

中山大學天琴中心

TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU



New dark matter production mechanism and the gravitational wave signals

Fa Peng Huang (黄发朋)

Sun Yat-sen University, TianQin center

based on our recent work

Dayun Qiu, Siyu Jiang, **FPH**, arXiv: [2508.04314](#)

Siyu Jiang, **FPH**, JCAP06 (2025) 023

Siyu Jiang, **FPH**, Pyungwon Ko, JHEP 07 (2024) 053

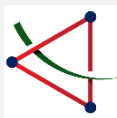
Siyu Jiang, Aidi Yang, Jiucheng Ma, **FPH**

Class.Quant.Grav. 41 (2024) 6, 065009

Siyu Jiang, **FPH**, Chong Sheng Li, Phys.Rev.D 108(2023)6, 063508

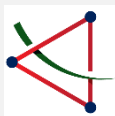
The XIX International Conference on Topics in Astroparticle and Underground Physics
(TAUP2025)@Xichang, 2025.08.28





Outline

- 1. Motivation for new dark matter (DM) mechanism**
- 2. pNGB DM from Primordial black hole (PBH) radiation and superradiance with its gravitational wave (GW) signals**
- 3. DM from first-order phase transition (FOPT) and GW**
 - Case I: Q-ball and gauged Q-ball DM**
 - Case II: filtered DM**
- 4. Summary and outlook**

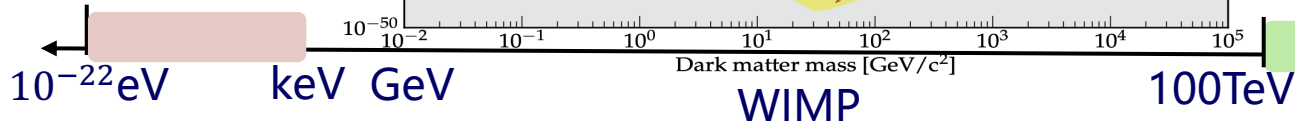


Motivation DM theory and experiments status

What is the microscopic nature of DM?

How DM relic density is produced?

Ultralight DM:
Axion/ALP

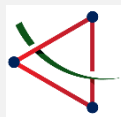


arXiv: 1904.07915
Snowmass 2021,
arXiv: 2209.07426

Heavy DM: Q-
ball/filtered/
PBH (radiation,
superradiance)

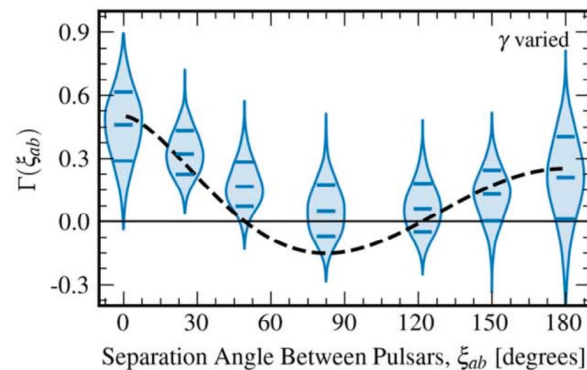
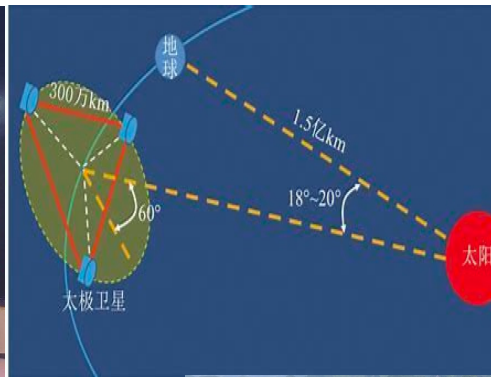
- new DM mechanism beyond thermal freeze out: **cosmic phase transition, PBH Hawking radiation, superradiance...**
- new detection method: **various GW detector** (LISA, TianQin, Taiji, aLIGO, FAST, SKA, NanoGrav, Cosmic Explorer...)





GW experiments

LISA/TianQin/Taiji ~2034



**“TianQin”
“Harpe in space”**

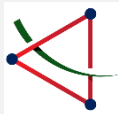
2023 June 29th: NANOGrav,
EPTA, InPTA, Parkes PTA, CPTA



FAST



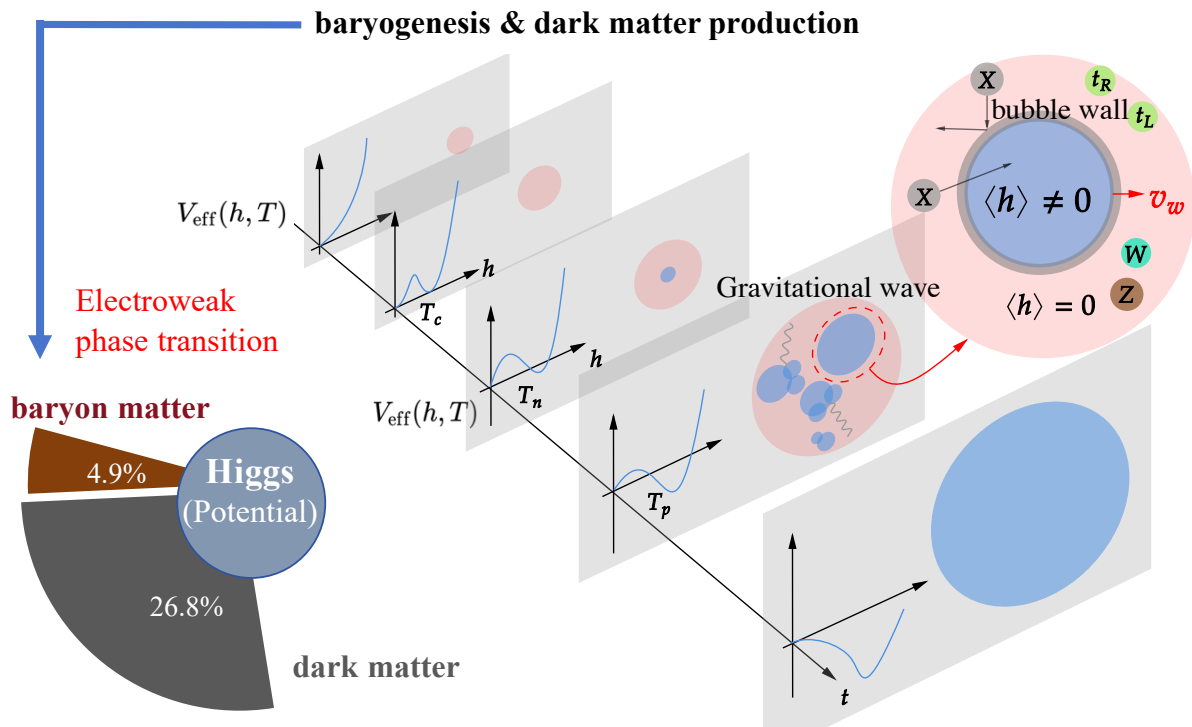
SKA



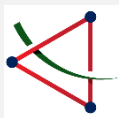
Motivation DM in post-Higgs and GW Era

The observation of **Higgs@LHC** and **GW@LIGO** initiates new era of exploring DM by GW.

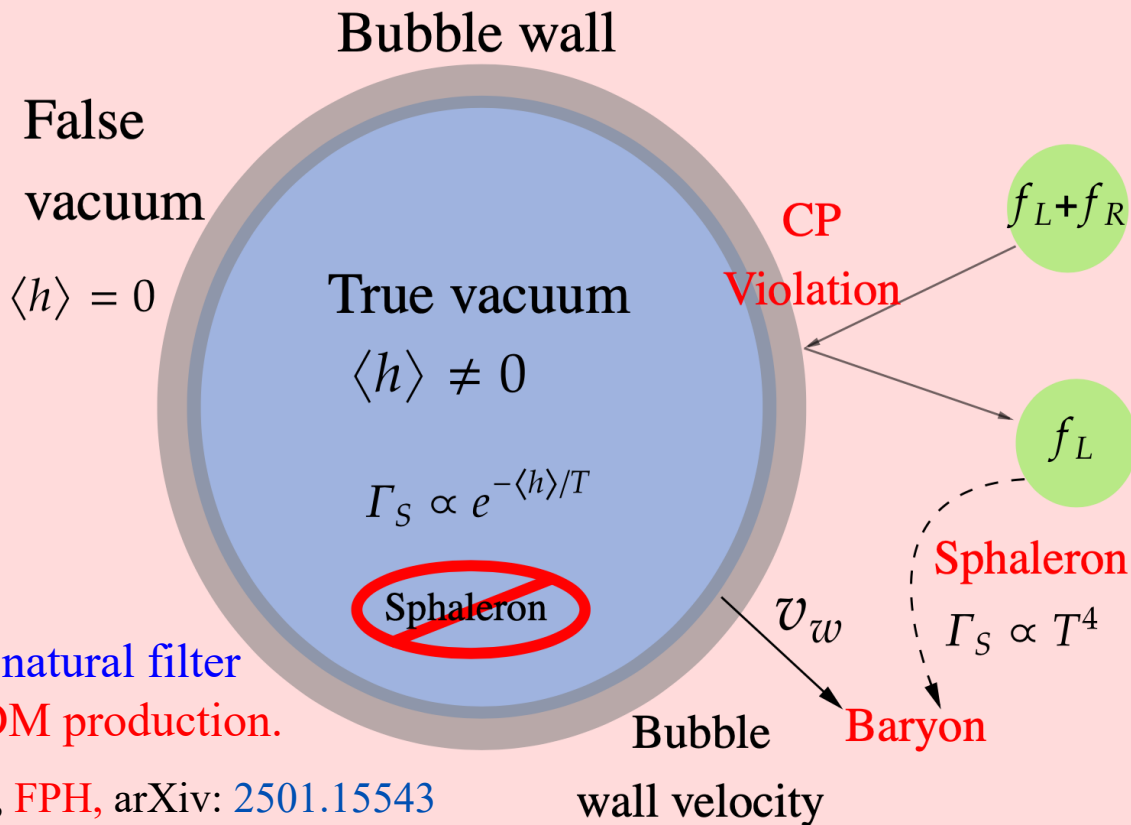
FOPT by Higgs could provide a new approach for DM production.



The First Particles, **FPH**, arXiv: [2501.15543](https://arxiv.org/abs/2501.15543)

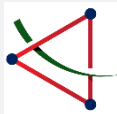


DM from cosmic phase transition

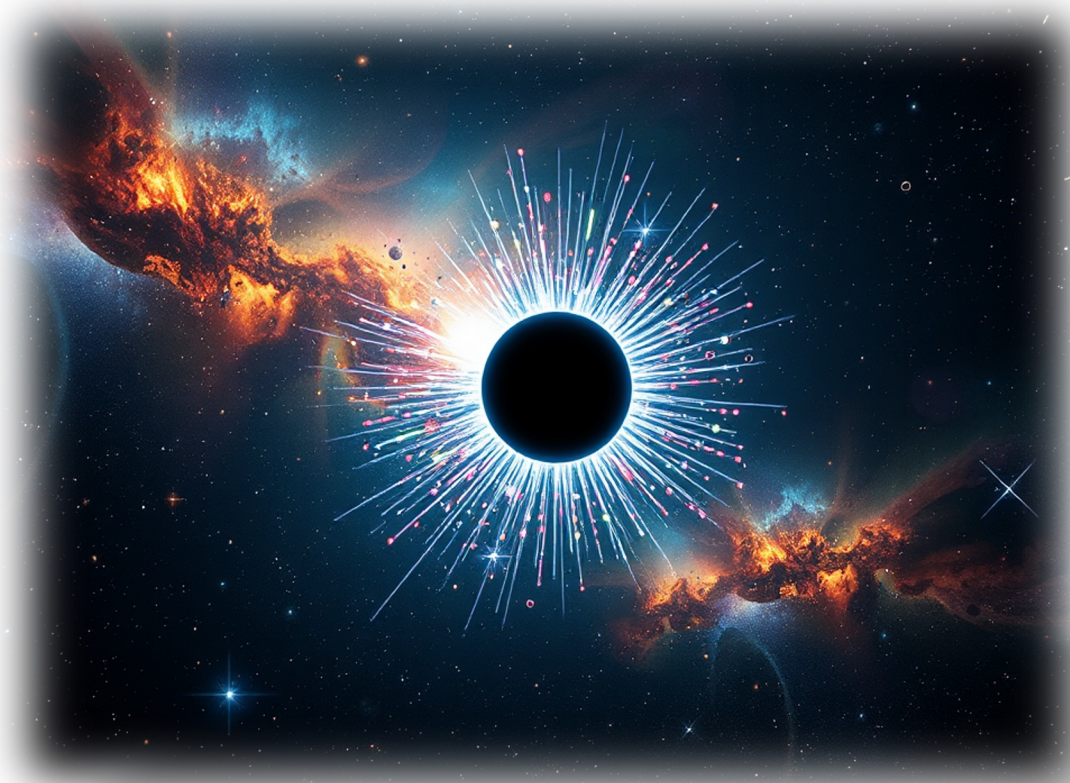


Bubble wall is a natural filter
for baryon and DM production.

The First Particles, FPH, arXiv: 2501.15543



DM from PBH radiation/superradiance



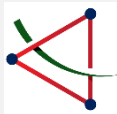
PBH can be the particle factory
in the early universe.

(Kerr) PHB can produce SM
particles and DM through
(superradiance) Hawking radiation.

