Compact Four-degree-of-freedom Seismometer with Capacitive readout

PHYS. REV. APPLIED 23, 044030 (2025)

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2025.08.28 Xichang Sichuan

Outline

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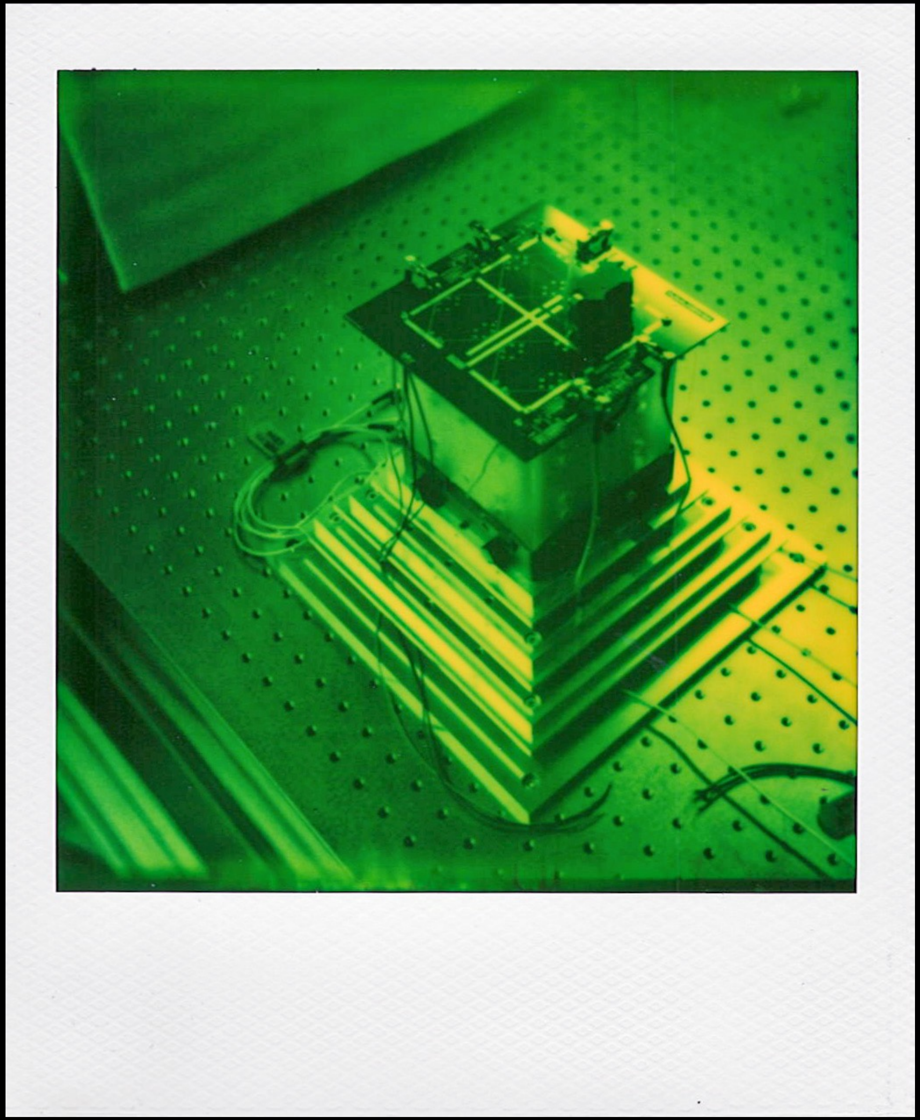
- Background
- Mechanical design
- Motion Sensing design
- Noise Budget and Measurement
- Conclusion



Background

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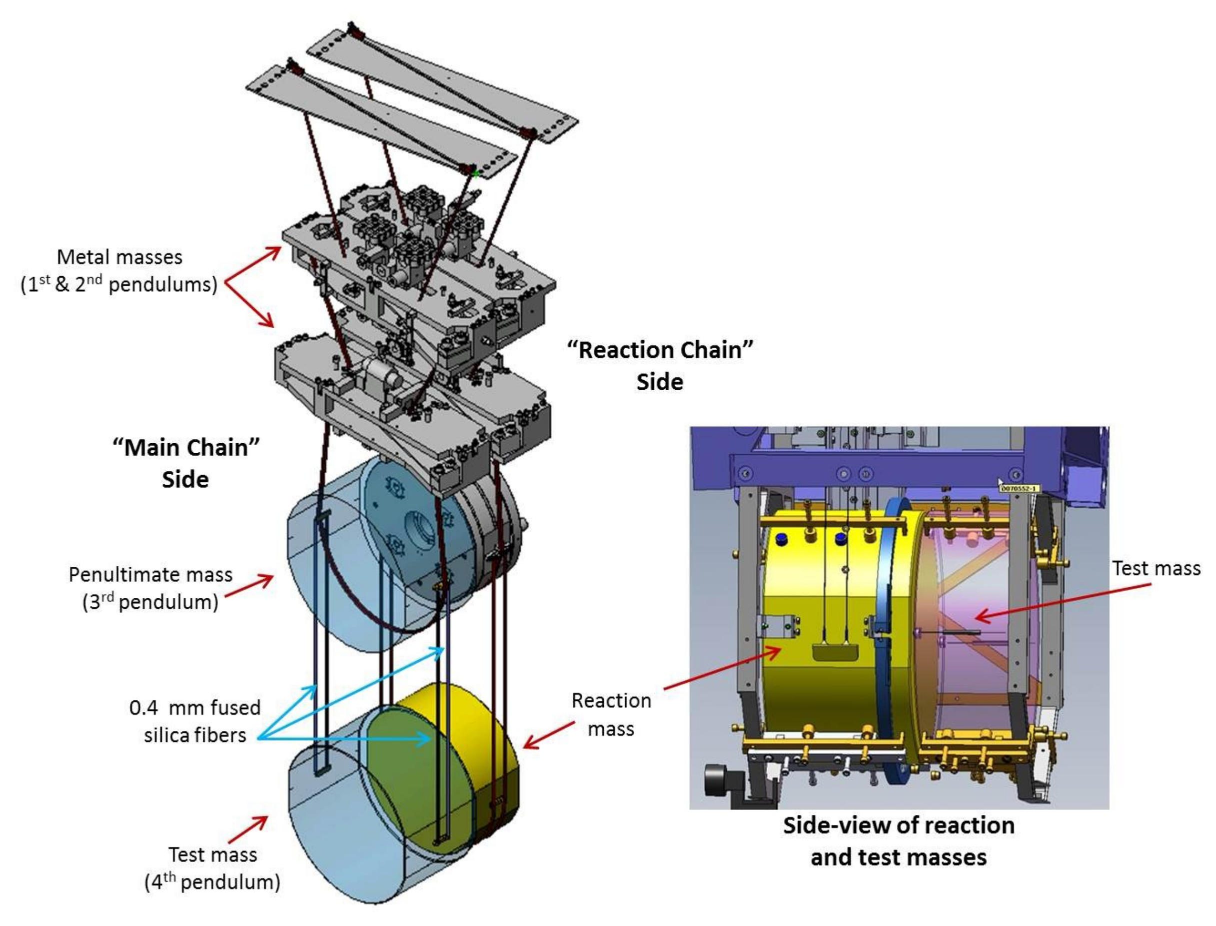
- Ground-base high-precision
- Gravitational-wave observatory
  - Advance LIGO 10^{-14} m RMS
  - Seismic noise 10^{-6} m RMS

- Active vibration isolation system
  - Seismometer



Cr. [Seismic Isolation | LIGO Lab | Caltech]

