3}$

monopoles mass $\quad M_{\rm mono} \simeq \dfrac{\Lambda_{\rm GUT}}{\alpha_{\rm GUT}} \sim 10T_{\rm Rh}$

$$\Lambda_{\rm GUT}\sim T_{\rm Rh},\quad \alpha_{\rm GUT}\sim g^2/(4\pi)\sim 0.1$$

energy density proportion of monopoles today

$$\Omega_{\rm mono} = \frac{8\pi G_{\rm N}}{3H_0^3}M_{\rm mono}\left(\frac{a_{\rm Rh}e^{-N_\star}}{a_0}\right)^3 n_\star$$

CMB observation $\quad \Omega_{\rm mono} \lesssim 10^{-5}$

Monopoles problem limitation

$$\frac{\Omega_{\rm mono}}{10^{-5}} \sim \left(\frac{T_{\rm Rh}}{10^{15}\ {\rm GeV}}\right)^4 e^{-3(N_\star-15)} \lesssim 1$$