Gravitational interaction is universal.

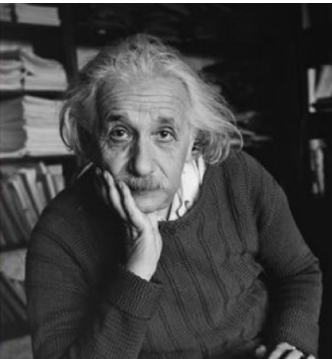
S.W. Hawking, Black hole explosions, Nature 248
(1974) 30

S.W. Hawking, Particle Creation by Black Holes,
Commun. Math. Phys. 43 (1975) 199



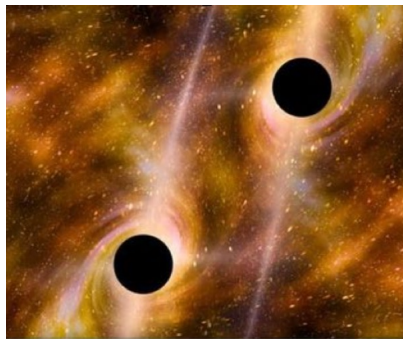
What is GW ?

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$



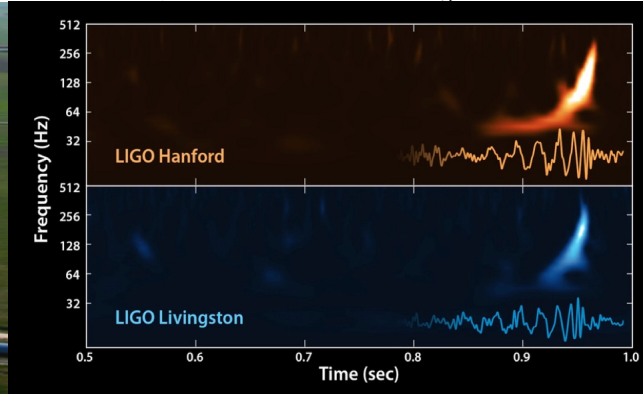
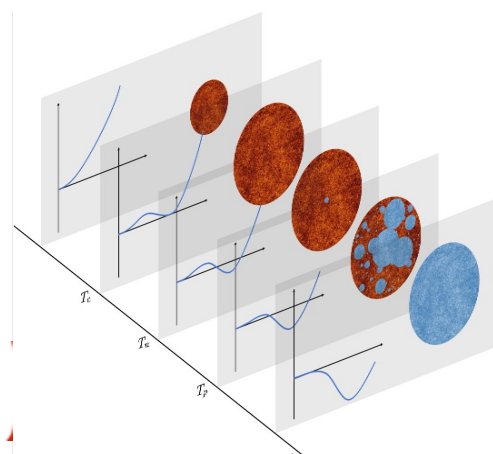
Isolated
sources:
quadrupole
radiation

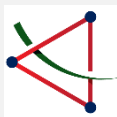
$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



Stochastic
sources:
anisotropic
stress
tensor

$$\Pi_{ij}(\mathbf{x}, t)$$





General GW in the early universe

$$\ddot{h}_{ij}(\mathbf{x}, t) + 3H \dot{h}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x}, t) = 16\pi G \Pi_{ij}(\mathbf{x}, t)$$

- ✓ phase transition: TeV physics (focus)
- ✓ cosmic defects: cosmic string, domain wall...

Possible sources of **tensor anisotropic stress** in the early universe

- Scalar field gradients $\Pi_{ij} \sim [\partial_i \phi \partial_j \phi]^{TT}$ Collisions of bubble walls, cosmic string
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2 (\rho + p) v_i v_j]^{TT}$ Sound waves and turbulence
- Gauge fields $\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$ Primordial magnetic fields (MHD turbulence)
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i \Psi, \partial_i \Phi$ induced GW in PBH models
- ... [arXiv:1801.04268](https://arxiv.org/abs/1801.04268)



Phase transition in a nutshell



calculate the finite-temperature effective potential using the thermal field theory:

这世上的热闹，源自隧穿

$$\Gamma = \Gamma_0 e^{-S(T)}$$

$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}}(\phi, T) \right]$$

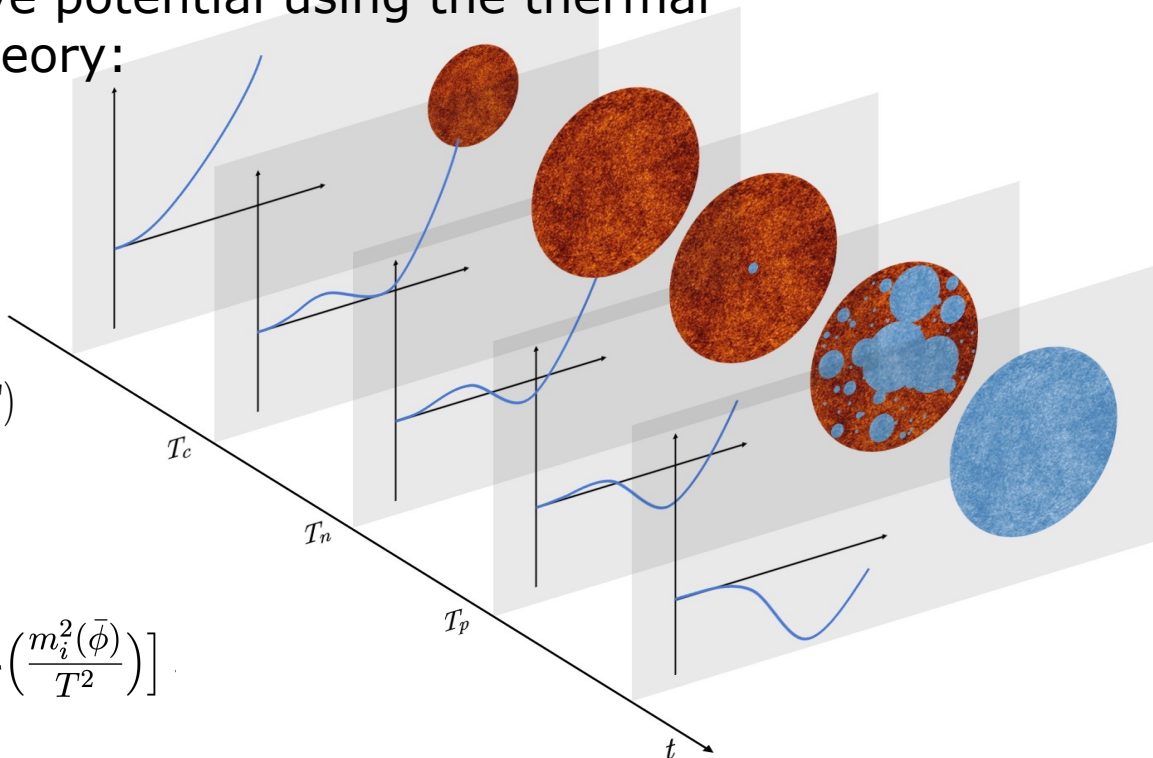
$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \left[\int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m_i^2(\bar{\phi})) + J_{\text{B,F}} \left(\frac{m_i^2(\bar{\phi})}{T^2} \right) \right]$$

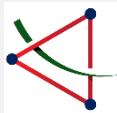
Xiao Wang, **FPH**, Xinmin Zhang, JCAP05(2020)045

2025/08/28

Fa Peng Huang, New dark matter production mechanism and the gravitational wave signals

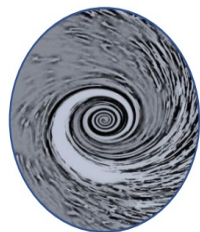
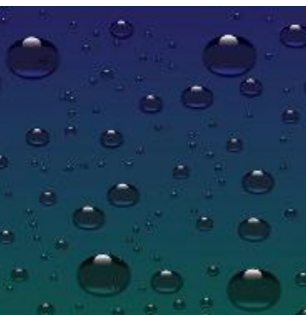
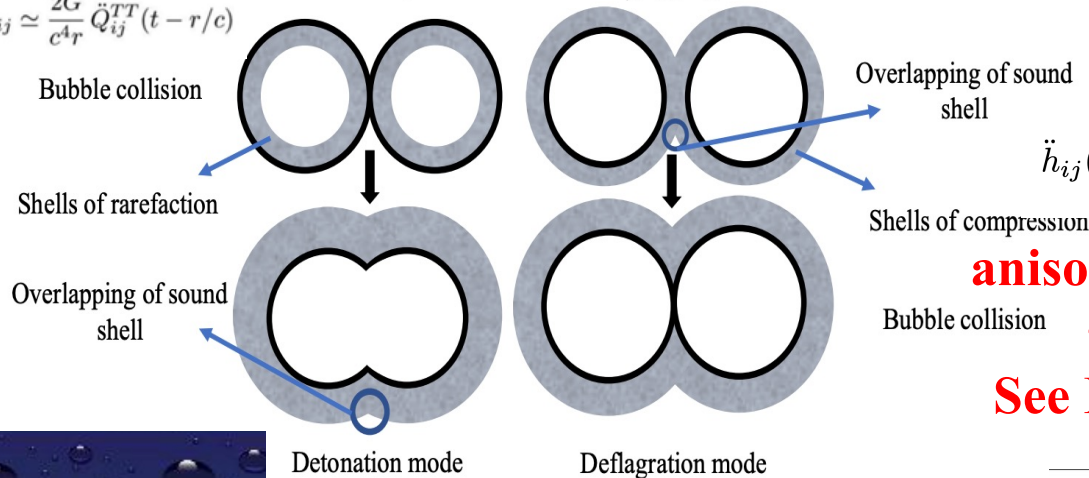
10





Phase transition GW in a nutshell

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



Turbulence

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{x}, t) + 3H \dot{h}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x}, t) = 16\pi G \Pi_{ij}(\mathbf{x}, t)$$

anisotropic stress tensor:

source of GW

See Haipeng An's talk

General form Π_{ij}

$$[\partial_i \phi \partial_j \phi]^{TT}$$

$$[\gamma^2 (\rho + p) v_i v_j]^{TT}$$

$$[-E_i E_j - B_i B_j]^{TT}$$

$$\partial_i \Psi, \partial_i \Phi$$

E. Witten, Phys. Rev. D 30, 272 (1984)

C. J. Hogan, Phys. Lett. B 133, 172 (1983);

M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

**EW phase transition
GW becomes more
interesting and
realistic after the
discovery of
Higgs by LHC and
GW by LIGO.**

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045



Phase transition dynamics

Theory: The most important and difficult phase transition parameter for GW, dynamical DM, baryogenesis is bubble wall velocity v_w

Experiment: GW experiment is most sensitive to bubble wall velocity v_w

arXiv: 2404.18703

Aidi Yang, **FPH**, JCAP 2025

Finite-temperature effective potential

$$V_{eff}(\phi, T)$$

α

T_p

$R_* H_*$

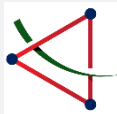
- (1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
- (2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
- (3). No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

*Bubble wall velocity
this talk* v_w

Energy budget κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
Xiao Wang, **FPH**, Xinmin Zhang, arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005...

F. Giese, T. Konstandin, K. Schmitz and J. van de Vis, arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006



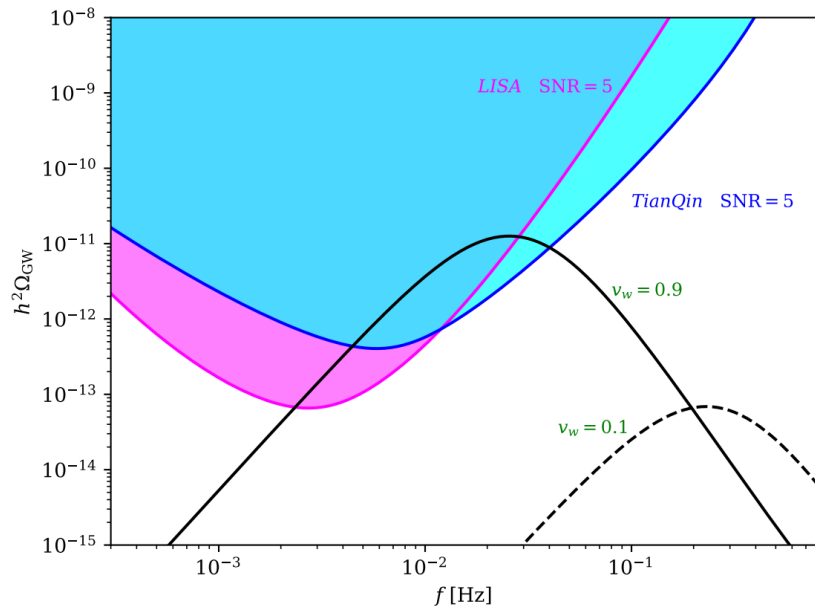
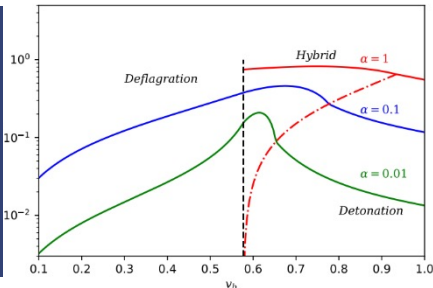
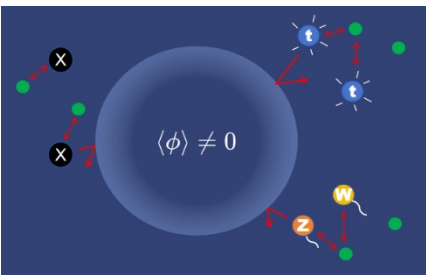
Bubble wall is essential (like a filter)

The most essential parameter for
phase transition GW, phase
transition DM, baryogenesis v_w

GW detection favor larger v_w
EW baryogenesis favor smaller v_w
Dynamical DM is sensitive to v_w

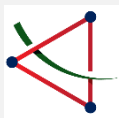
S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith,
arXiv:2009.14295v2

Xiao Wang, **FPH**, Xinmin Zhang, arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005



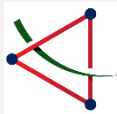
$$\rho_{DM}^4 v_w^{3/4} = 73.5 (2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;



Outline

1. Motivation for new dark matter (DM) mechanism
2. pNGB DM from Primordial black hole (PBH) radiation and superradiance with its gravitational wave (GW) signals
3. DM from first-order phase transition (FOPT) and GW
 - Case I: Q-ball and gauged Q-ball DM
 - Case II: filtered DM
4. Summary and outlook



Pseudo-Goldstone DM

In many well-motivated new physics models, the pseudo-Nambu-Goldstone boson (pNGB) from U(1) symmetry breaking emerges as a promising DM candidate.