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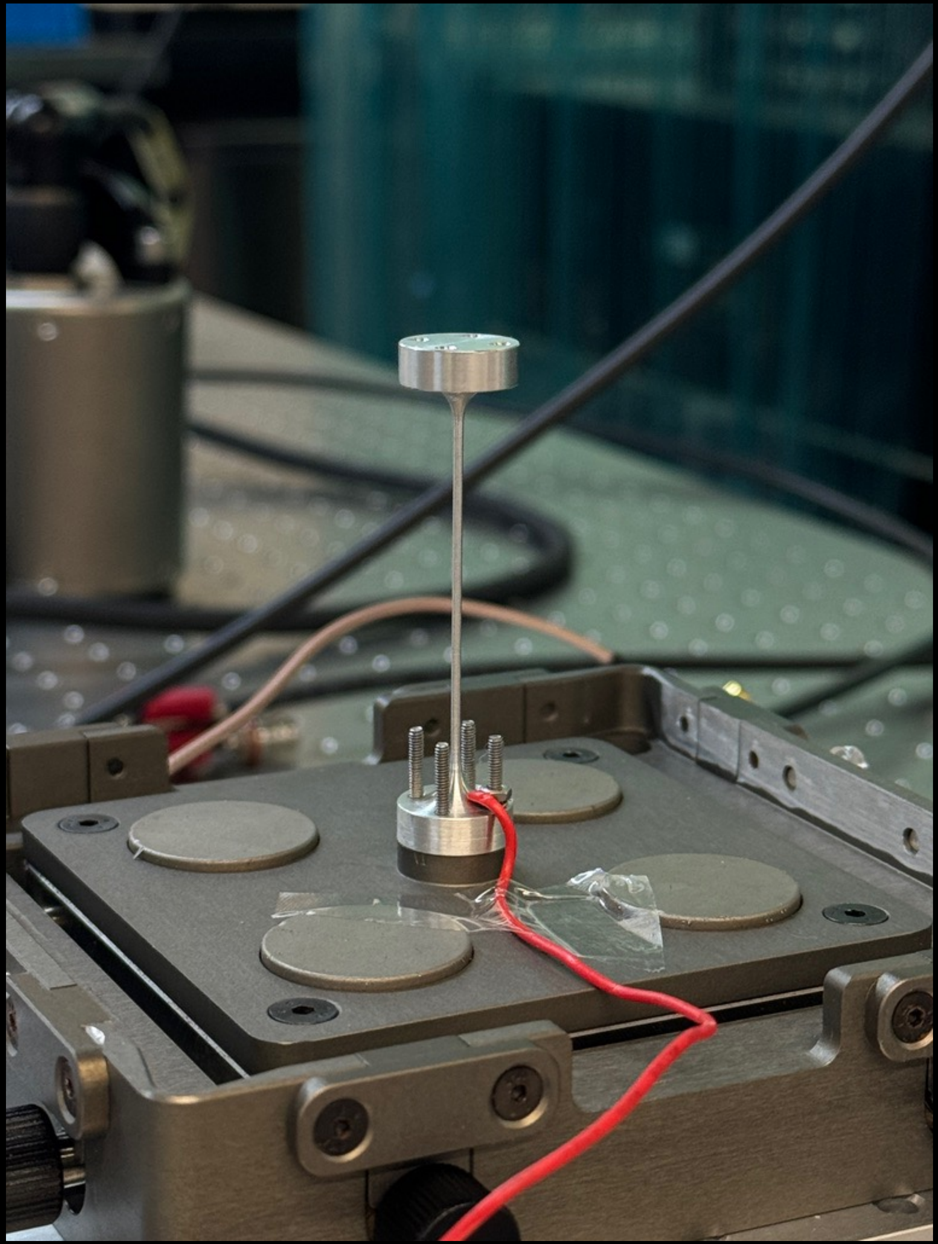
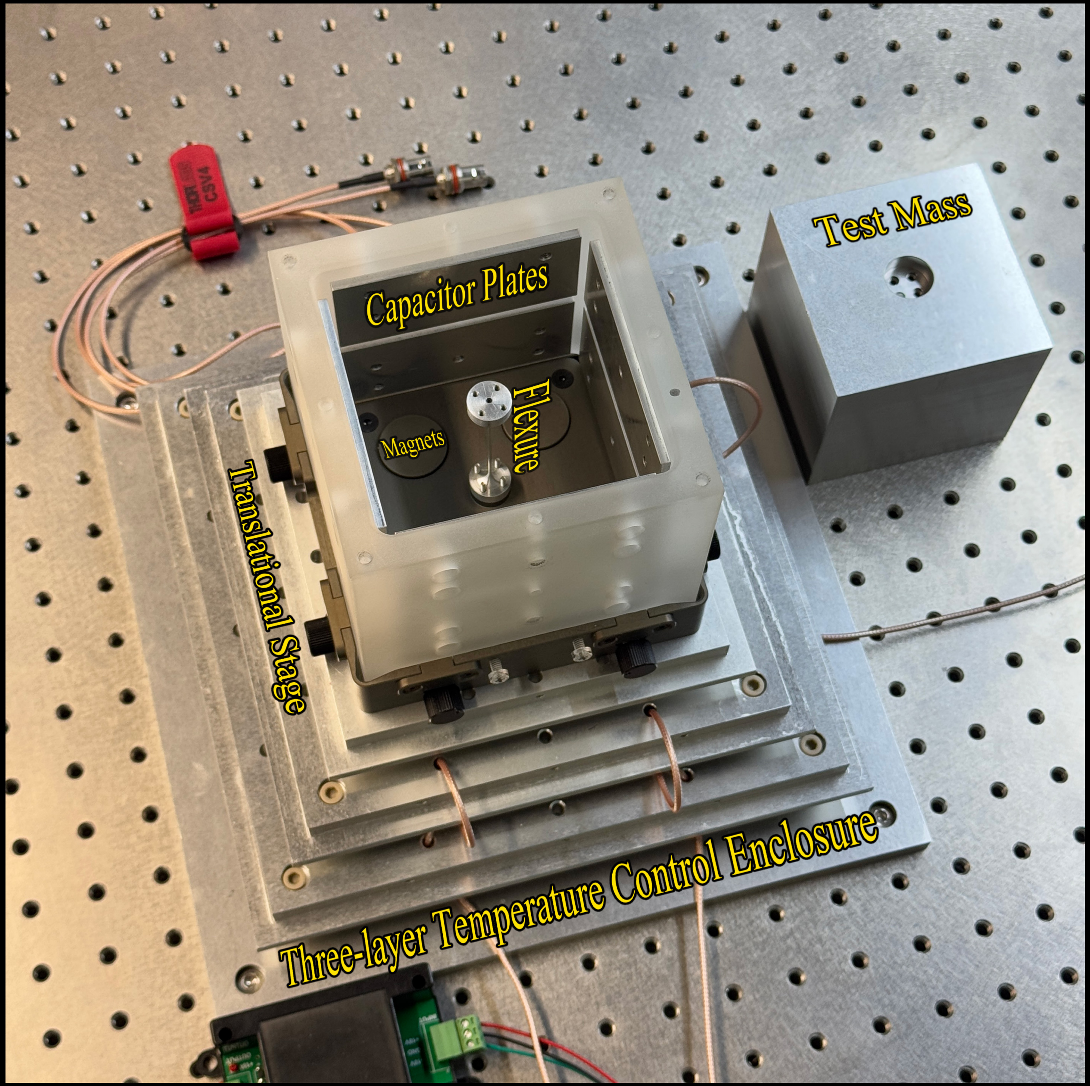


Test Mass design

- Inverted pendulum
  - Size: 8.8cm cube
  - Eigenmode:
  - 0.86Hz, 3.75Hz

- Four-degree-of-freedom response
  - (X, RY) & (Y, RX)



Test Mass design

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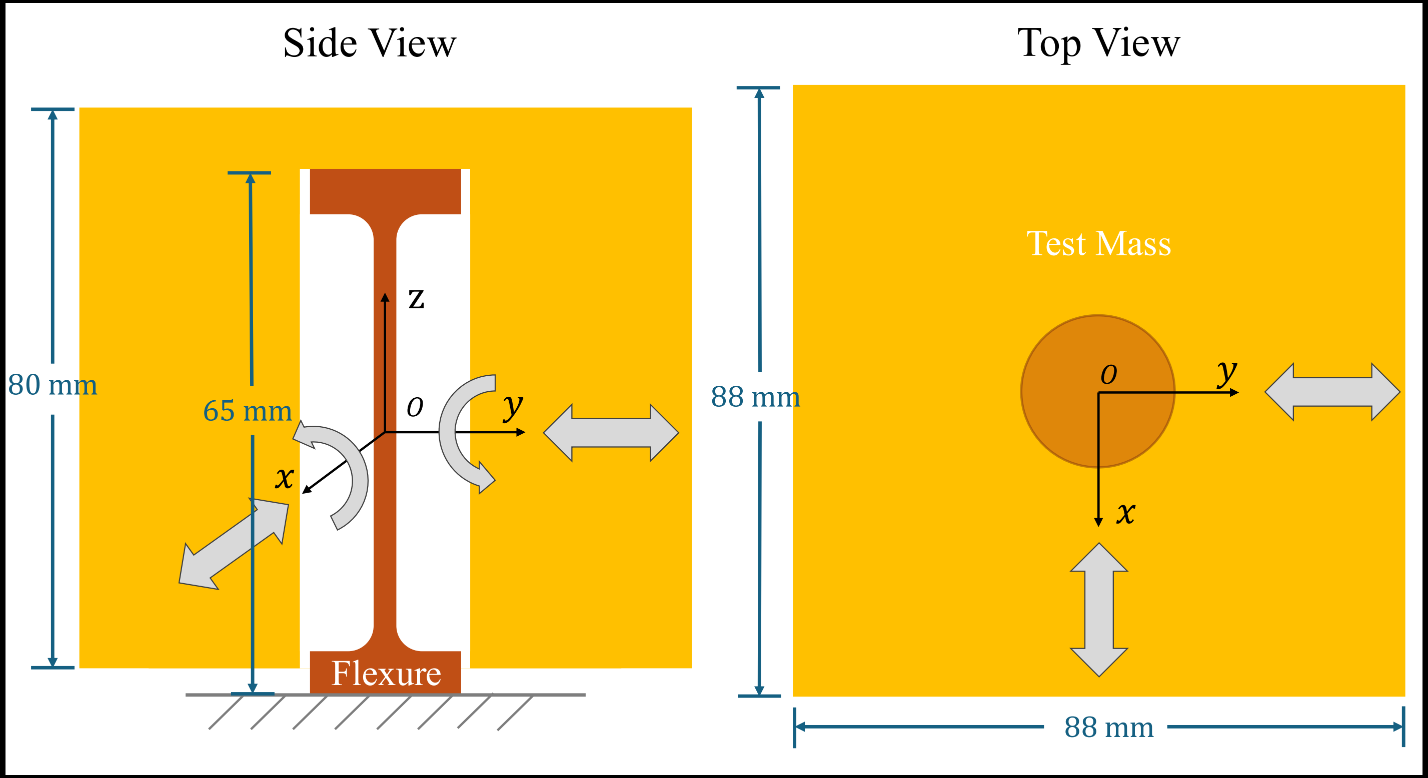
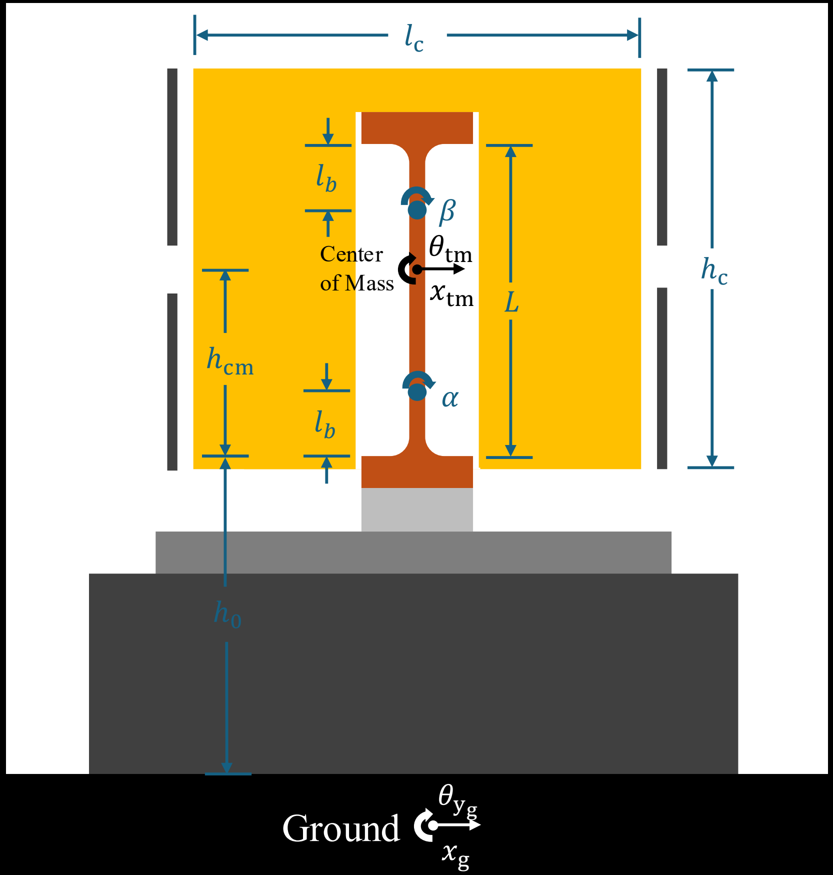
- Inverted pendulum design



