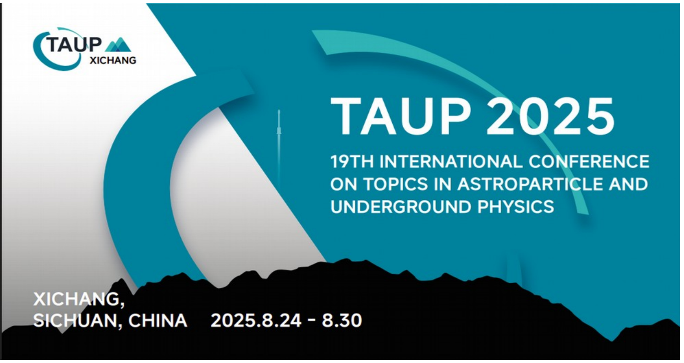


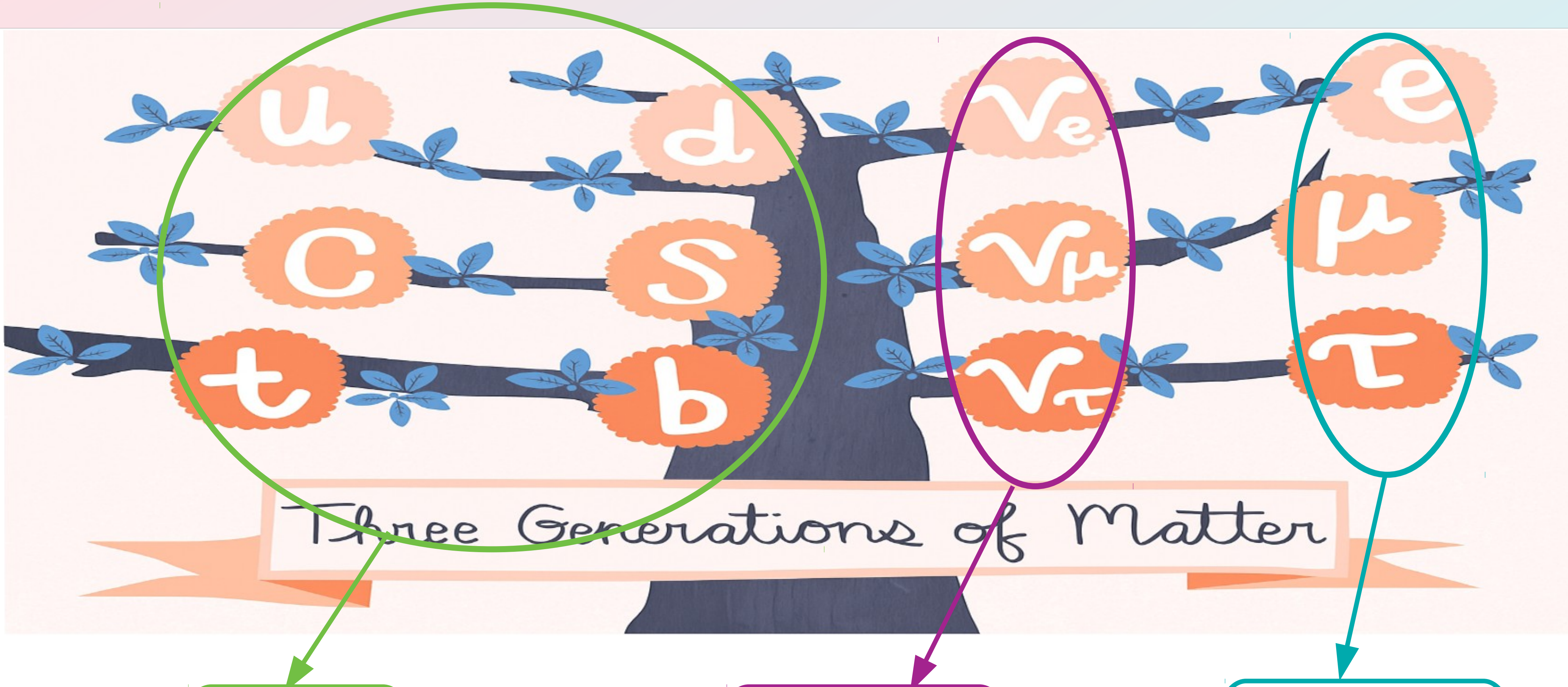
Charged lepton flavor violation with light dark matter and muonium invisible decay