Meanwhile, pNGB DM candidate naturally suppress the direct detection signals!

$$\mathcal{L} \supset \frac{v_s}{2} \chi^2 (\kappa_{\chi\chi h_1} h_1 + \kappa_{\chi\chi h_2} h_2)$$

$$\kappa_{\chi\chi h_1} = + m_{h_1}^2 / v_s^2 \sin \theta, \quad \kappa_{\chi\chi h_2} = - m_{h_2}^2 / v_s^2 \cos \theta,$$

$$\mathcal{L} \supset -(h_1 \cos \theta + h_2 \sin \theta) \sum_f \frac{m_f}{v} \bar{f} f.$$

$$\mathcal{A}_{dd}(t) \propto \sin \theta \cos \theta \left(\frac{m_{h_2}^2}{t - m_{h_2}^2} - \frac{m_{h_1}^2}{t - m_{h_1}^2} \right) \simeq \sin \theta \cos \theta \frac{t (m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2 m_{h_2}^2} \simeq 0$$

Phys.Rev.Letts.119.191801

$$t \simeq 0.$$

Minimal Pseudo-Goldstone DM model

$$V(H, S) = -\frac{\mu_H^2}{2}|H|^2 + \frac{\lambda_H}{2}|H|^4 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2 - \boxed{\frac{m^2}{4}(S^2 + S^{*2})}$$

Explicit
symmetry
breaking term
to give DM mass

CP symmetry $S \rightarrow S^*$.

$\downarrow$
 $\chi \rightarrow -\chi$
DM candidate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{v_s + s}{\sqrt{2}} e^{i\chi/v_s}$$

pNGB coupling is small due to the suppression of high symmetry-breaking scale, then how to produce enough DM relic density?

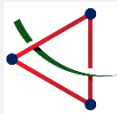
$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}.$$

$$m_{h_1}^2 = \frac{1}{2} \left[\lambda_H v^2 + \lambda_S v_s^2 - \sqrt{(\lambda_S v_s^2 - \lambda_H v^2)^2 + 4\lambda_{HS}^2 v^2 v_s^2} \right],$$

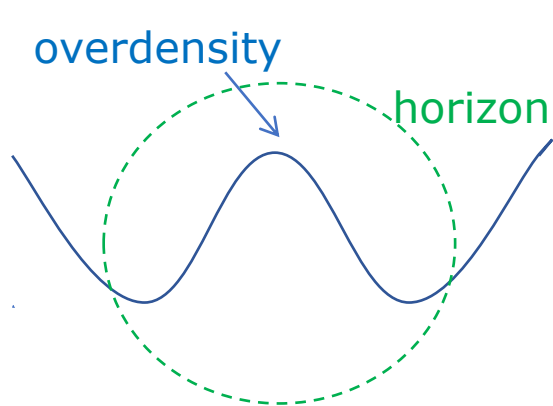
$$m_{h_2}^2 = \frac{1}{2} \left[\lambda_H v^2 + \lambda_S v_s^2 + \sqrt{(\lambda_S v_s^2 - \lambda_H v^2)^2 + 4\lambda_{HS}^2 v^2 v_s^2} \right],$$

$$\tan 2\alpha = \frac{2\lambda_{HS} v v_s}{\lambda_S v_s^2 - \lambda_H v^2}$$

$v \ll v_s$ mixing can be ignored



Basic properties of PBH



Initial PHB mass

$$M_{\text{in}} \equiv M_{\text{PBH}}(T_{\text{in}}) = \frac{4\pi}{3} \gamma \frac{\rho_R(T_{\text{in}})}{H^3(T_{\text{in}})}$$

Plank limit

$$H(T_{\text{in}}) \leq 5 \times 10^{13} \text{ GeV}$$

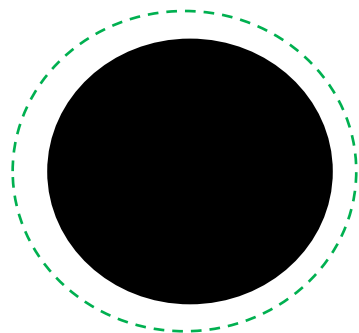
→ $M_{\text{in}} \geq 0.5 \text{ g.}$



collapse

Hubble rate

$$H = \sqrt{\frac{8\pi}{3} \frac{\rho_R}{M_{\text{Pl}}^2}} = \sqrt{\frac{4\pi^3 g_\star}{45}} \frac{T^2}{M_{\text{Pl}}}$$

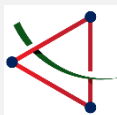


Formation temperature of PHB

$$T_{\text{in}} = \frac{1}{2} \left(\frac{5}{g_\star \pi^3} \right)^{1/4} \left(\frac{3\gamma M_{\text{Pl}}^3}{M_{\text{in}}} \right)^{1/2}$$

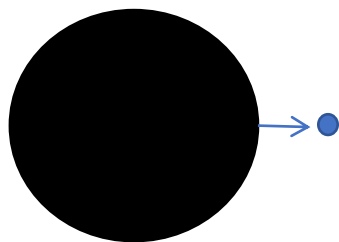
Initial energy fraction of PBH

$$\beta \equiv \frac{\rho_{\text{PBH}}(T_{\text{in}})}{\rho_R(T_{\text{in}})} = \frac{n_{\text{PBH}}(T_{\text{in}}) M_{\text{in}}}{\rho_R(T_{\text{in}})}$$



pNGB from PBH

PBH can radiate particles lighter than the Hawking temperature.



$$T_{\text{PBH}} = \frac{M_{\text{Pl}}^2}{4\pi M_{\text{PBH}}} f(a_\star) = \frac{M_{\text{Pl}}^2}{4\pi M_{\text{PBH}}} \frac{\sqrt{1-a_\star}}{1+\sqrt{1-a_\star}}$$

$$a_\star \equiv JM_{\text{Pl}}^2/M_{\text{PBH}}^2$$

$$\Gamma_{\text{PBH} \rightarrow i} \equiv \frac{dN_i}{dt} = \frac{27g_i M_{\text{Pl}}^2}{1024\pi^4 M_{\text{PBH}}} \begin{cases} 2\zeta(3), & \text{Bosons} \\ \frac{3}{2}\zeta(3), & \text{Fermions} \end{cases}$$

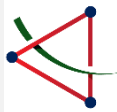
Particle spectrum from Hawking radiation

absorption cross section

$$\frac{d^2 N_i}{dp dt} = \frac{g_i}{2\pi^2} \sum_{l=s_i} \sum_{m=-l}^l \frac{\sigma_{s_i}^{lm}(M_{\text{PBH}}, p, a_\star)}{\exp[(E_i(p) - m\Omega)/T_{\text{PBH}}] - (-1)^{2s_i}} \frac{p^3}{E_i(p)} \quad E_i(p) = \sqrt{p^2 + m_i^2}$$

horizon angular velocity

$$\Omega = (a_\star M_{\text{Pl}}^2 / (2M_{\text{PBH}})) \left(1 / \left(1 + \sqrt{1 - a_\star^2} \right) \right)$$



Schwarzschild PBH

geometric limit $\frac{dM_{\text{PBH}}}{dt} = -\varepsilon (M_{\text{PBH}}) \frac{M_{\text{Pl}}^4}{M_{\text{PBH}}^2} \approx -\frac{27}{4} \frac{g_{* \text{PBH}}}{30720\pi} \frac{M_{\text{Pl}}^4}{M_{\text{PBH}}^2}$

$$M_{\text{PBH}}(t) = M_{\text{in}} \left(1 - \frac{t - t_{\text{in}}}{\tau}\right)^{1/3} \quad \text{Lifetime of PBH} \quad \tau = \frac{40960\pi}{27g_{* \text{PBH}}} \frac{M_{\text{in}}^3}{M_{\text{Pl}}^4}$$

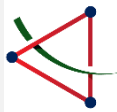
$\beta < \beta_c$ If the universe were always radiation dominated

$$H(\tau) = 1/(2\tau) \quad H = \sqrt{\frac{8\pi}{3} \frac{\rho_R}{M_{\text{Pl}}^2}} \quad T_{\text{evap}} \simeq \frac{9}{128} \left(\frac{1}{5g_*\pi^5}\right)^{1/4} \left(\frac{g_{* \text{PBH}} M_{\text{Pl}}^5}{2M_{\text{in}}^3}\right)^{1/2}$$

$\beta > \beta_c$ Since BHs evolve as matter, if they dominate universe before fully decaying

$$H(\tau) = 2/(3\tau) \quad H = \sqrt{\frac{8\pi}{3} \frac{M_{\text{in}} n_{\text{PBH}}(T_{\text{in}}) T_{\text{evap}}^3 / T_{\text{in}}^3}{M_{\text{Pl}}^2}} \quad \beta_c \equiv T_{\text{evap}} / T_{\text{in}}$$

$$T_{\text{evap}}|_{\text{PBHdom}} \simeq \frac{9}{256} \left(\frac{g_{* \text{PBH}}^2}{2\beta}\right)^{1/3} \left(\frac{1}{5^5 \pi^{17} g_*^3}\right)^{1/12} \left(\frac{M_{\text{Pl}}^{17}}{3\gamma M_{\text{in}}^{11}}\right)^{1/6}$$



Schwarzschild PBH

If particle mass is smaller than initial Hawking temperature: $M_i < \frac{M_{\text{Pl}}^2}{8\pi M_{\text{in}}}$

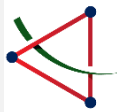
$$N_i = \int_0^{M_{\text{in}}} \frac{1}{\varepsilon(M_{\text{PBH}})} \frac{M_{\text{PBH}}^2}{M_{\text{Pl}}^4} \frac{27g_i \zeta(3) M_{\text{Pl}}^2}{512\pi^4 M_{\text{PBH}}} dM_{\text{PBH}} = \frac{120\zeta(3)}{\pi^3} \frac{g_i}{g_{*\text{PBH}}} \left(\frac{M_{\text{in}}}{M_{\text{Pl}}} \right)^2$$

If particle mass is heavier than the initial temperature, PBHs firstly radiate other lighter particles and then the temperature increases:

$$N_i = \int_0^{\frac{M_{\text{Pl}}^2}{8\pi M_i}} \frac{1}{\varepsilon(M_{\text{PBH}})} \frac{M_{\text{PBH}}^2}{M_{\text{Pl}}^4} \frac{27g_i \zeta(3) M_{\text{Pl}}^2}{512\pi^4 M_{\text{PBH}}} dM_{\text{PBH}} = \frac{15\zeta(3)}{8\pi^5} \frac{g_i}{g_{*\text{PBH}}} \left(\frac{M_{\text{Pl}}}{M_i} \right)^2$$

If the PBHs dominate the Universe before evaporation, the radiation into SM particles will reheat the Universe and dilute the final DM relic.

$$\frac{\pi^2}{30} g_* T_{\text{evap}}^4 + M_{\text{in}} \frac{n_{\text{PBH}}(T_{\text{in}})}{s(T_{\text{in}})} s(T_{\text{evap}}) = \frac{\pi^2}{30} g_* \tilde{T}_{\text{evap}}^4 \longrightarrow Y_i \equiv \frac{N_i n_{\text{PBH}}(T_{\text{evap}})}{s(T_{\text{evap}})} \boxed{\frac{s(T_{\text{evap}})}{s(\tilde{T}_{\text{evap}})}}$$



Schwarzschild PBH

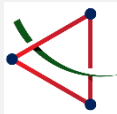
DM relic density

$$\Omega_{\text{DM}} = \frac{\rho_{\text{DM}}}{\rho_c} = \frac{M_{\text{DM}}}{\rho_c} \frac{n_{\text{DM}}}{s} (t_0) s_0 = \frac{M_{\text{DM}}}{\rho_c} Y_{\text{DM}} s_0$$

$$\begin{aligned} \beta < \beta_c \quad \Omega_{\text{DM}} &\simeq 1.61 \times 10^8 \beta \left(\frac{g_*}{106.75} \right)^{-1/4} \left(\frac{g_{\text{DM}}}{g_{*\text{PBH}}} \right) \left(\frac{M_{\text{in}}}{M_{\text{Pl}}} \right)^{1/2} \left(\frac{M_{\text{DM}}}{\text{GeV}} \right), \quad M_{\text{DM}} < \frac{M_{\text{Pl}}^2}{8\pi M_{\text{in}}} \\ &\simeq 2.55 \times 10^5 \beta \left(\frac{g_*}{106.75} \right)^{-1/4} \left(\frac{g_{\text{DM}}}{g_{*\text{PBH}}} \right) \left(\frac{M_{\text{Pl}}^7}{M_{\text{in}}^3 M_{\text{DM}}^4} \right)^{1/2} \left(\frac{M_{\text{DM}}}{\text{GeV}} \right), \quad M_{\text{DM}} > \frac{M_{\text{Pl}}^2}{8\pi M_{\text{in}}} \end{aligned}$$