- Two Bending Point Model



Dynamical Model

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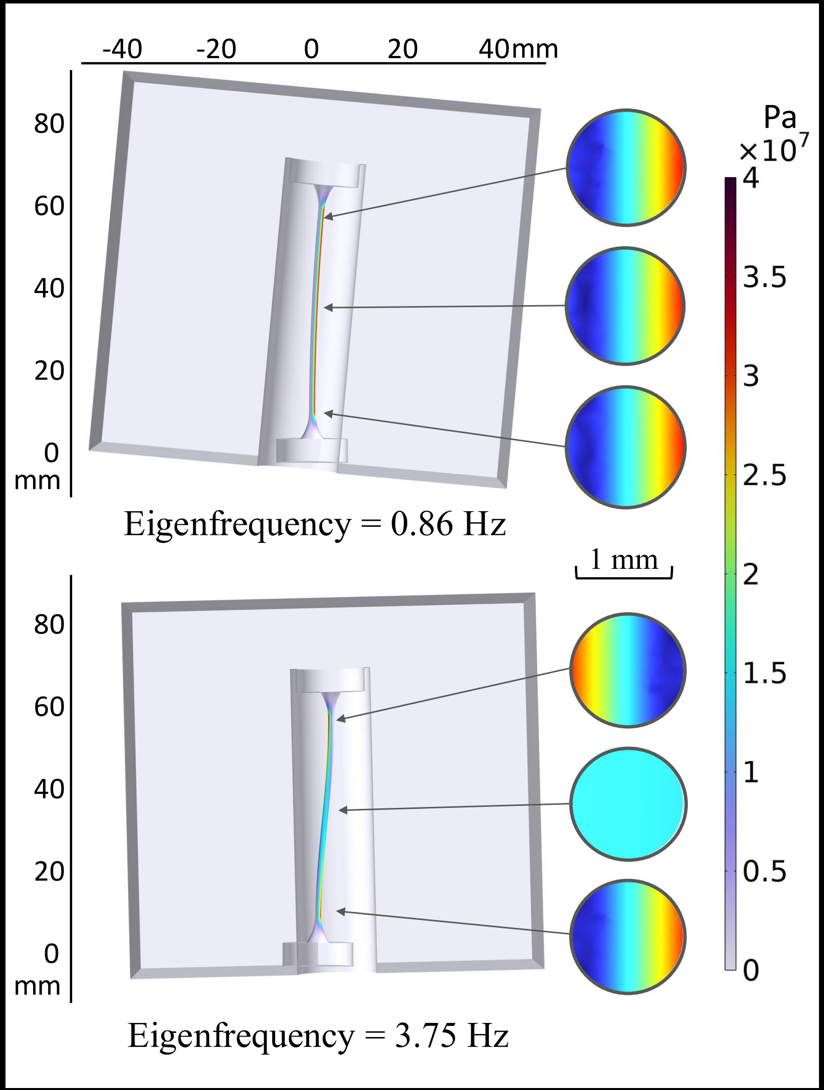
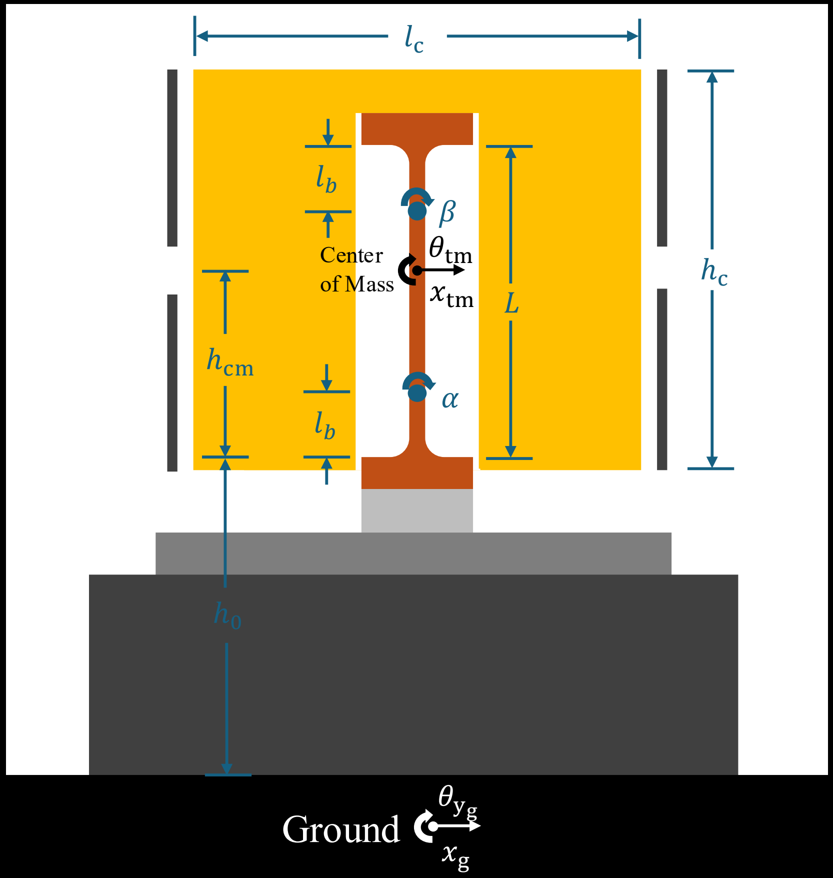


First Mode:
$\alpha * \beta > 0$

Second Mode:
$\alpha * \beta < 0$

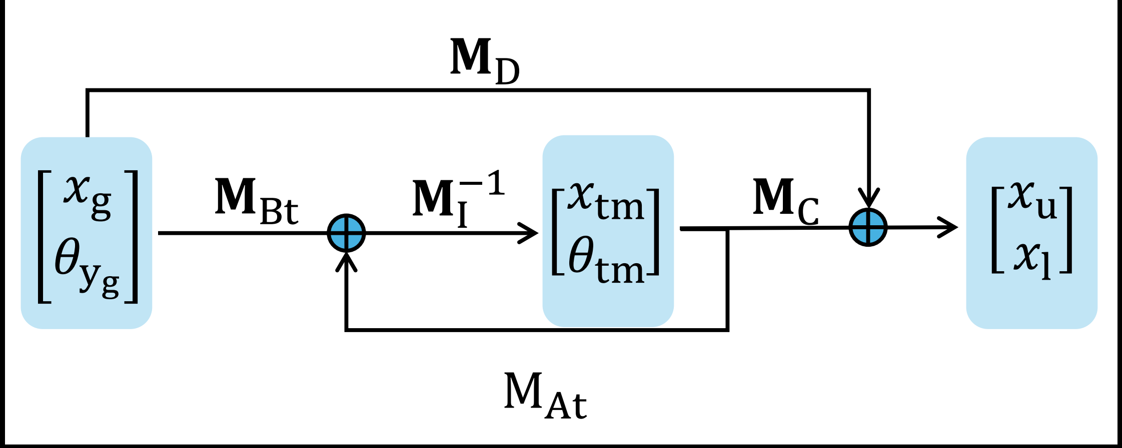
- Two Bending Point Model

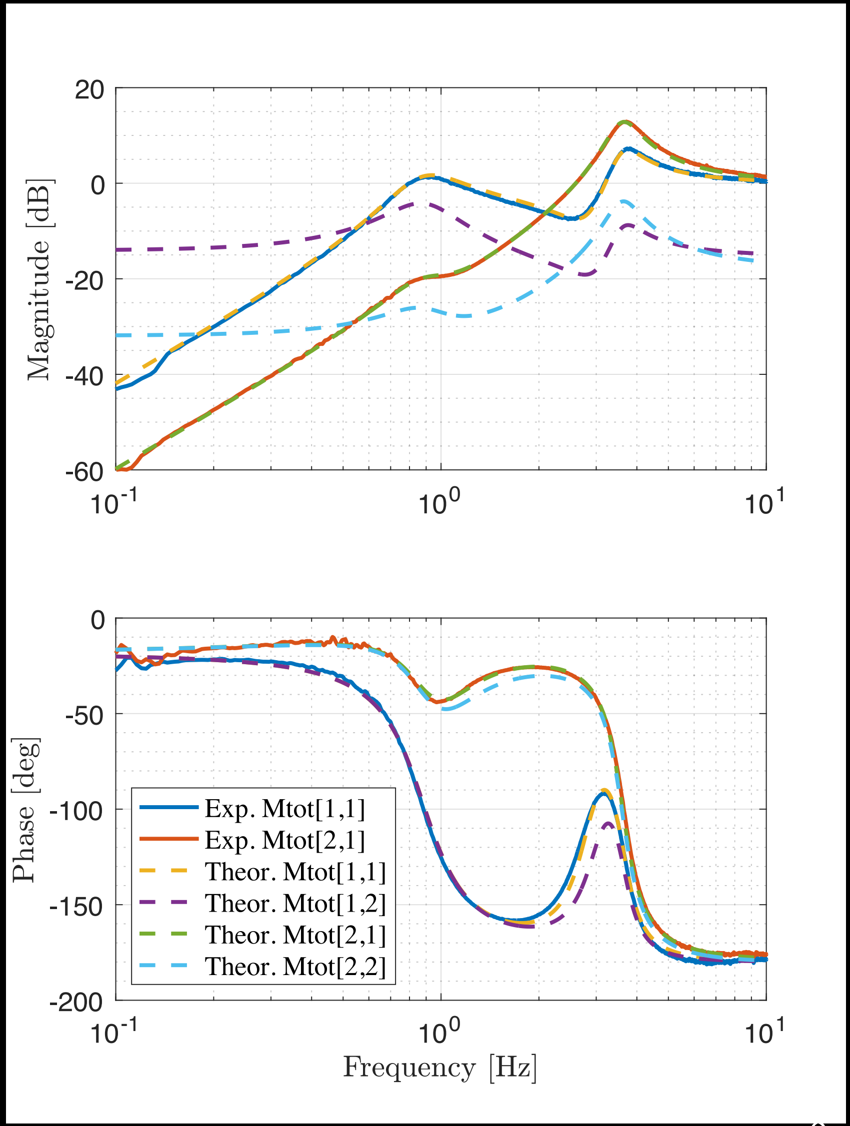


Dynamical Model

- Mechanical Transfer Function matrix

$$\begin{bmatrix} x_u \\ x_l \\ y_u \\ y_l \end{bmatrix} = \begin{bmatrix} \mathrm{M_{tot}} & 0 \\ 0 & \mathrm{M_{tot}} \end{bmatrix} \begin{bmatrix} x_g \\ \theta_{y_g} \\ y_g \\ \theta_{x_g} \end{bmatrix}$$





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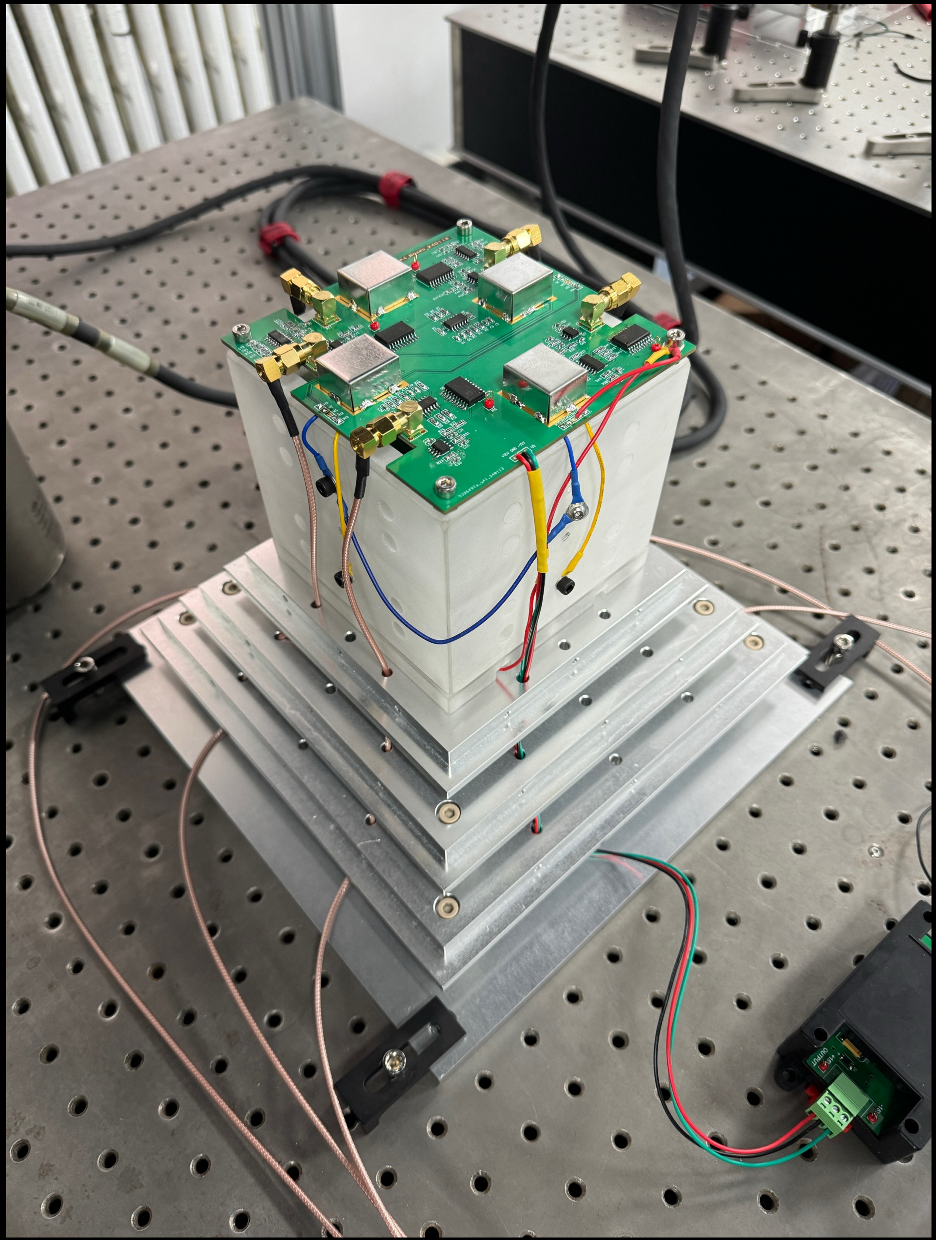

Motion Sensing design

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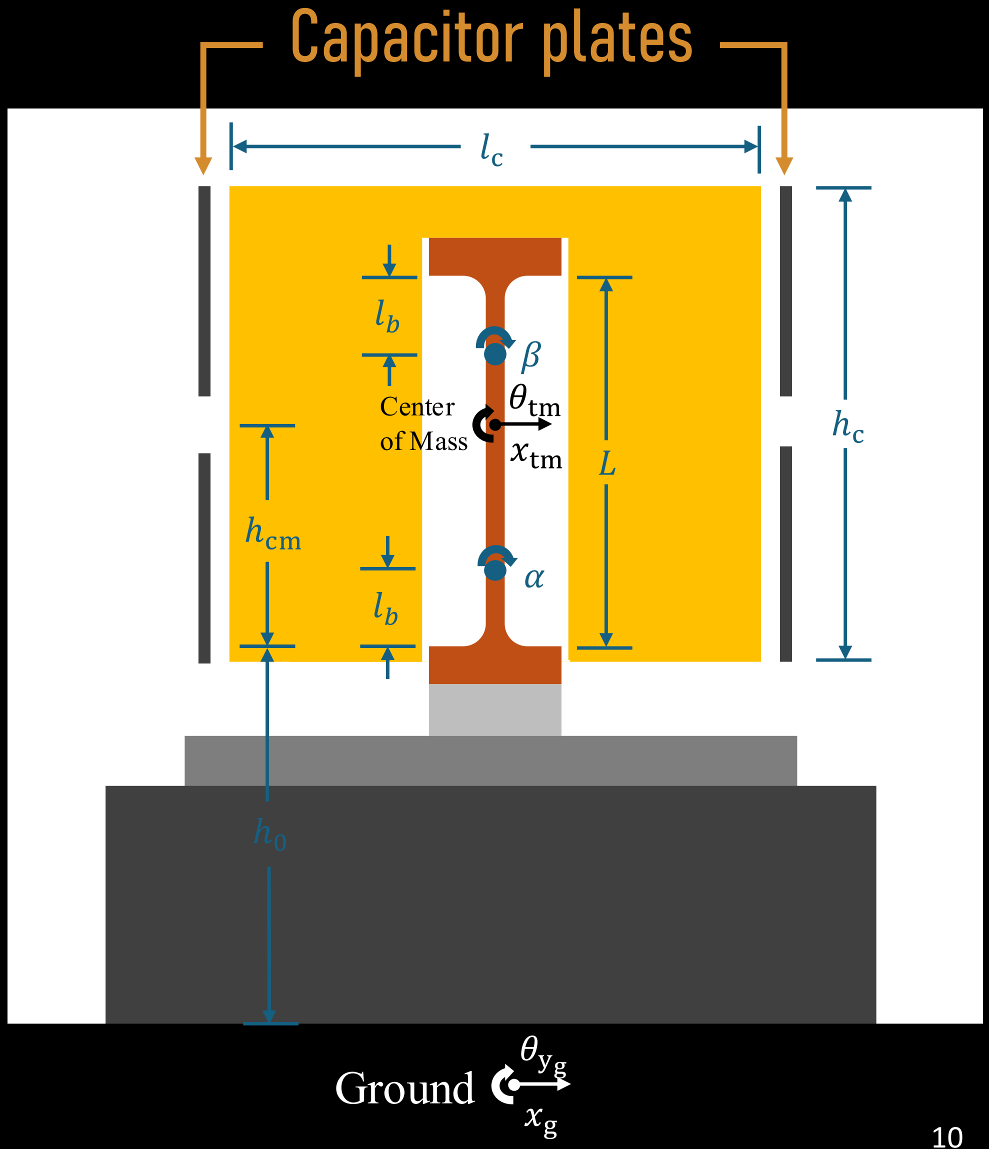
- Capacitive sensing
  - Capacitor pair
  - $C_1 \approx C_0\left(1-\frac{\Delta d}{d_0}\right), C_2 \approx C_0\left(1+\frac{\Delta d}{d_0}\right)$
- Differential capacitor bridge
  - $V \propto |C_1 - C_2| = 2\Delta C \approx \frac{2C_0}{d_0}\Delta d.$



Motion Sensing design

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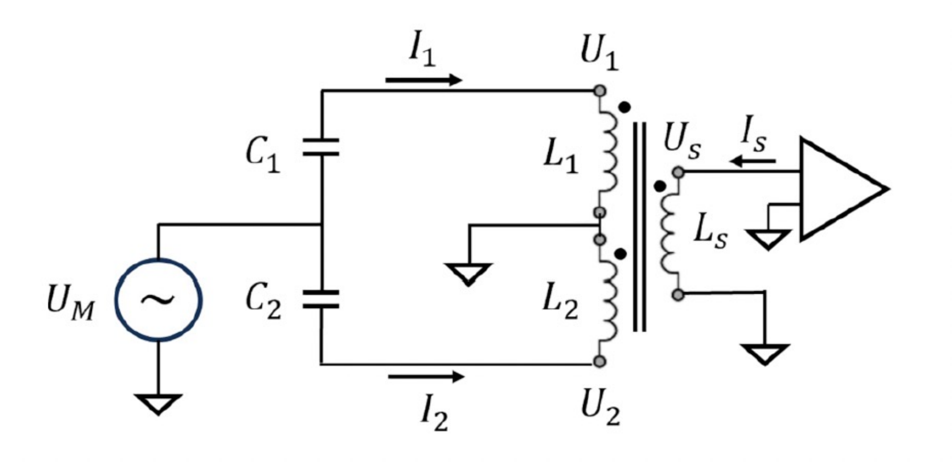
- High-frequency Modulated Bridge

For two differential capacitors $C_1 = C_0 + \Delta C$ and $C_2 = C_0 - \Delta C$, then

$$U_S = -\frac{2\omega^2 L U_M}{1 - 2\omega^2 L C_0}\Delta C.$$



And choose a modulation frequency $\omega = \frac{1}{\sqrt{LC_0}}$,

$$U_S = 2U_M\frac{\Delta C}{C_0} = 2U_M\frac{\Delta d}{d_0}.$$

Then obtain $\frac{U_S}{\Delta d} = 1.80\times10^4$ V/m as $d_0 = 500$ μm and $U_M = 4.5$ V.

Motion Sensing design

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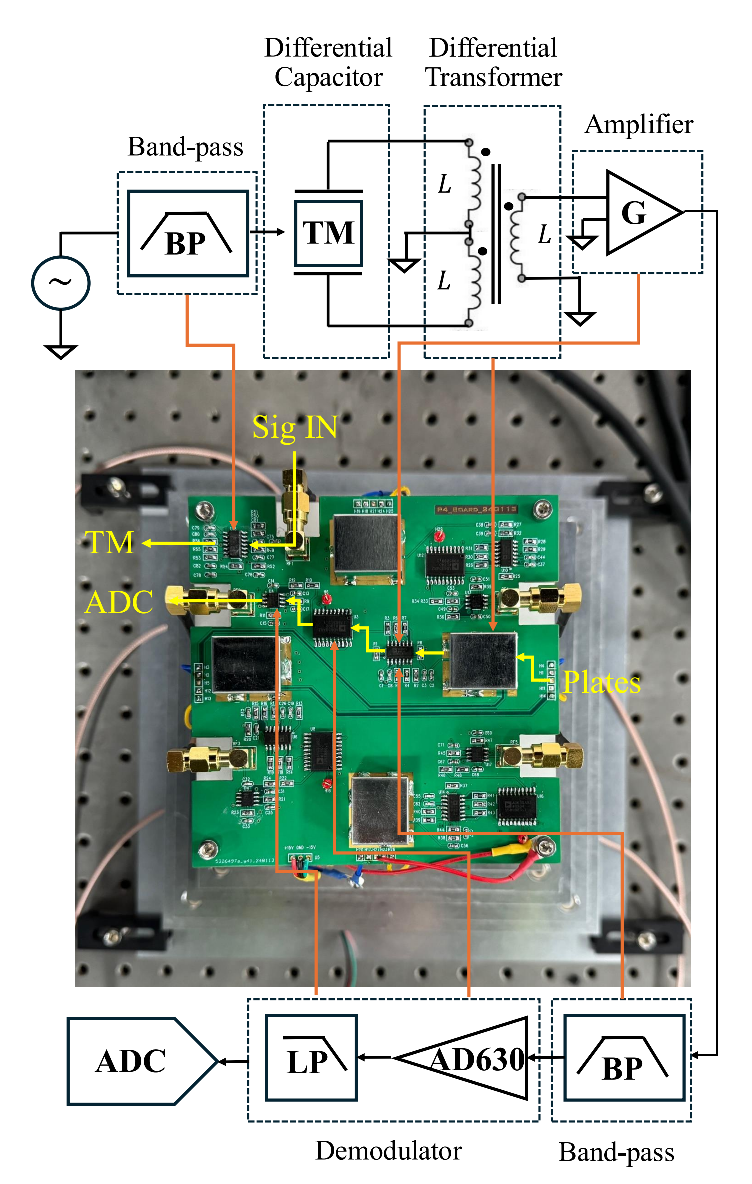