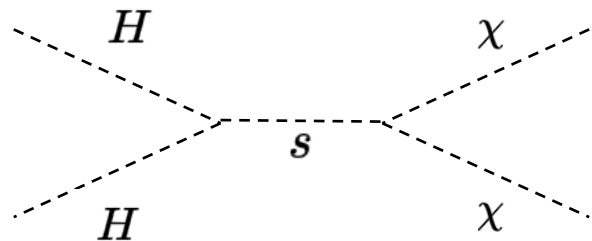
$$\begin{aligned} \beta > \beta_c \quad \Omega_{\text{DM}} &\simeq 6.46 \times 10^7 \left(\frac{g_*}{106.75} \right)^{-1/4} \left(\frac{g_{*\text{PBH}}}{115} \right)^{1/2} \left(\frac{g_{\text{DM}}}{g_{*\text{PBH}}} \right) \left(\frac{M_{\text{Pl}}}{M_{\text{in}}} \right)^{1/2} \left(\frac{M_{\text{DM}}}{\text{GeV}} \right), \quad M_{\text{DM}} < \frac{M_{\text{Pl}}^2}{8\pi M_{\text{in}}} \\ &\simeq 1.02 \times 10^5 \left(\frac{g_*}{106.75} \right)^{-1/4} \left(\frac{g_{*\text{PBH}}}{115} \right)^{1/2} \left(\frac{g_{\text{DM}}}{g_{*\text{PBH}}} \right) \left(\frac{M_{\text{Pl}}^9}{M_{\text{in}}^5 M_{\text{DM}}^4} \right)^{1/2} \left(\frac{M_{\text{DM}}}{\text{GeV}} \right), \quad M_{\text{DM}} > \frac{M_{\text{Pl}}^2}{8\pi M_{\text{in}}} \end{aligned}$$

N.B. If PBHs dominate the energy density of the early Universe before they evaporate, the relic density is independent of β .



UV freeze-in

UV freeze-in

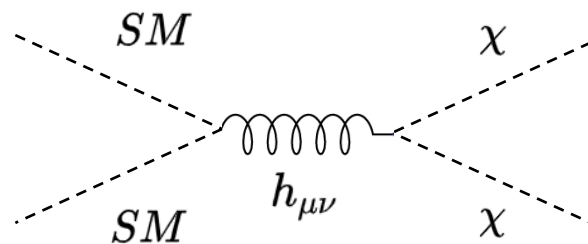


$$\frac{dY_{\chi}^{\text{FI}}(x)}{dz} = \frac{2}{s(z)H(z)z} \gamma_{\chi\chi \rightarrow H^{\dagger}H}$$

$$\gamma_{\chi\chi \rightarrow H^{\dagger}H} \equiv \frac{g_H}{2!2!} \frac{T\lambda_{HS}^2}{2^9\pi^5} \int_{4M_{\chi}^2}^{\infty} ds \sqrt{s - 4M_{\chi}^2} K_1(\sqrt{s}/T) \frac{s^2}{(s - M_s^2)^2}$$

$$\Omega_{\chi}^{\text{FI}} = 3.34 \times 10^{22} \lambda_{HS}^2 \left(\frac{100}{g_s}\right) \left(\frac{100}{g_*}\right)^{1/2} \left(\frac{T_{\text{reh}}}{M_s}\right)^4 \left(\frac{M_{\chi}}{T_{\text{reh}}}\right)$$

Gravitational freeze-in

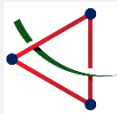


Phys.Lett.B 774 (2017) 676-681; Phys.Rev.D 97 (2018) 11, 115020; Phys.Rev.D 105 (2022) 7, 075005

$$\frac{dY_{\chi}^{\text{GR}}(x)}{dz} = \frac{2}{s(z)H(z)z} \gamma_g$$

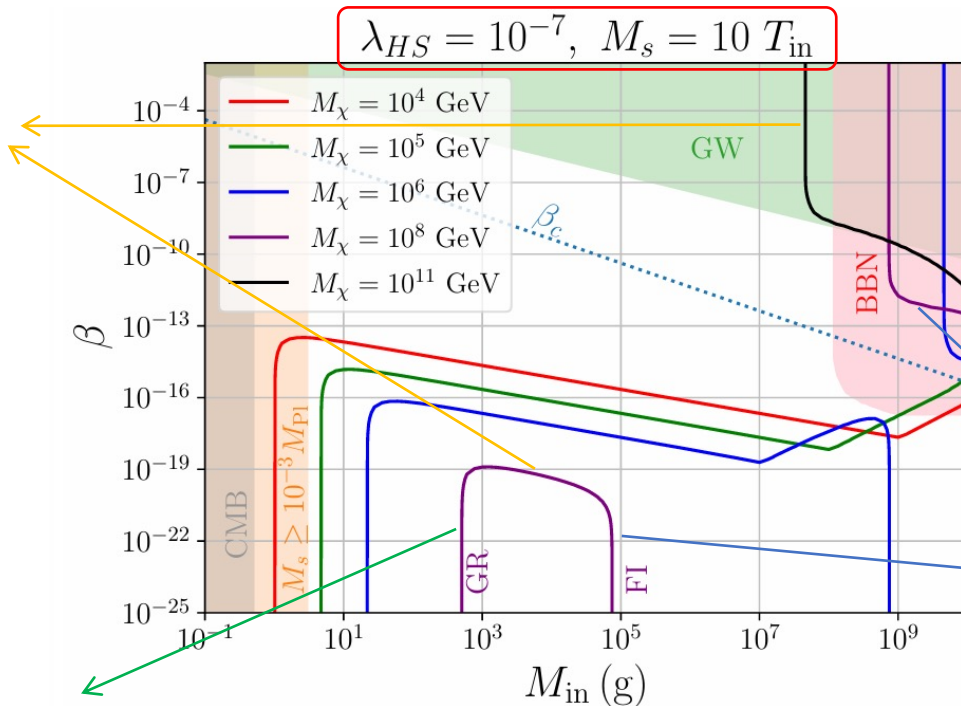
$$\gamma_g = 64\pi^2 \delta \frac{T_{\text{Pl}}^8}{M_{\text{Pl}}^4} \quad \delta = \frac{3997\pi^3}{41472000}$$

$$Y_{\chi}^{\text{GR}} = \frac{720\delta}{\pi^2 g_{*s}} \sqrt{\frac{5\pi}{g_*}} \left(\frac{T_{\text{in}}}{M_{\text{Pl}}}\right)^3 \frac{s(T_{\text{evap}})}{s(\tilde{T}_{\text{evap}})} \times \begin{cases} 1, & M_{\chi} \ll T_{\text{in}} \\ \left(\frac{T_{\text{in}}}{M_{\chi}}\right)^4, & M_{\chi} \gg T_{\text{in}} \end{cases}$$



Schwarzschild PBH: superheavy DM

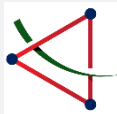
DM from
PBH
evaporation



DM from UV
freeze-in

DM from gravitational freeze-in

Siyu Jiang, FPH*, arXiv:2503.14332, JCAP06 (2025) 023



Kerr PBH: superheavy DM

Since the DM is pseudo-Goldstone boson, it can also be produced by superradiance process of Kerr BH.

The superradiance is efficient when the Compton length of the DM particle is comparable with the gravitational radius of Kerr BH.

The superradiant rate can be given approximately and analytically

$$\Gamma_{\text{SR}} = \frac{M_\chi}{24} \left(\frac{M_{\text{PBH}} M_\chi}{M_{\text{Pl}}^2} \right)^8 (a_\star - 2M_\chi r_+)$$

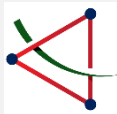
The event horizon

$$r_+ = r_g(1 + \sqrt{1 - a_\star^2})$$

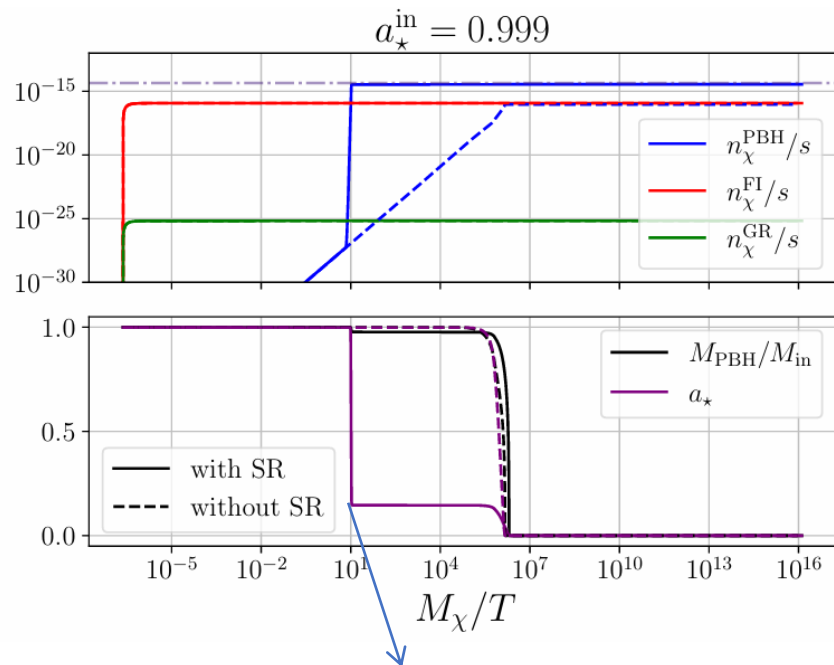
Or more accurately by solving the Teukolsky equation numerically

$$\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta S) + \left[a_\star^2 r_g^2 (\omega^2 - \mu^2) \cos^2 \theta - \frac{m^2}{\sin^2 \theta} + \lambda \right] S = 0,$$

$$\Delta \partial_r (\partial_r R) - \Delta \left[\mu^2 r^2 + a_\star^2 r_g^2 \omega^2 - 2\omega m a_\star r_g r + (\omega (r^2 + a_\star^2 r_g^2) - m a_\star r_g)^2 + \lambda \right] R = 0$$



Kerr PBH: superheavy DM



Superradiance reduces the BH angular momentum before the Hawking radiation, DM is mainly produced by superradiance.

$$Ha \frac{dN_\chi^{\text{SR}}}{da} = \Gamma_{\text{SR}} N_\chi^{\text{SR}},$$

$$Ha \frac{dM_{\text{PBH}}}{da} = -\varepsilon(M_{\text{PBH}}, a_\star) \frac{M_{\text{Pl}}^4}{M_{\text{PBH}}^2} - M_\chi \Gamma_{\text{SR}} N_\chi^{\text{SR}},$$

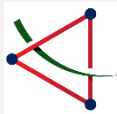
$$Ha \frac{da_\star}{da} = -a_\star [\gamma(M_{\text{PBH}}, a_\star) - 2\varepsilon(M_{\text{PBH}}, a_\star)] \frac{M_{\text{Pl}}^4}{M_{\text{PBH}}^3}$$

$$- \left[\sqrt{2} - 2\alpha a_\star \right] \Gamma_{\text{SR}} N_\chi^{\text{SR}} \frac{M_{\text{Pl}}^2}{M_{\text{PBH}}^2},$$

$$Ha \frac{dN_\chi^{\text{PBH}}}{da} = \frac{\varrho_{\text{PBH}}}{M_{\text{PBH}}} \left[\Gamma_{\text{PBH} \rightarrow \chi} + \Gamma_{\text{SR}} N_\chi^{\text{SR}} \right],$$

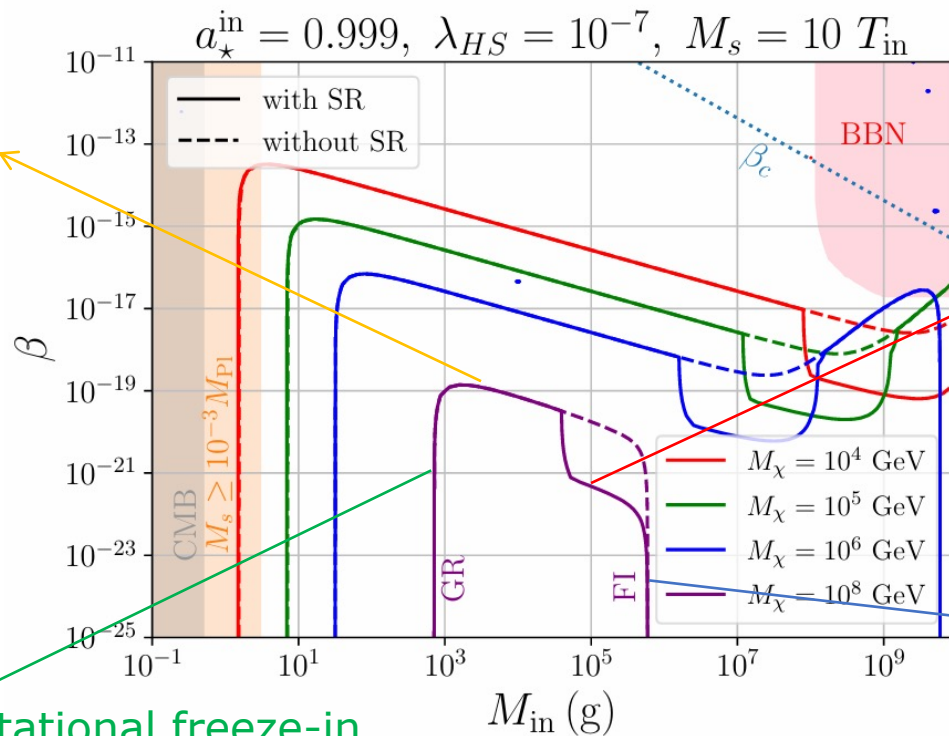
$$Ha \frac{dN_\chi^{\text{FI}}}{da} = 2a^3 \gamma_{\chi\chi \rightarrow H^\dagger H}, \quad Ha \frac{dN_\chi^{\text{GR}}}{da} = 2a^3 \gamma_g$$

Solve by using ULYSSES (2301.05722)



Kerr PBH: superheavy DM

DM from
PBH
evaporation

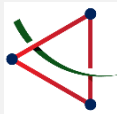


DM from
superradiance

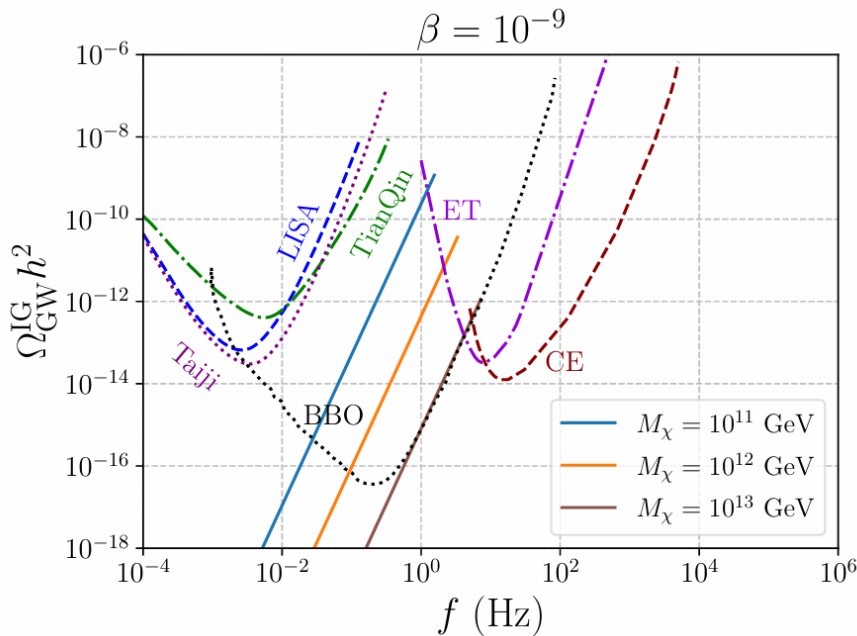
DM from UV
freeze-in

DM from gravitational freeze-in

Siyu Jiang, FPH*, arXiv:2503.14332, JCAP06 (2025) 023



Induced GW from superheavy DM



Siyu Jiang, FPH*, arXiv:2503.14332,
JCAP06 (2025) 023

When the PBHs begin to dominate the energy density of the Universe, the inhomogeneous distribution of PBHs leads to curvature perturbations. Subsequently, at second order, these perturbations can generate GWs.

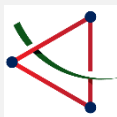
$$f_p^{\text{IG}} \simeq 1.7 \times 10^3 \text{ Hz} \left(\frac{M_{\text{in}}}{10^4 \text{ g}} \right)^{-5/6}$$

$$\simeq 0.33 \text{ Hz} \left(\frac{M_\chi}{10^9 \text{ GeV}} \right)^{1/3}$$

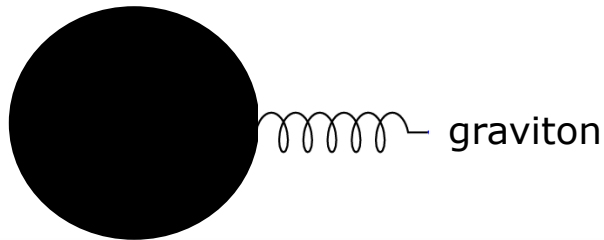
$$\Omega_p^{\text{IG}} h^2 \simeq 9 \times 10^{-7} \left(\frac{\beta}{10^{-8}} \right)^{16/3} \left(\frac{M_{\text{in}}}{10^7 \text{ g}} \right)^{34/9}$$

$$\Omega_{\text{GW}}^{\text{IG}} h^2 \simeq \Omega_p^{\text{IG}} h^2 \left(\frac{f}{f_p^{\text{IG}}} \right)^{11/3} \Theta(f_p^{\text{IG}} - f)$$

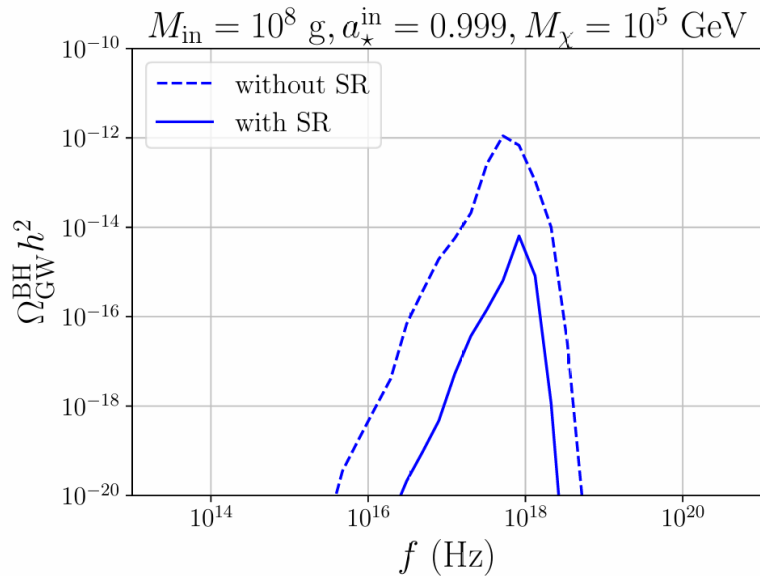
JCAP 04 (2021) 062; JCAP 03 (2021) 053; JHEP 03 (2023) 127



GW from Hawking radiation



graviton



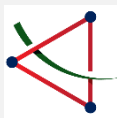
$$\frac{d\rho_{\text{GW}}}{dt dp} = n_{\text{PBH}}(t) p \frac{dN_{\text{grav}}}{dt dp}$$



public code
BlackHawk

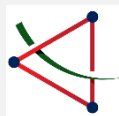
$$\frac{d\rho_{\text{GW},0}}{d\ln p_0} = 2 \times 10^{32} \beta M_{\text{in}}^{-3/2} p_0^4 \int_{\frac{a_0}{a_{\text{evap}}}}^{\frac{a_0}{a_{\text{in}}}} dx \frac{\sigma_{s_i}^{lm}}{\exp[(xp_0 - m\Omega)/T_{\text{PBH}}] - 1}$$

$$\Omega_{\text{GW}}^{\text{BH}} h^2 = \frac{h^2}{\rho_c} \frac{d\rho_{\text{GW},0}}{d\ln p_0}$$



Outline

- 1. Motivation for new dark matter (DM) mechanism**
- 2. pNGB DM from Primordial black hole (PBH) radiation and superradiance with its gravitational wave (GW) signals**
- 3. DM from first-order phase transition (FOPT) and GW**
 - Case I: Q-ball and gauged Q-ball DM**
 - Case II: filtered DM**
- 4. Summary and outlook**



Heavy DM from cosmic phase transition

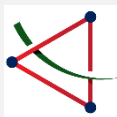
Renaissance of quark nugget DM idea by E. Witten.

Recently, dynamical DM formed by phase transition has become a new idea for heavy. Bubble wall in FOPT can be the “filter” to obtain the needed heavy DM when avoiding the unitarity constraints.



FOPT in the early universe	Coffee making process
Bubble wall	filter
Case I:(gauged) Q-ball DM	Large coffee beans
Case II: filtered DM	Coffee
Phase transition GW	Aroma

E. Krylov, A. Levin, V. Rubakov, *Phys.Rev.D* 87 (2013) 8, 083528
FPH, Chong Sheng Li, *Phys.Rev. D*96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
Phys.Rev.Lett. 125 (2020) 15, 151102, M. J. Baker, J. Kopp, and A. J. Long
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi, Eric D. Kramer, Eric Kuflik, Gregory W.
Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi, Eric D. Kramer, Eric Kuflik, Gregory W.
Ridgway, Tracy R. Slatyer, J. Smirnov
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509
more than 100 papers in recent 5 years



Case I: Q-ball DM

What is Q-ball?

PHYSICS REPORTS (Review Section of Physics Letters) 221, Nos. 5 & 6 (1992) 251-350. North-Holland

PHYSICS REPORTS

Nontopological solitons*

T.D. Lee

Department of Physics, Columbia University, New York, NY 10027, USA

and

Y. Pang

Brookhaven National Laboratory, Upton, NY 11973, USA

Received May 1992; editor: D.N. Schramm

Nuclear Physics B262 (1985) 263-283

© North-Holland Publishing Company

Q-BALLS*

Sidney COLEMAN

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Q-ball is the most typical non-topological soliton, initially proposed by Prof. Tsung-Dao Lee and Sidney Coleman. In quantum field theory, a spherically symmetric extended body that forms a non-topological soliton structure with a conserved global quantum number Q is called a Q-ball.

$$\phi = (\phi_R + i\phi_I)/\sqrt{2} \quad Q = \int j^0 dx = \int (\phi_I \dot{\phi}_R - \phi_R \dot{\phi}_I) dx.$$

$$\delta(E - \omega Q) = 0$$



$$E = \int \left\{ \frac{1}{2} [\dot{\phi}_R^2 + \dot{\phi}_I^2 + (\nabla \phi_R)^2 + (\nabla \phi_I)^2] + U \left[\frac{1}{2} (\phi_R^2 + \phi_I^2) \right] \right\} dx$$

$$\phi = f(r)e^{-i\omega t}$$

Q-ball production mechanism

Q-ball production:

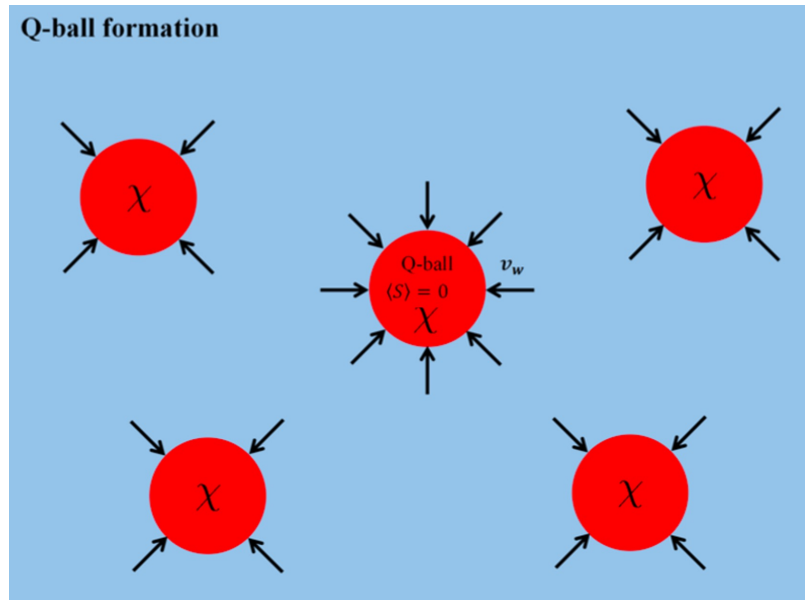
- (1) produce the charge asymmetry (i.e. locally produce lots of particles with the same charge to form Q-ball)
- (2) and packet the same sign charge in the small size after overcoming the Coulomb repulsive interaction.

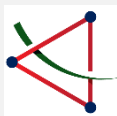
1. Supersymmetry? Affleck-Dine mechanism.

We do not observe the supersymmetry until now!