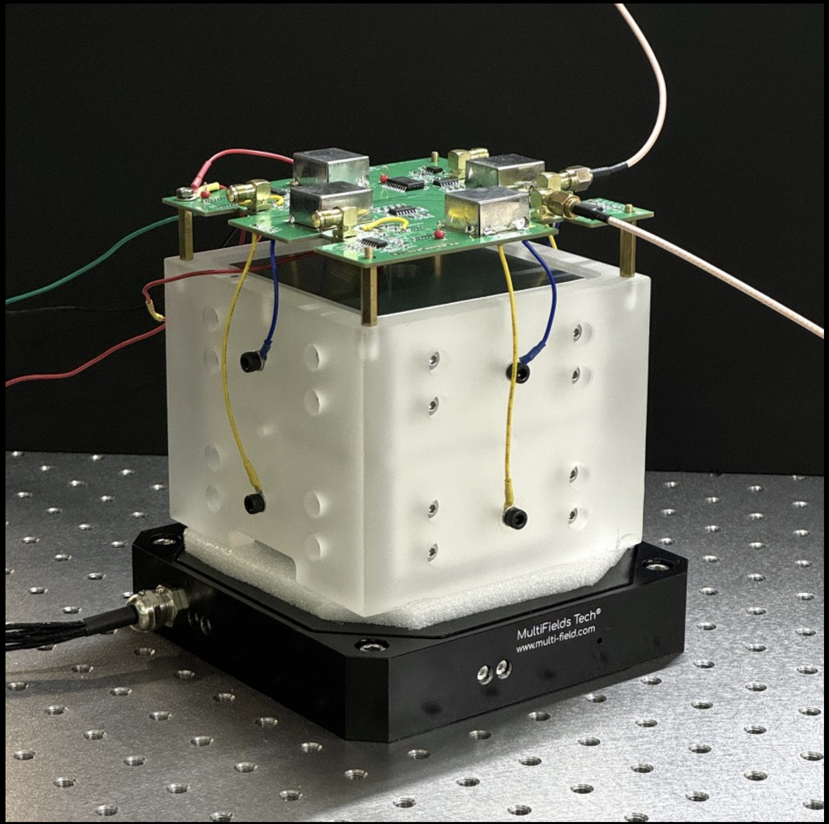
- High-frequency Modulated Bridge
  - Modulation Frequency @178 kHz
  - Amplifier Gain = 400

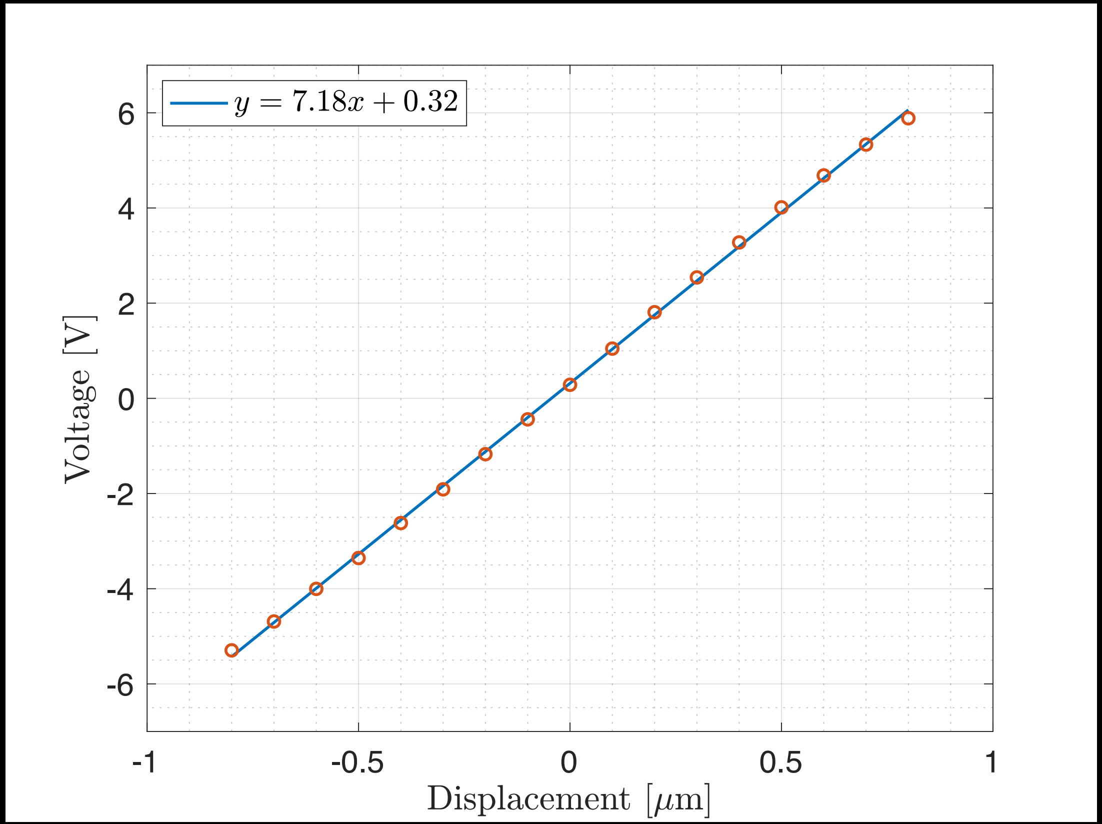
  - Analog Demodulation (AD630 Chip)
  - Physical Parameters:
    - Driving Voltage U_M = 4.5V
    - Gap = 0.5 mm

Motion Sensing design

- Displacement-to-Voltage Calibration

Theor. $7.20\ \mathrm{V/\mu m}$ Exp. $7.18 \pm 0.08\ \mathrm{V/\mu m}$



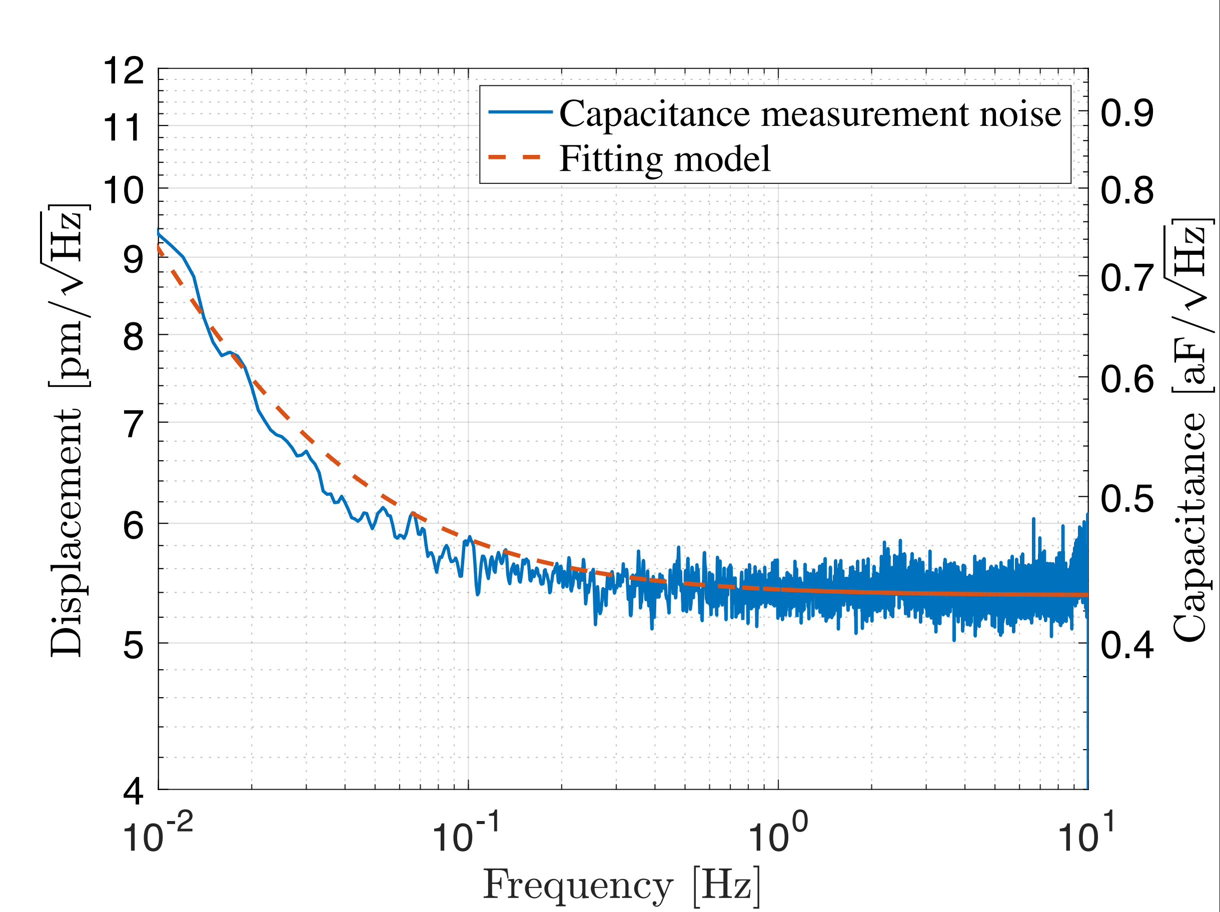


Motion Sensing design

- Range: $\pm 1\,\mu\text{m}$
- Resolution: 16 pm @10Hz
- Dynamic Range: 102 dB

1/f low-frequency component and a flat high-frequency white-noise

$$\text{PSD} = 5.38^2 \left(1 + \frac{0.02\ \text{Hz}}{f}\right) \left[\frac{\text{pm}^2}{\text{Hz}}\right].$$