2. Q-ball formation based on FOPT.

This talk

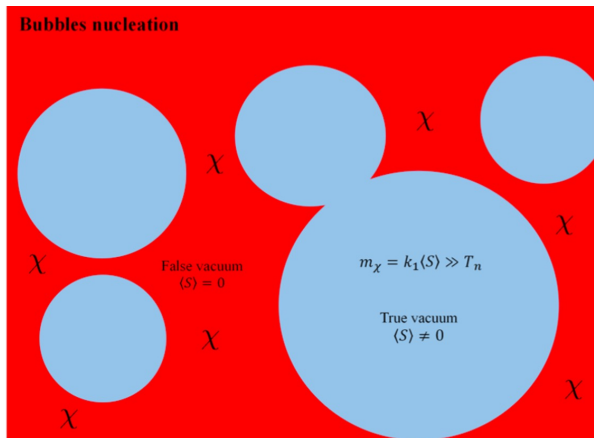




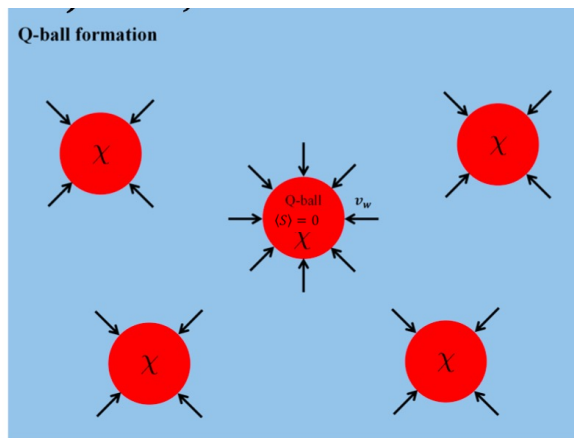
Case I: Q-ball DM

Global Q-ball DM: The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously.

$$\rho_{DM}^4 v_w^{3/4} = 73.5 (2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

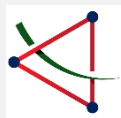
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;



New DM production scenario by the bubbles.
The global Q-ball model proposed by T.D. Lee

Friedberg-Lee-Sirlin model

R. Friedberg, T.D. Lee and A. Sirlin.
Rev. D 13 (1976) 2739



Case I: Gauged Q-ball DM

$$\langle h \rangle \neq 0$$

$$\langle \phi \rangle = 0$$

$$\langle h \rangle = 0$$

$$\langle \phi \rangle \neq 0$$

$$\langle A \rangle \neq 0$$

When the conserved U(1) symmetry is **local**,
This introduces an extra **gauge field A**.

The **minimal model** achieving

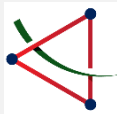
$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{4} \tilde{A}_{\mu\nu} \tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4} (h^2 - v_0^2)^2$$

Interestingly, this portal coupling also naturally induces a strong FOPT.

$$J_\mu = i \left(\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + 2i\tilde{g}\tilde{A}_\mu |\phi|^2 \right) \quad Q = \int d^3x J^0$$

Conserved charge



Gauged Q-ball

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{4} \tilde{A}_{\mu\nu} \tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4} (h^2 - v_0^2)^2$$

$$\tilde{A}_t(r) = v_0 \frac{\tilde{g}}{\sqrt{2\lambda_h}} \mathcal{A}(\rho), \quad \phi(t, r) = \frac{v_0}{\sqrt{2}} \Phi(\rho) e^{-i\omega t}, \quad h(r) = v_0 \mathcal{H}(\rho)$$

Friedberg-Lee-Sirlin-Maxwell model

$$\frac{1}{\rho^2} \partial_\rho (\rho^2 \partial_\rho \mathcal{A}) + (\nu - \alpha^2 \mathcal{A}) \Phi^2 = 0,$$

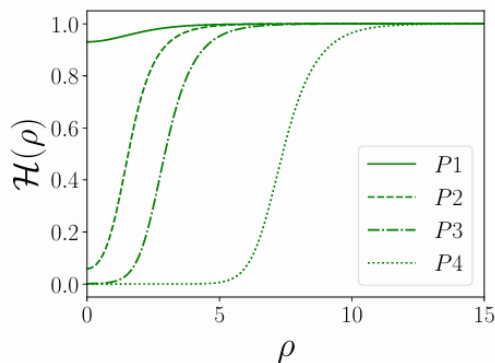
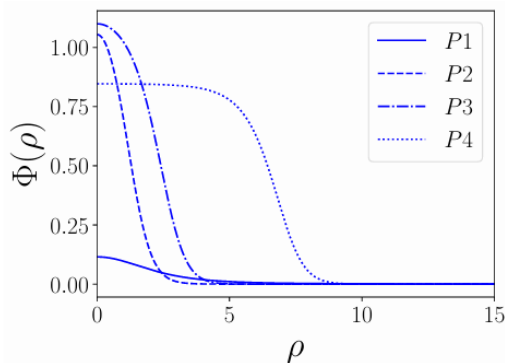
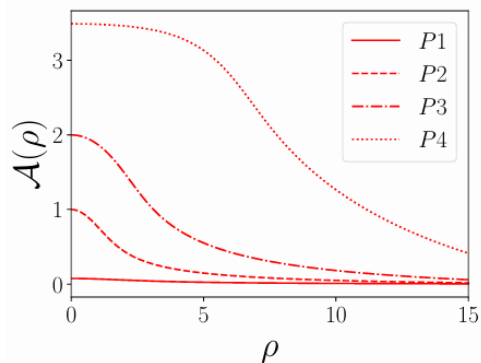
$$\alpha \equiv \frac{|\tilde{g}|}{\sqrt{2\lambda_h}}, \quad k \equiv \frac{\sqrt{\lambda_{\phi h}}}{2\sqrt{\lambda_h}} = \frac{m_\phi}{m_h}$$

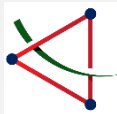
$$\frac{1}{\rho^2} \partial_\rho (\rho^2 \partial_\rho \Phi) + [(\nu - \alpha^2 \mathcal{A})^2 - k^2 \mathcal{H}^2] \Phi = 0,$$

$$\nu \equiv \frac{\omega}{\sqrt{2\lambda_h} v_0}$$

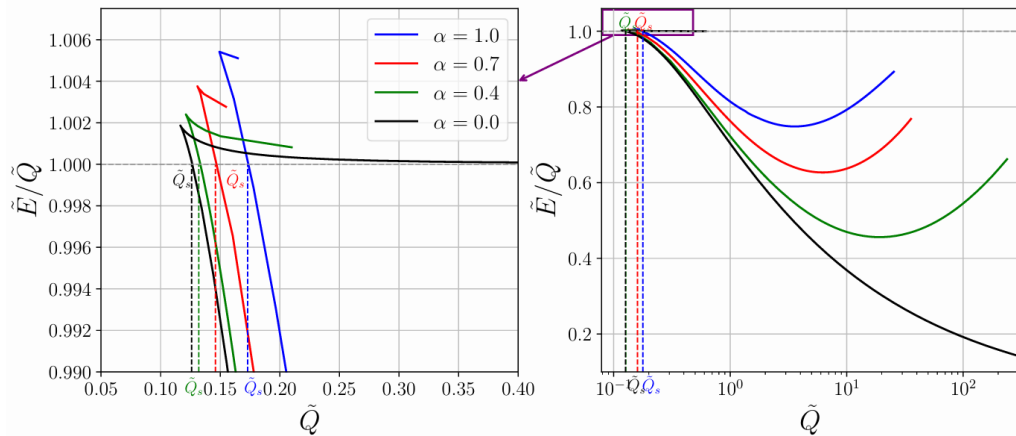
relaxation method

$$\frac{1}{\rho^2} \partial_\rho (\rho^2 \partial_\rho \mathcal{H}) - k^2 \mathcal{H} \Phi^2 - \frac{1}{2} \mathcal{H} (\mathcal{H}^2 - 1) = 0.$$





Gauged Q-ball stability

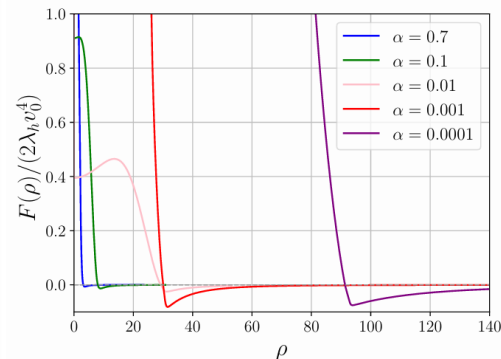
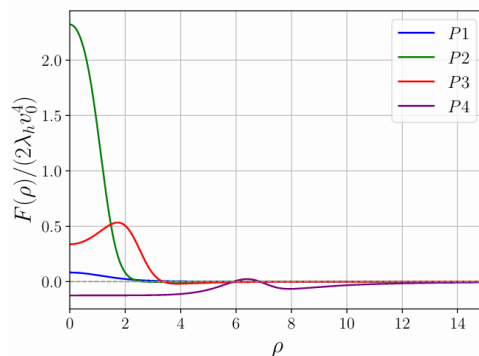


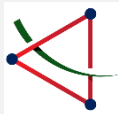
Quantum stability

$$E < m_\phi Q \quad \text{or} \quad \tilde{E}/\tilde{Q} < 1$$

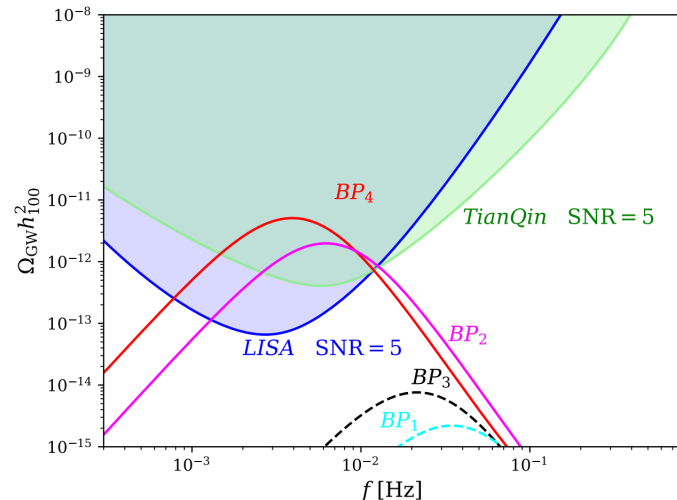
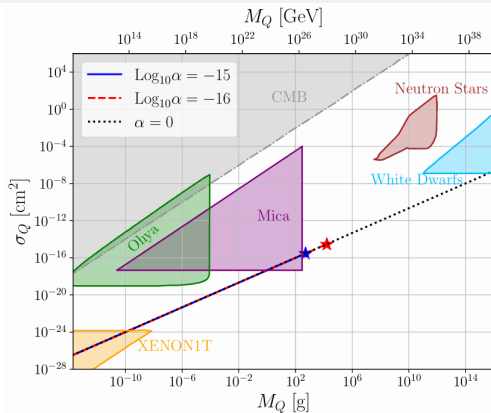
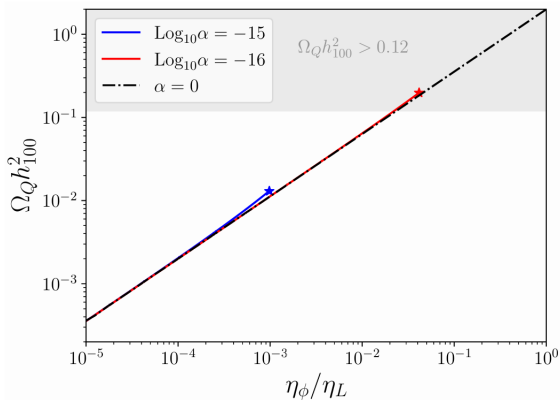
Stress stability

$$F(r) = \frac{2}{3}s(r) + p(r) > 0$$





Gauged Q-ball DM from a FOPT

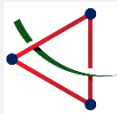


$$\Omega_Q h_{100}^2 \simeq 2.81 \times \left(\frac{s_0 h_{100}^2}{\rho_c} \right) \left(\frac{\Gamma(T_\star)}{v_w} \right)^{3/16} s_\star^{-1/4} (F_\phi^{\text{trap}} \eta_\phi)^{3/4} \lambda_h^{1/4} v_0 \left(1 + \frac{108^{1/4} \tilde{g}^2 F_\phi^{\text{trap}} \eta_\phi s_\star v_w^{3/4}}{5.4 \pi^{7/4} \Gamma(T_\star)^{3/4}} \right)$$

F_ϕ^{trap} : The fraction of particles trapped into the false vacuum. It is determined by the phase transition dynamics.

	$\lambda_{\phi h}$	T_p [GeV]	α_p	β/H_p	v_w	F_ϕ^{trap}	η_ϕ/η_L	$\delta\sigma_{Zh}$	GW
BP_1	6.8	69.8	0.12	540	0.1	0.932	0.48	-0.36%	●
BP_2	6.8	70.4	0.12	578	0.6	0.805	3.0	-0.36%	●
BP_3	7.0	63.0	0.15	372	0.1	0.965	3.4	-0.37%	●
BP_4	7.0	63.9	0.15	403	0.6	0.858	20.8	-0.37%	●

Siyu Jiang, **FPH**,
Pyungwon Ko, JHEP 07 (2024) 053



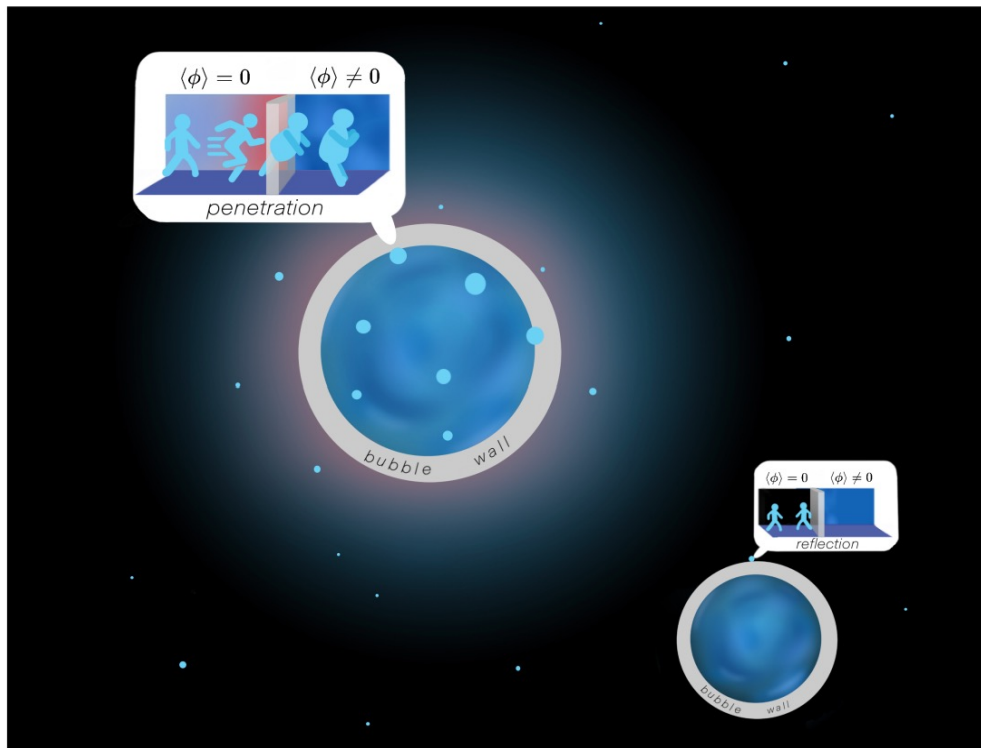
Case II: filtered DM from a FOPT

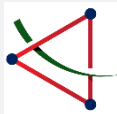


Bubble wall plays an essential role in the filtered DM mechanism.

DM

Siyu Jiang, FPH, Chong Sheng Li,
Phys.Rev.D 108 (2023) 6, 063508



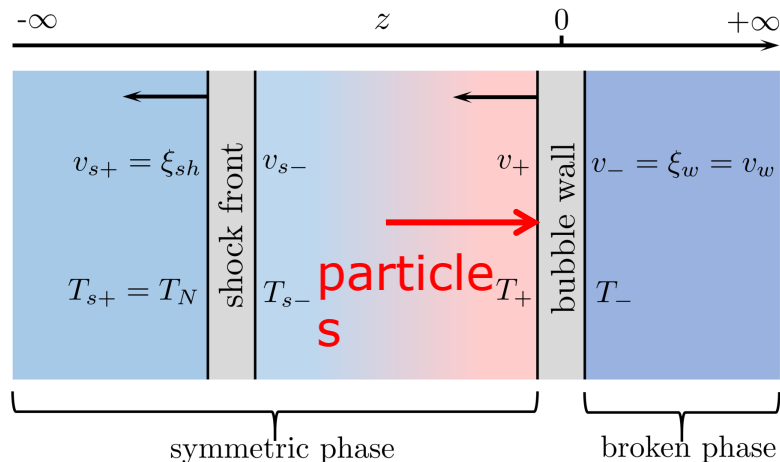


Case II: filtered DM

Original work:

$$\tilde{v}_{\text{pl}} = v_w, \quad T = T' = T_n$$

Phys.Rev.Lett. 125 (2020)
15, 151102, M. J. Baker, J.
Kopp, and A. J. Long



$$\tilde{v}_{\text{pl}} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \quad (\text{this work with hydrodynamic effects}).$$

$$J_w^{\text{in}} = \frac{g_\chi}{(2\pi)^2} \int_0^{-1} d\cos\theta \cos\theta \int_{-\frac{m_\chi^{\text{in}}}{\cos\theta}}^\infty dp \frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3 (1 + \tilde{\gamma}_+ m_\chi^{\text{in}} (1 - \tilde{v}_+)/T_+)}{4\pi^2 \tilde{\gamma}_+^3 (1 - \tilde{v}_+)^2} e^{-\tilde{\gamma}_+ m_\chi^{\text{in}} (1 - \tilde{v}_+)/T_+}.$$

$$n_\chi^{\text{in}} = \frac{J_w^{\text{in}}}{\gamma_w v_w} \quad \Omega_{\text{DM}}^{(\text{hy})} h^2 = \frac{m_\chi^{\text{in}} (n_\chi^{\text{in}} + n_{\bar{\chi}}^{\text{in}})}{\rho_c/h^2} \frac{g_{*0} T_0^3}{g_*(T_-) T_-^3} \simeq 6.29 \times 10^8 \frac{m_\chi^{\text{in}}}{\text{GeV}} \frac{(n_\chi^{\text{in}} + n_{\bar{\chi}}^{\text{in}})}{g_*(T_-) T_-^3}$$



Case II: filtered DM

$$T_{\phi}^{\mu\nu} = \partial^{\mu}\phi\partial^{\nu}\phi - g^{\mu\nu} \left[\frac{1}{2}(\partial\phi)^2 - V_{T=0}(\phi) \right] \quad \text{Energy-momentum tensor of scalar field}$$

$$T_{\text{pl}}^{\mu\nu} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k^{\mu} k^{\nu} f_i^{\text{eq}}(k) \quad \text{Energy-momentum tensor of fluid}$$

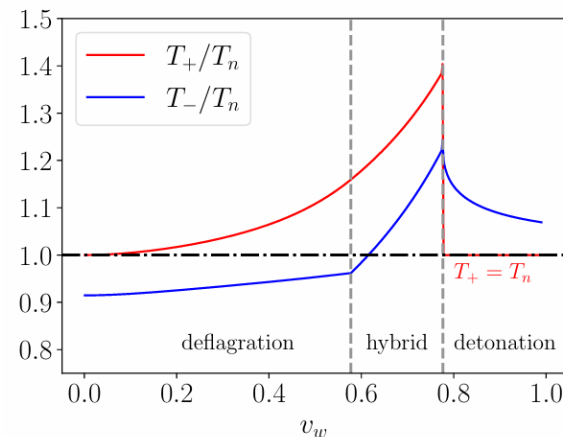
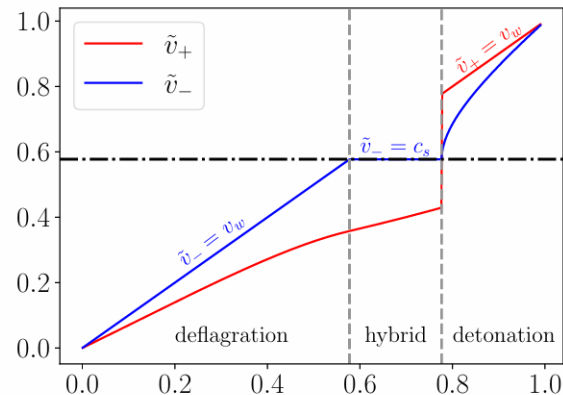
$$T_{\text{fl}}^{\mu\nu} = T_{\phi}^{\mu\nu} + T_{\text{pl}}^{\mu\nu} = \omega u^{\mu} u^{\nu} - p g^{\mu\nu} \quad \text{Energy-momentum conservation}$$

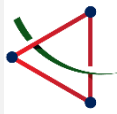
$$\omega_+ \tilde{v}_+^2 \tilde{\gamma}_+^2 + p_+ = \omega_- \tilde{v}_-^2 \tilde{\gamma}_-^2 + p_-, \quad \omega_+ \tilde{v}_+ \tilde{\gamma}_+^2 = \omega_- \tilde{v}_- \tilde{\gamma}_-^2$$

$$\alpha_+ \equiv \epsilon / (a_+ T_+^4)$$

$$r_{\omega} = \omega_+ / \omega_- = (a_+ T_+^4) / (a_- T_-^4)$$

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} j_{\xi}^v &= \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1 \right] \partial_{\xi} v \\ \frac{\partial_{\xi} \omega}{\omega} &= \left(1 + \frac{1}{c_s^2} \right) \gamma^2 \mu \partial_{\xi} v . \end{aligned}$$





Case II: filtered DM

Boltzmann equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

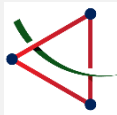
$$f_\chi = \mathcal{A}(z, p_z) f_{\chi,+}^{\text{eq}} = \mathcal{A}(z, p_z) \exp\left(-\frac{\tilde{\gamma}_+(E - \tilde{v}_+ p_z)}{T_+}\right)$$

$$\mathbf{L}[f_\chi] = \frac{p_z}{E} \frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E} \frac{\partial m_\chi}{\partial z} \frac{\partial f_\chi}{\partial p_z} \quad m_\chi(z) \equiv \frac{m_\chi^{\text{in}}(\phi_-)}{2} \left(1 + \tanh \frac{2z}{L_w}\right)$$

$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{L}[f_\chi] \approx \left[\left(\frac{p_z}{m_\chi} \frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z} \right) \frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z} \right) \frac{\tilde{\gamma}_+ \tilde{v}_+}{T_+} \right) \mathcal{A}(z, p_z) \right] \frac{g_\chi m_\chi T_+}{2\pi \tilde{\gamma}_+} e^{\tilde{\gamma}_+(\tilde{v}_+ p_z - \sqrt{m_\chi^2 + p_z^2})/T_+}$$

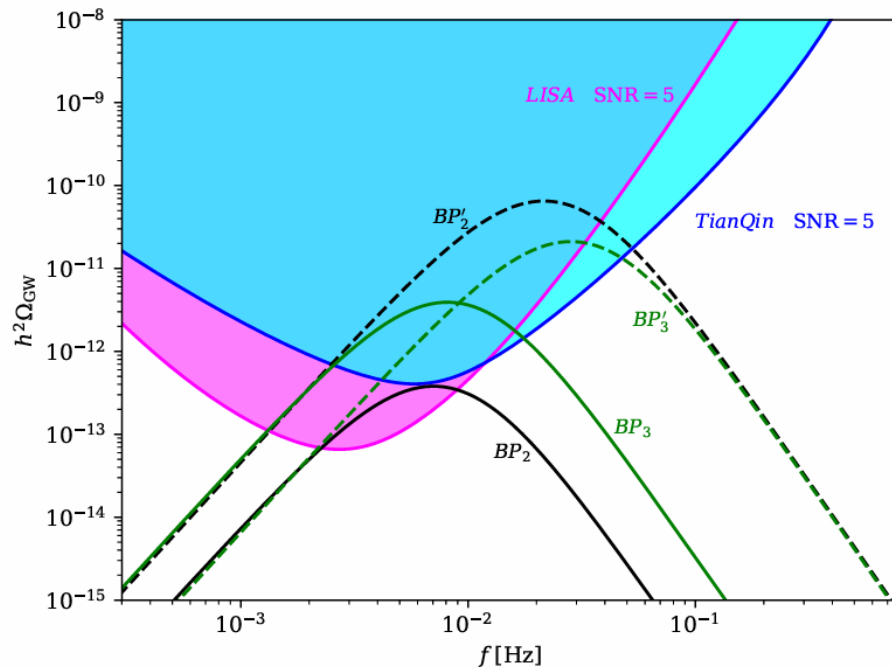
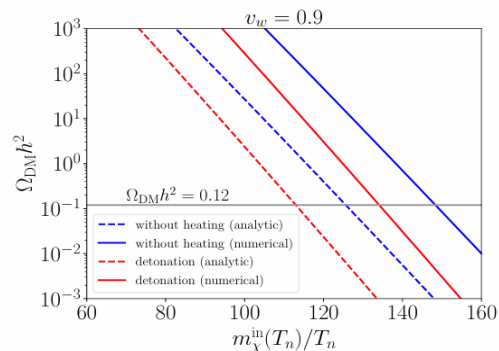
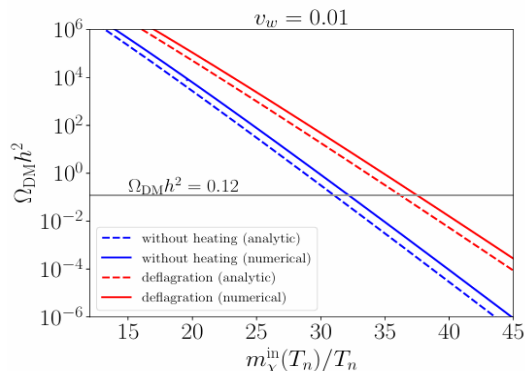
including $\chi\bar{\chi} \leftrightarrow \phi\phi, \chi\phi \leftrightarrow \chi\phi, \chi\chi \leftrightarrow \chi\chi, \chi\bar{\chi} \leftrightarrow \chi\bar{\chi}, \dots$

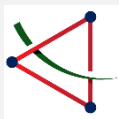
$$\begin{aligned} g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{C}[f_\chi] &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F \sigma_{\chi\bar{\chi} \rightarrow \phi\phi} \left[f_{\chi_p} f_{\bar{\chi}_q,+}^{\text{eq}} - f_{\chi_p}^{\text{eq}} f_{\bar{\chi}_q}^{\text{eq}} \right] \\ &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F \sigma_{\chi\bar{\chi} \rightarrow \phi\phi} \left[\mathcal{A} f_{\chi_p,+}^{\text{eq}} f_{\bar{\chi}_q,+}^{\text{eq}} - f_{\chi_p}^{\text{eq}} f_{\bar{\chi}_q}^{\text{eq}} \right] \\ &\equiv \Gamma_{\text{P}}(z, p_z) \mathcal{A}(z, p_z) - \Gamma_{\text{I}}(z, p_z) , \end{aligned}$$



Case II: filtered DM

$$n_{\chi}^{\text{in}} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp \left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_{\chi}^{\text{in}})^2} \right) / T_+ \right] \left(\sqrt{p_z^2 + (m_{\chi}^{\text{in}})^2} + \frac{T_+}{\tilde{\gamma}_+} \right)$$