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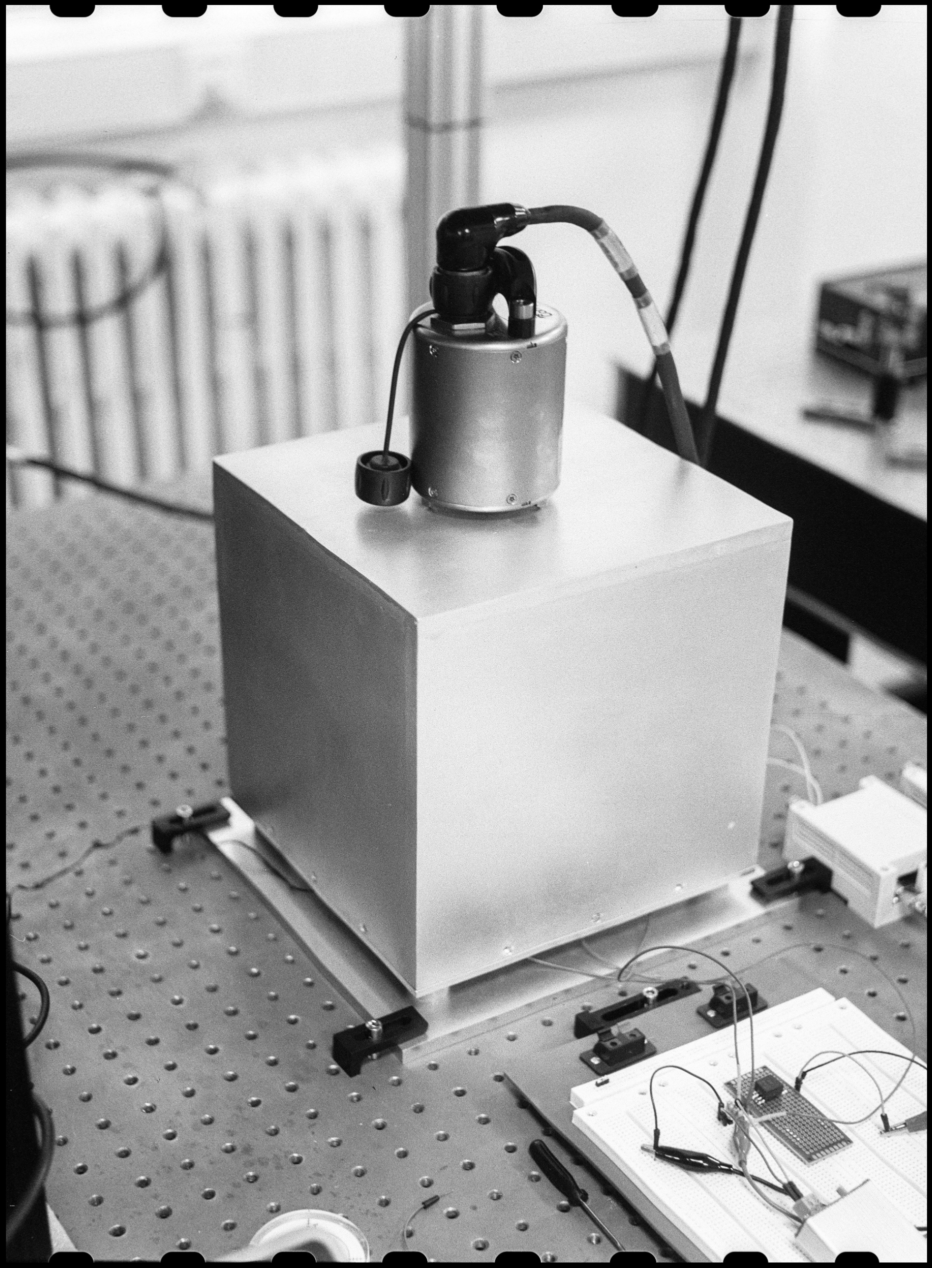

Noise Budget and Measurement

- Two Usage Scenario:
  - Low-angular-motion Scenario: 2D seismometer

$$\begin{bmatrix} x_u \\ x_l \end{bmatrix} \approx \begin{bmatrix} \mathrm{M_{tot}}[1,1] \\ \mathrm{M_{tot}}[2,1] \end{bmatrix} x_g, \qquad \begin{bmatrix} y_u \\ y_l \end{bmatrix} \approx \begin{bmatrix} \mathrm{M_{tot}}[1,1] \\ \mathrm{M_{tot}}[2,1] \end{bmatrix} y_g.$$

Low-angular-motion approximation: $\frac{g}{\omega^2}\theta_{x_g} \ll y_g,\ \frac{g}{\omega^2}\theta_{y_g} \ll x_g.$

  - High-angular-motion Scenario: 4D seismometer

$$\begin{bmatrix} x_g \\ \theta_{y_g} \\ y_g \\ \theta_{x_g} \end{bmatrix} = \begin{bmatrix} \mathrm{M_{tot}^{-1}} & 0 \\ 0 & \mathrm{M_{tot}^{-1}} \end{bmatrix} \begin{bmatrix} x_u \\ x_l \\ y_u \\ y_l \end{bmatrix}$$

Noise Budget and Measurement

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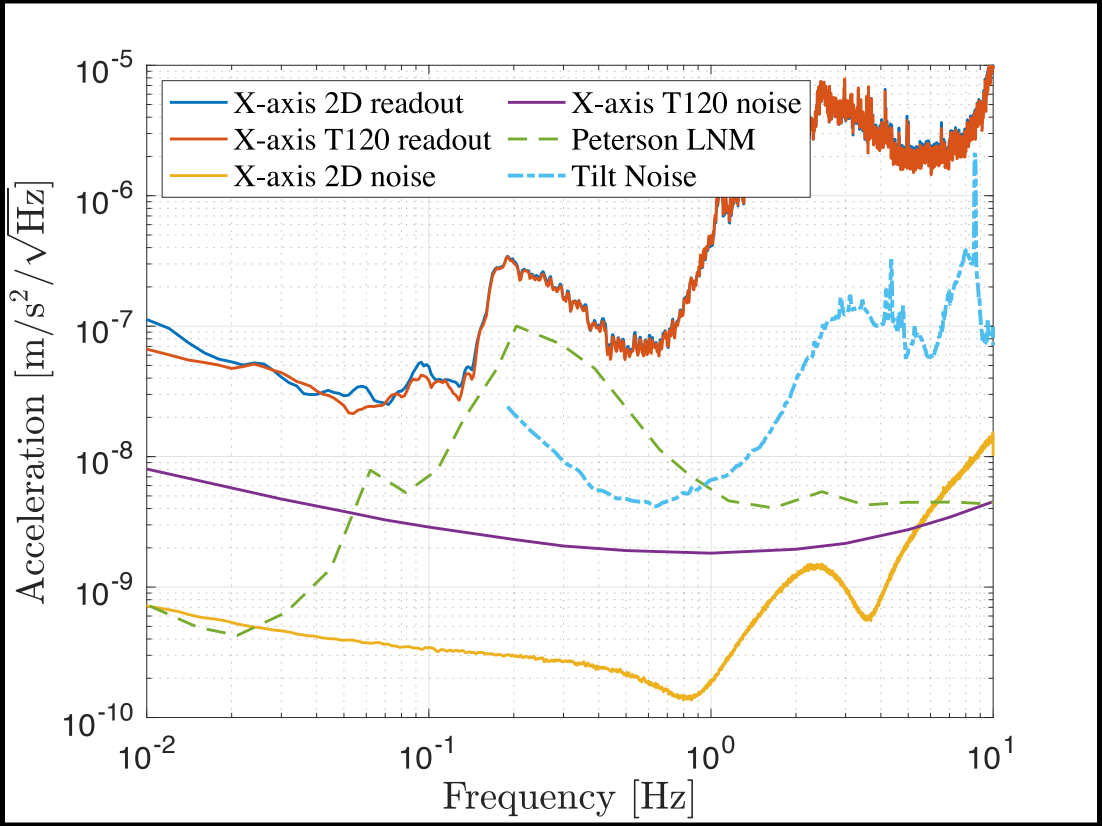
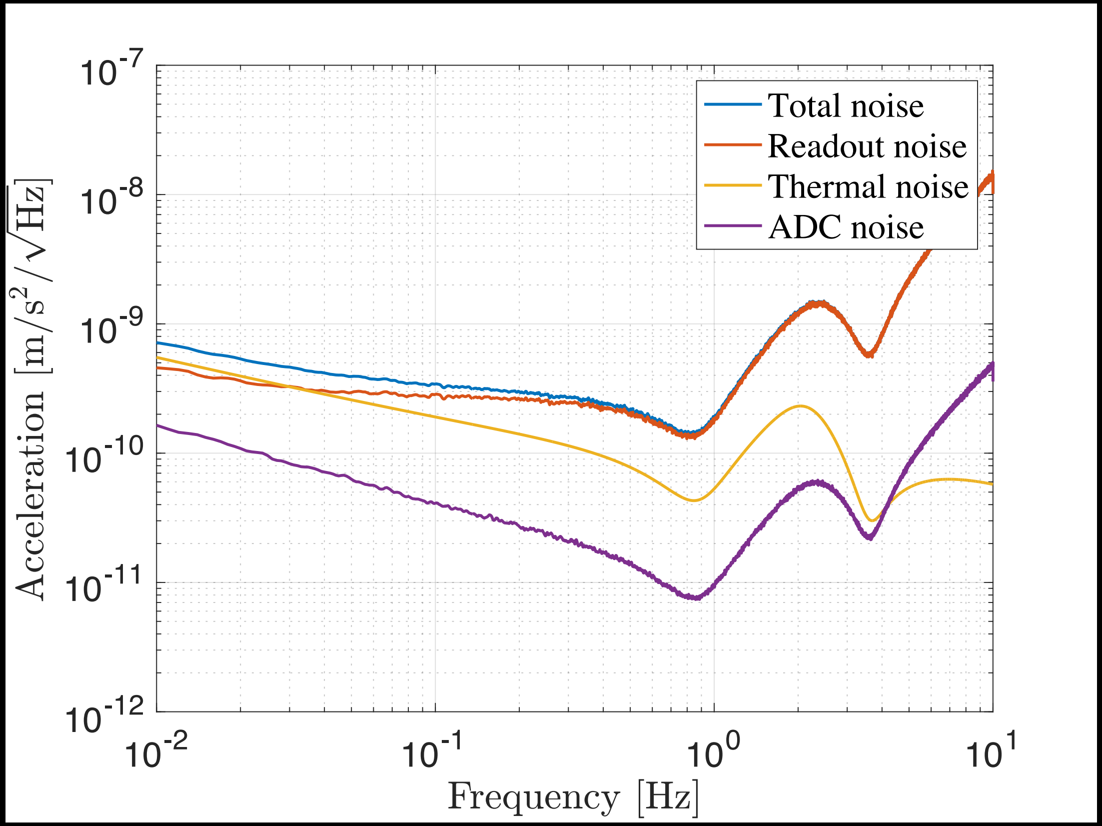
- Low-angular-motion Usage: 2D Mode





Noise Budget and Measurement

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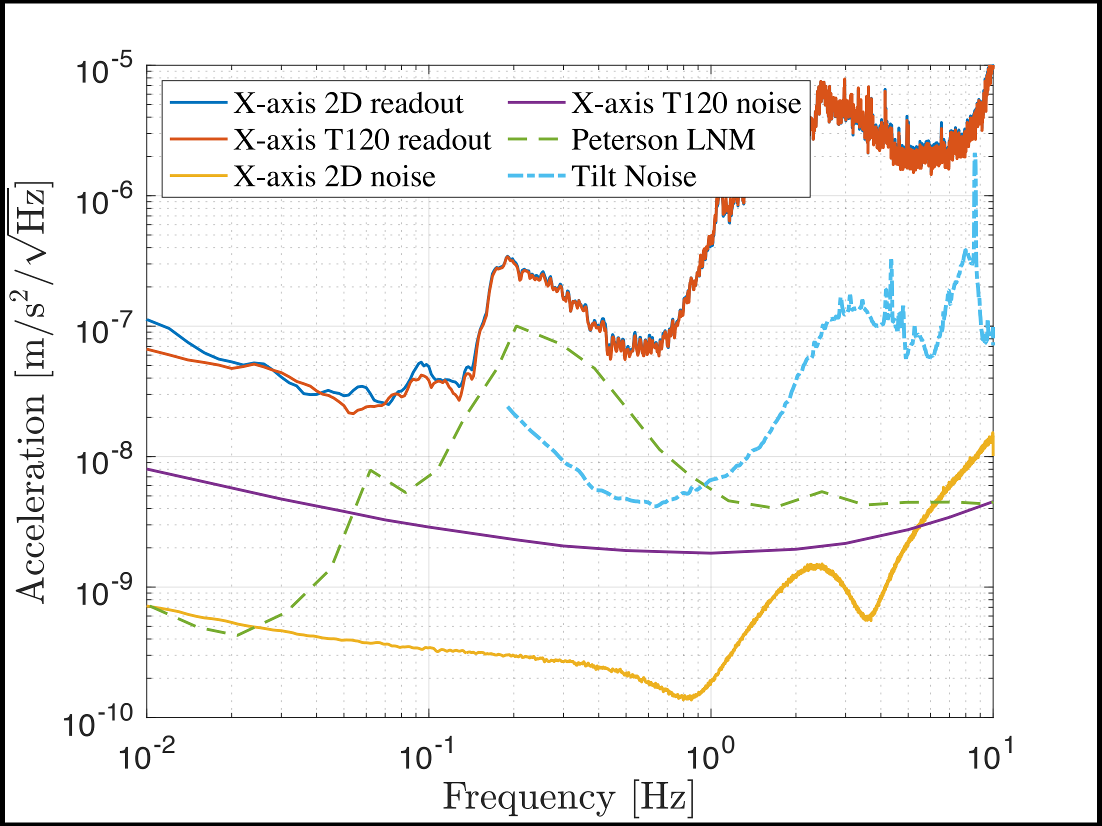
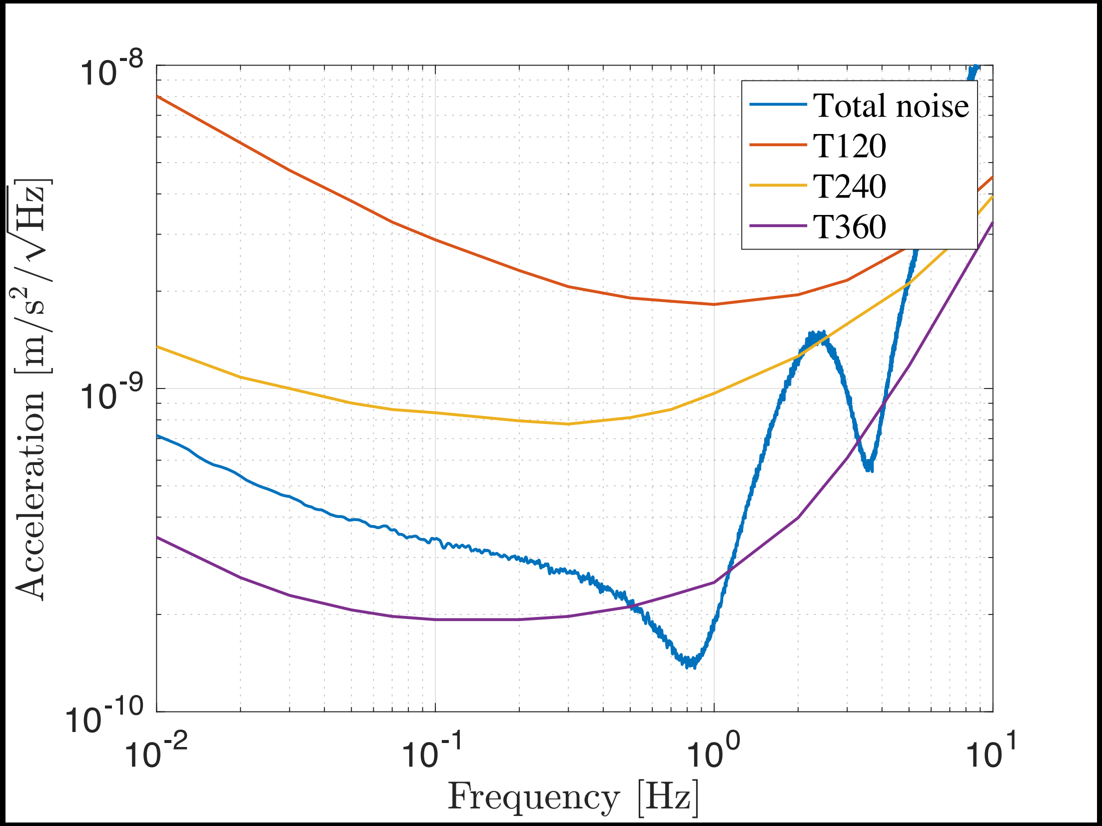
- Noise comparison in 2D Mode





Noise Budget and Measurement

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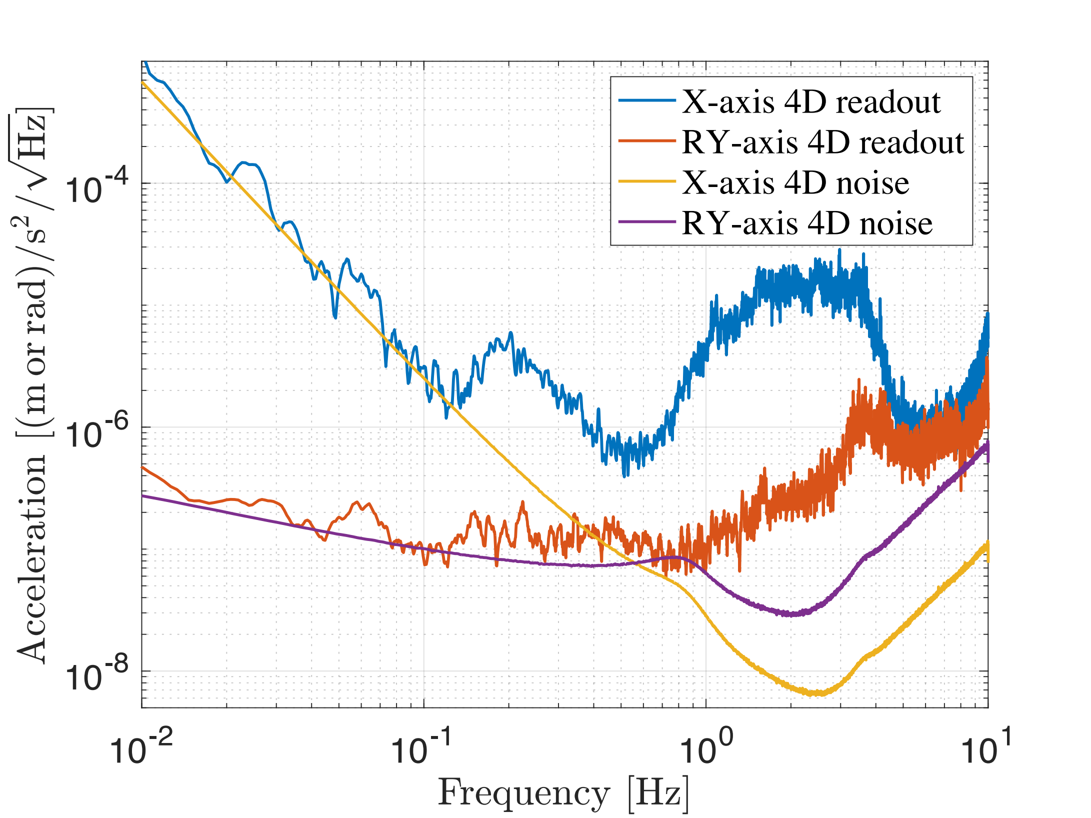
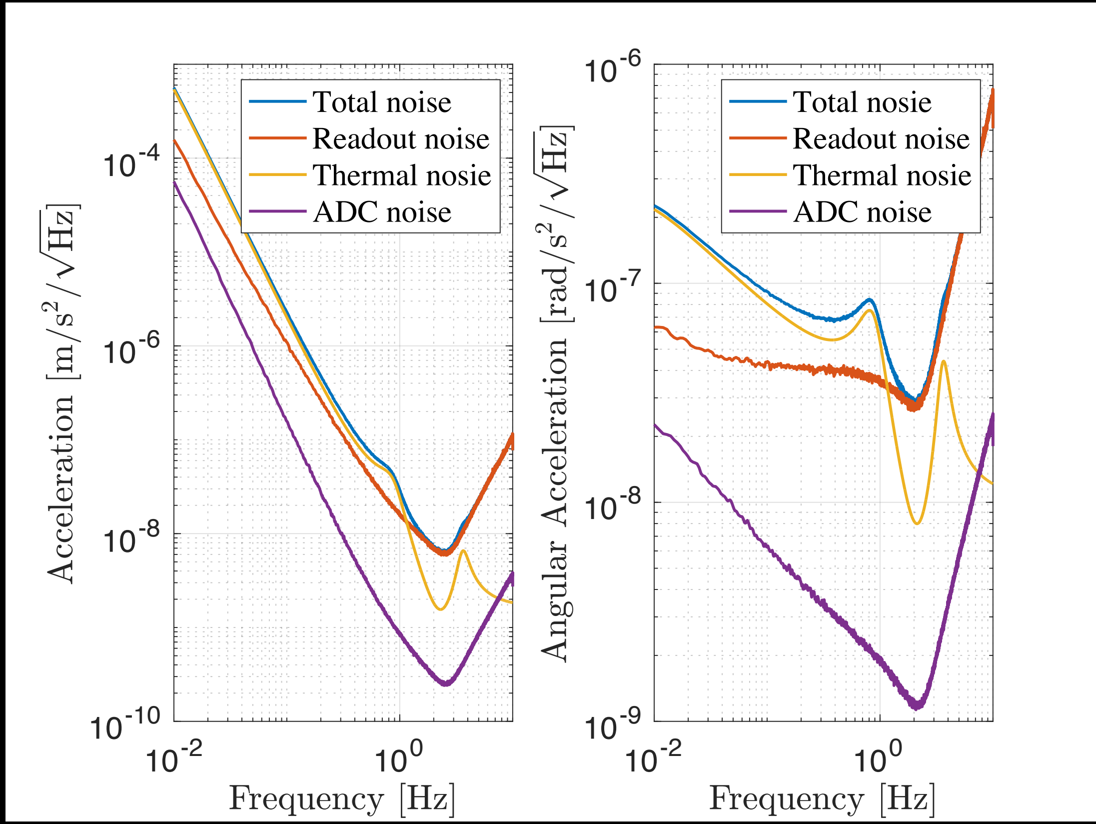
- High-angular-motion Usage: 4D Mode