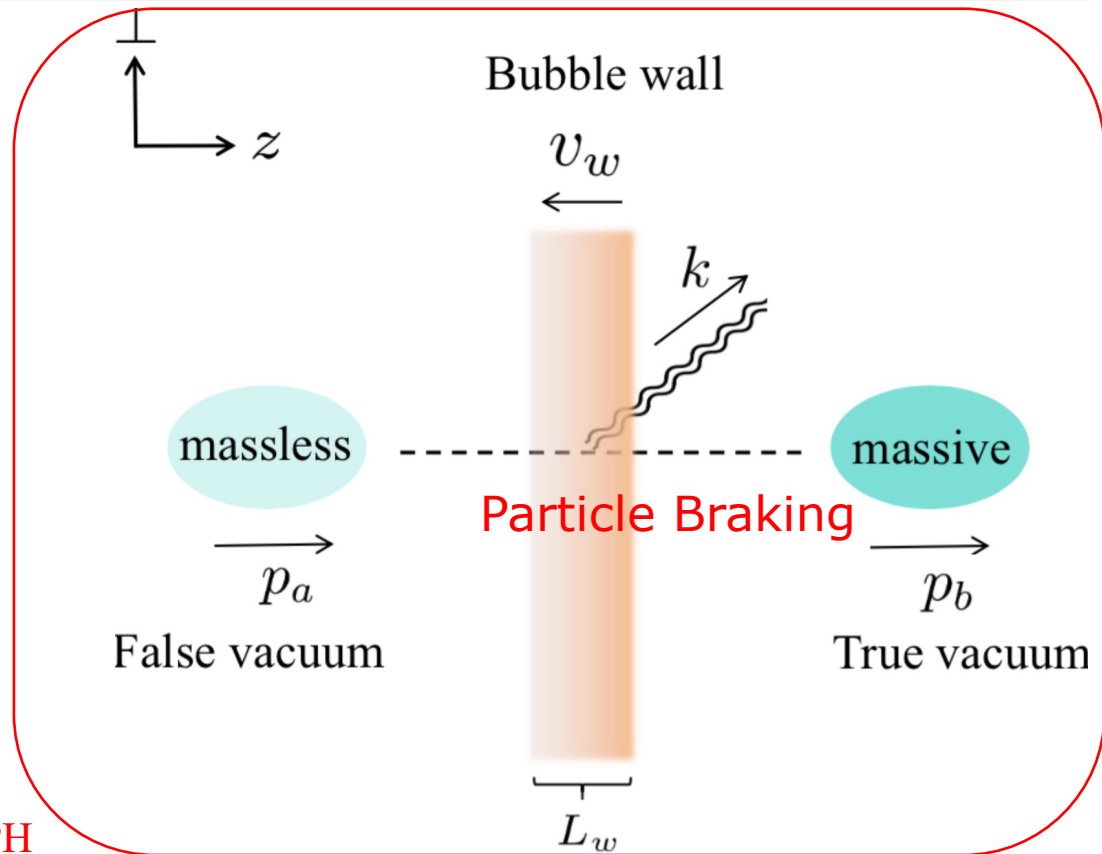
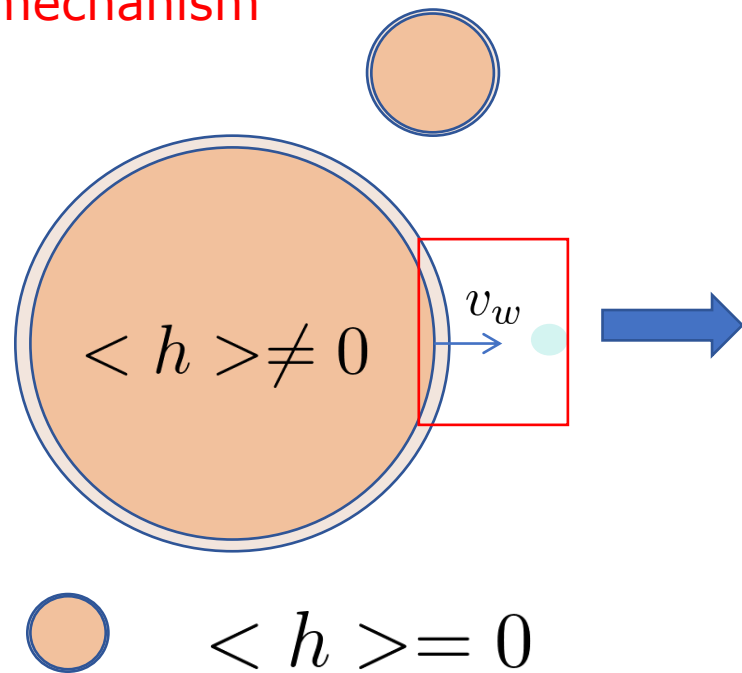


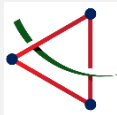


The missing GW source ?

Braking GW from phase transition

New phase transition GW mechanism

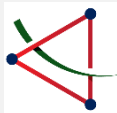




Bremsstrahlung probability

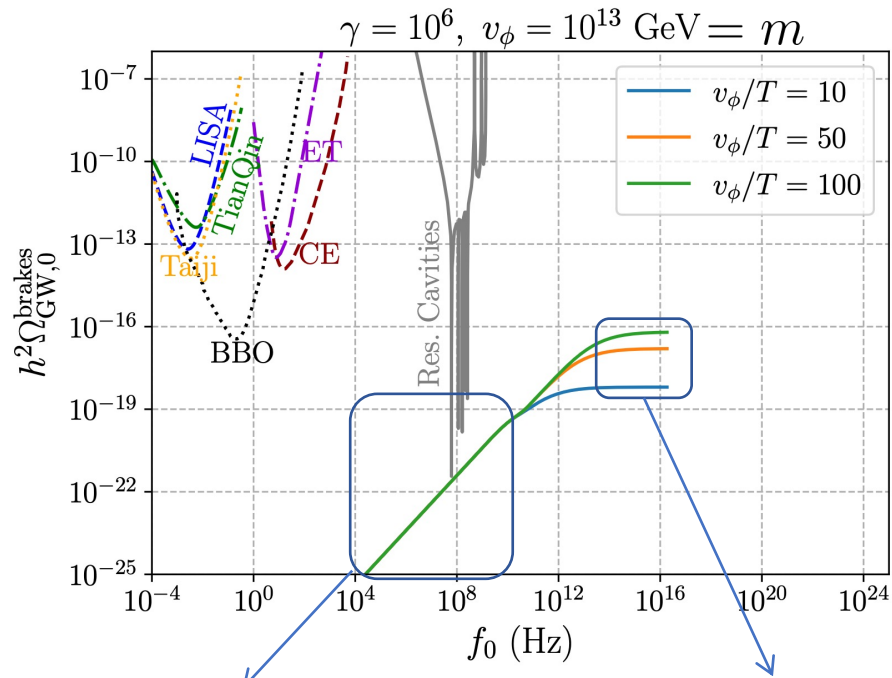
Now, we can calculate the **interaction matrix element**.

$$\begin{aligned}
 \langle \vec{p}_b^{I,\text{out}}, \vec{k} | \mathcal{T} | \vec{p}_a^R \rangle &= \int d^4x \langle \vec{p}_b^{I,\text{out}}, \vec{k} | \mathcal{H}_{\text{int}} | \vec{p}_a^R \rangle && \text{Feynman amplitude} \\
 &= \int dz \int \frac{d^3 p'_a}{(2\pi)^3} \int \frac{d^3 p'_b}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} V^\dagger(z) \chi_R(z, p'_a{}^z) \zeta_I^*(z, p'_b{}^z) \chi^*(z, k'^z) \\
 &\quad \times (2\pi)^3 \delta(E'_a - E'_b - E'_k) \delta^{(2)}(\vec{p}'_{a,\perp} - \vec{p}'_{b,\perp} - \vec{k}'_\perp) \langle \vec{p}_b^{I,\text{out}}, \vec{k} | a_k^\dagger a_{I,b}^\dagger a_{R,a} | \vec{p}_a^R \rangle \\
 &= (2\pi)^3 \delta\left(\sum E\right) \delta^{(2)}\left(\sum \vec{p}_\perp\right) \mathcal{M}_I, \\
 \mathcal{M}_I &= \int_{-\infty}^{+\infty} dz V^\dagger(z) \chi_R(z, p_a^z) \zeta_I^*(z, p_b^z) \chi^*(z, k^z).
 \end{aligned}$$



GW spectrum

arXiv: [2508.04314](#), [Dayun Qiu](#), [Siyu Jiang](#), [FPH](#)

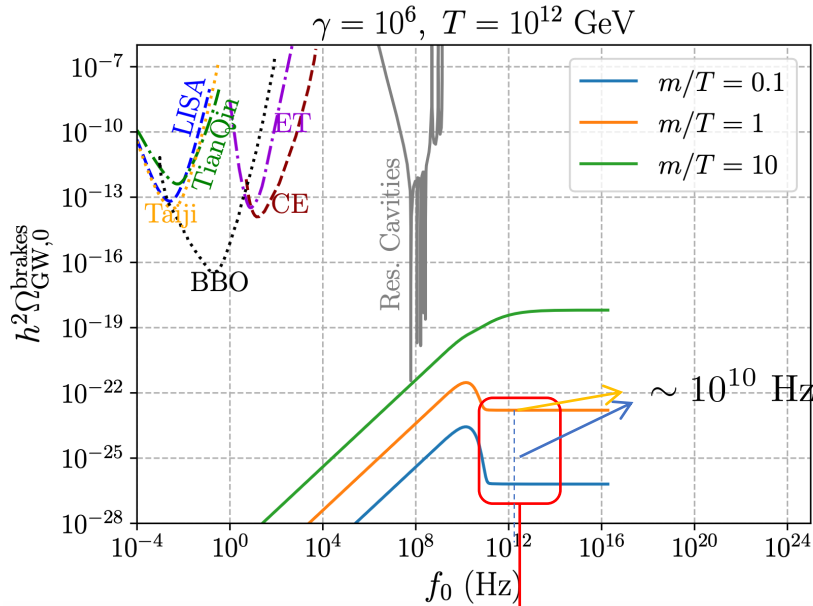


$$h^2 \Omega_{\text{GW},0}^{\text{brakes}}(f_0) \propto m^2 f_0,$$

collinear gravitons

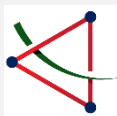
$$h^2 \Omega_{\text{GW},0}^{\text{brakes}}(f_{\text{peak}}) \propto m^4 / T^2.$$

non-collinear gravitons



when $T \gtrsim m$,

double-peaked structure



GW spectrum recap

The GW power spectrum exhibits two distinct behaviors across different frequency regimes.

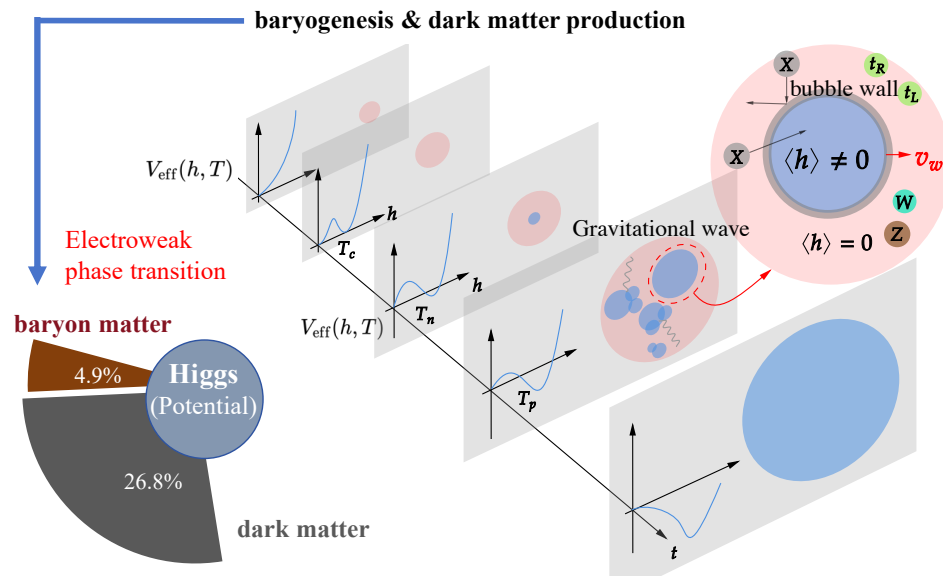
arXiv: [2508.04314](#), [Dayun Qiu](#), [Siyu Jiang](#), FPH

- In the low-frequency regime, the spectrum scales linearly with frequency and is **proportional to the square of the mass**, primarily sourced from ultra-collinear radiation emitted as particles traverse the bubble wall.
- In contrast, the high-frequency regime displays an approximately flat spectrum up to a **cutoff frequency** and the amplitude **scales with the fourth power of the mass**, dominated by non-collinear gravitons.
↓
proportional to the Lorentz factor of the bubble wall

These distinct behaviors may help to more directly to extract the new particle information.



-
- A collage of four images. The top left shows a spiral galaxy with a bright yellow core and blue and white stars. The top right shows a blue and orange galaxy with a bright yellow core and blue and white stars. The bottom left shows a pile of brown coffee beans. The bottom right shows a silver coffee pot with a handle and a spout, filled with coffee, and a silver coffee mug filled with coffee beans.



Thanks! Comments and collaborations are welcome!