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- Noise comparison in 4D Mode

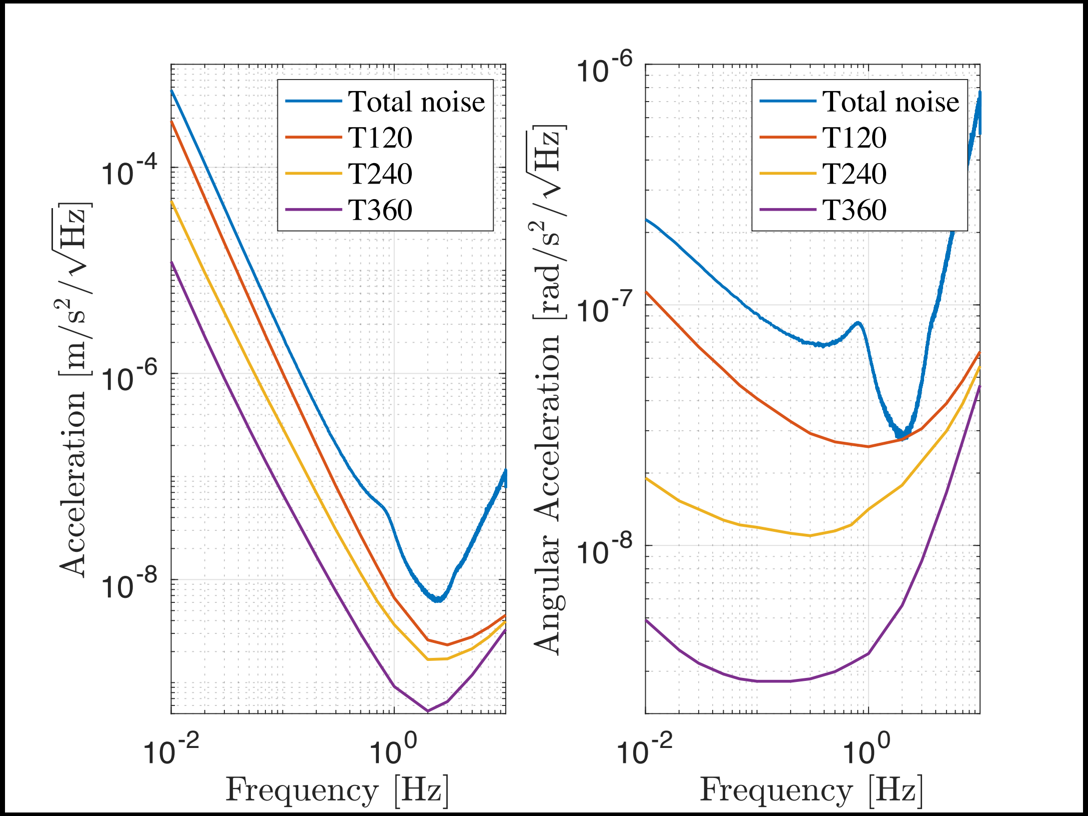
To obtain angular motion, two T120s are spatially separated along x-axis, separated by distance ΔL. Then the angular motion is estimated by $\theta_{T120} = (z_1 - z_2)/\Delta L$. The two-degree-of-freedom transfer function matrix can be written as

$$\begin{bmatrix} x_{\mathrm{T120}} \\ \theta_{\mathrm{T120}} \end{bmatrix} = \begin{bmatrix} 1 & \frac{g}{\omega^2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_g \\ \theta_{y_g} \end{bmatrix}.$$

The decoupled noise spectrum for the translational and angular motion can be obtained by inverting the above matrix, leading to

$$S^x_{\mathrm{T120}} = S_{\mathrm{T120}} \left(1 + \frac{2g^2}{\omega^4 \Delta L^2}\right),$$

$$S^\theta_{\mathrm{T120}} = S_{\mathrm{T120}} \frac{2}{\Delta L^2}.$$



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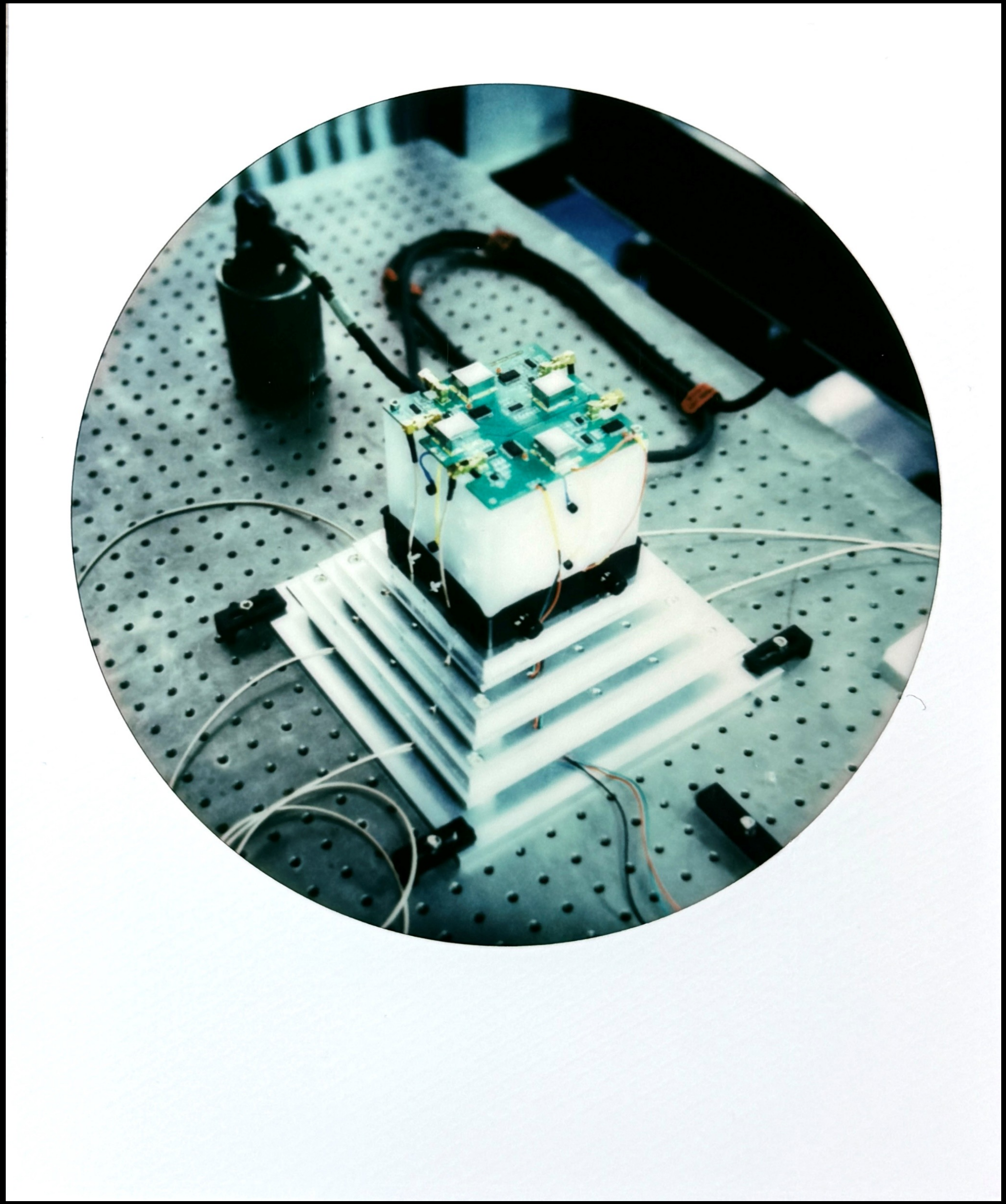

Conclusion

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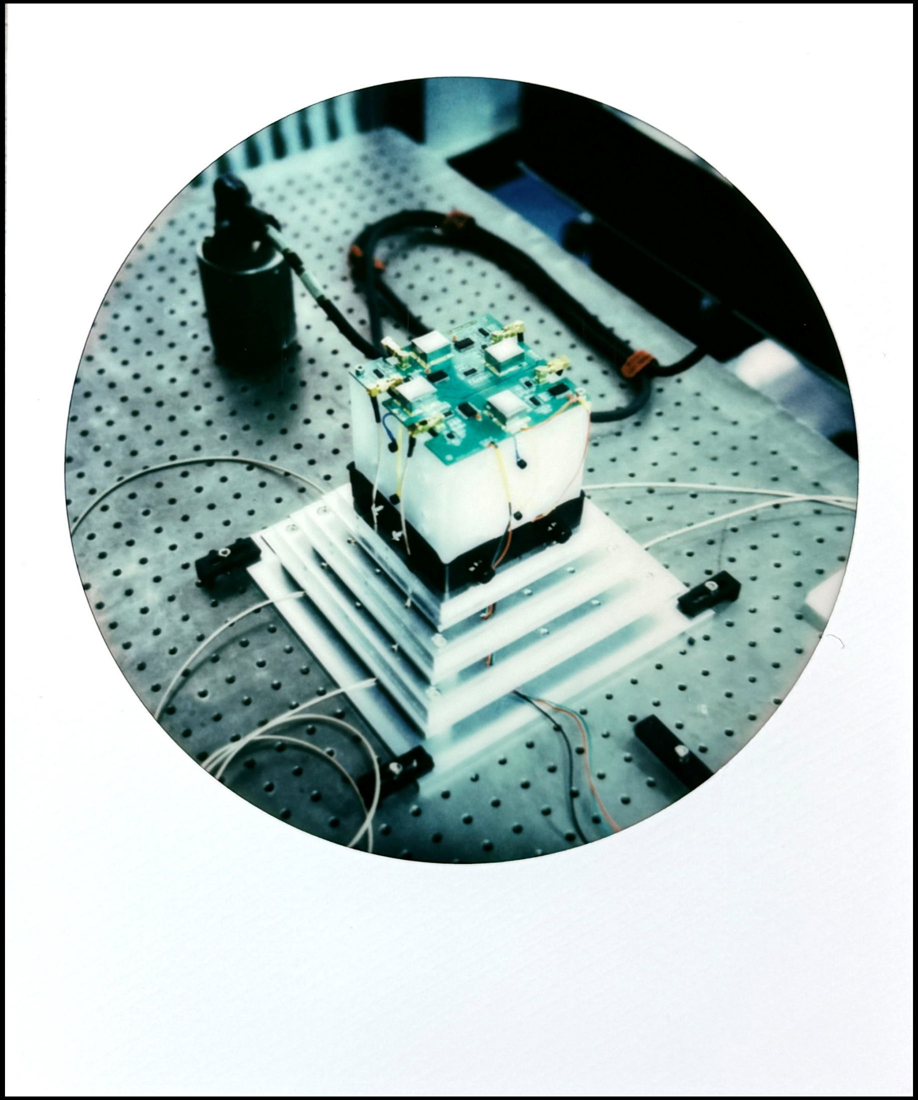
- Compact size
- Low frequency performance
- Four-degree-of-freedom response

- Active vibration isolation system
- Inertial control of satellites



Thank you