(Based on **2507.13875**)
In collarobation with Yi-Liao and Xiao-Dong Ma

Sahabub Jahedi
Institute of Quantum Matter
South China Normal University

Date : 26th August, 2025

Three Flavors in Standard Model

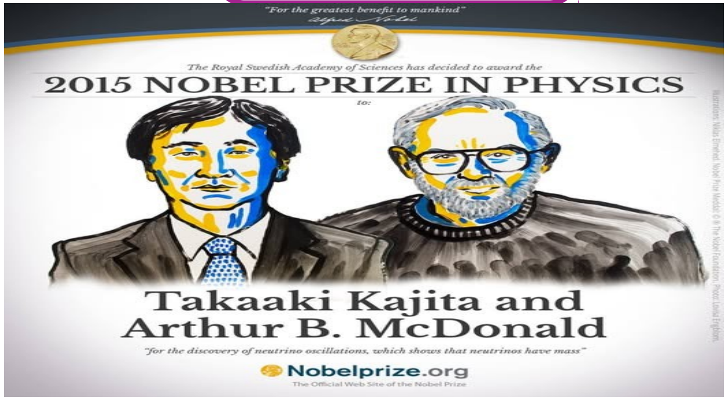


CKM Matrix

Neutrino Oscillation

Any mixing???

Charged lepton Flavor Violation !!!



"For the greatest benefit to mankind"
Alfred Nobel

The Royal Swedish Academy of Sciences has decided to award the

2015 NOBEL PRIZE IN PHYSICS

to:

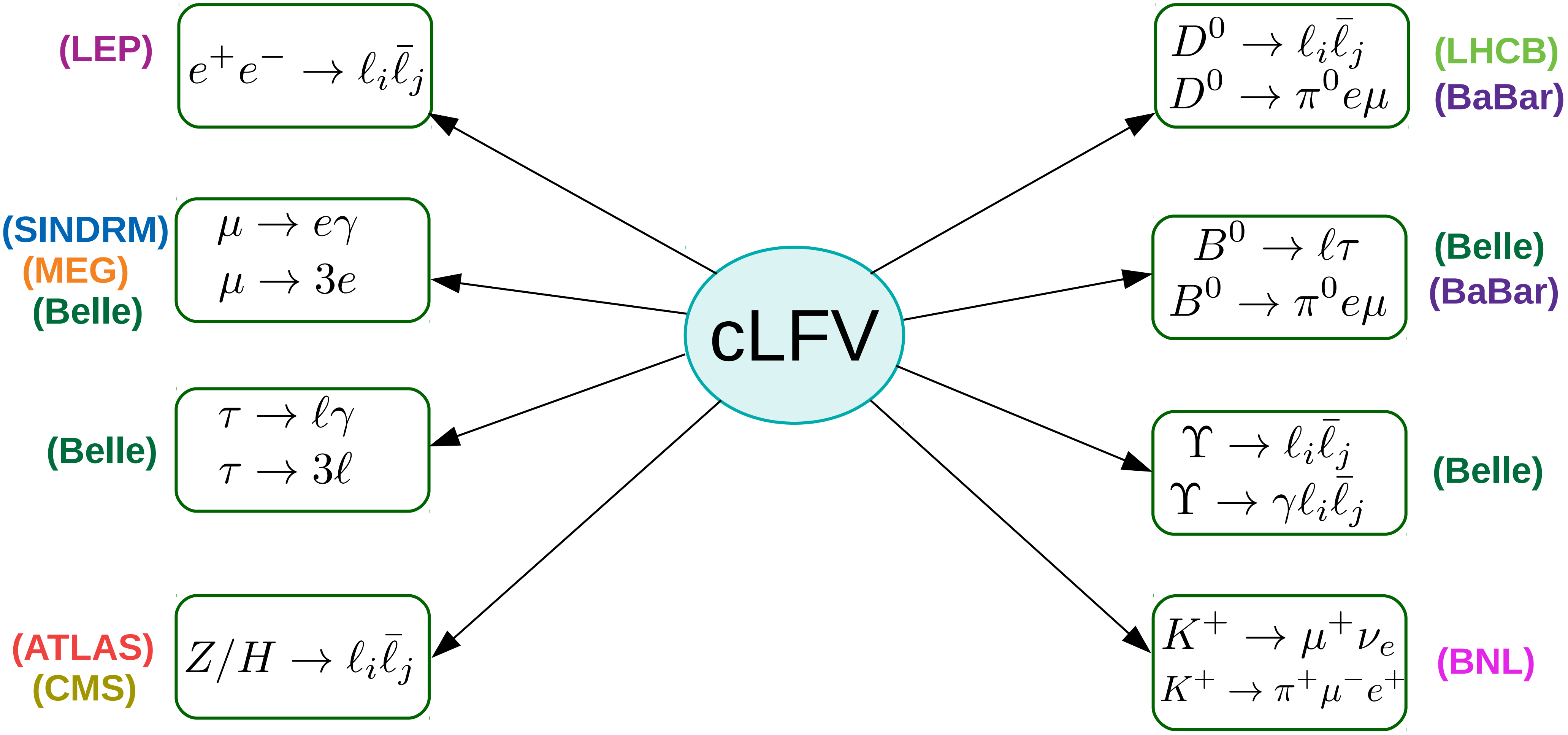
Takaaki Kajita and
Arthur B. McDonald

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

Nobelprize.org
The Official Web Site of the Nobel Prize

Charged Lepton Flavor Violation

Experiments



cLFV

(LEP) $e^+e^- \to \ell_i\bar{\ell}_j$

(SINDRM) (MEG) (Belle) $\mu \to e\gamma$ $\mu \to 3e$

(Belle) $\tau \to \ell\gamma$ $\tau \to 3\ell$

(ATLAS) (CMS) $Z/H \to \ell_i\bar{\ell}_j$

$D^0 \to \ell_i\bar{\ell}_j$ $D^0 \to \pi^0 e\mu$ (LHCB) (BaBar)

$B^0 \to \ell\tau$ $B^0 \to \pi^0 e\mu$ (Belle) (BaBar)

$\Upsilon \to \ell_i\bar{\ell}_j$ $\Upsilon \to \gamma\ell_i\bar{\ell}_j$ (Belle)

$K^+ \to \mu^+\nu_e$ $K^+ \to \pi^+\mu^-e^+$ (BNL)

Lepton invisible decays in the SM and beyond

Standard Model: **Flavor conserving**

$$\mathcal{B}(\mu \to e\bar{\nu}_e\nu_\mu) \sim 100\%$$

$$\left.\begin{array}{l}\mathcal{B}(\tau \to e\bar{\nu}_e\nu_\tau) = (17.82 \pm 0.04)\% \\ \mathcal{B}(\tau \to \mu\bar{\nu}_\mu\nu_\tau) = (17.39 \pm 0.04)\%\end{array}\right\}$$

PDG Average

Upper bound: **Flavor violating**

$$\mathcal{B}(\mu \to e\nu_e\bar{\nu}_\mu) < 1.2 \times 10^{-2} (95\% \text{ C.L.}) \Big\}$$

Neutrino oscillation data

$$\left.\begin{array}{l}\mathcal{B}(\tau \to e + \text{inv}) < 9 \times 10^{-4} (95\% \text{ C.L.}) \\ \mathcal{B}(\tau \to \mu + \text{inv}) < 6 \times 10^{-4} (95\% \text{ C.L.})\end{array}\right\}$$

Belle data

Aim :

Light Dark Matter phenomenology with in
Dark Sector Effective Field Theory (DSEFT) framework.

Dark Sector Effective Field Theory (DSEFT)

Symmetry: SU(3)$_c$ X U(1)$_{em}$

Scalar DM:

$$\mathcal{O}_{\ell\phi}^{S,ji} = (\overline{\ell_j}\ell_i)(\phi^\dagger\phi)$$

$$\mathcal{O}_{\ell\phi}^{P,ji} = (\overline{\ell_j}i\gamma_5\ell_i)(\phi^\dagger\phi),$$

$$\mathcal{O}_{\ell\phi}^{V,ji} = (\overline{\ell_j}\gamma^\mu\ell_i)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi),$$ **(X)**

$$O_{\ell\phi}^{A,ji} = (\overline{\ell_j}\gamma^\mu\gamma_5\ell_i)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi),$$ **(X)**

Fermion DM:

$$\mathcal{O}_{\ell\chi1}^{S,ji} = (\overline{\ell_j}\ell_i)(\overline{\chi}\chi),$$

$$\mathcal{O}_{\ell\chi2}^{S,ji} = (\overline{\ell_j}\ell_i)(\overline{\chi}i\gamma_5\chi)$$

$$\mathcal{O}_{\ell\chi1}^{P,ji} = (\overline{\ell_j}i\gamma_5\ell_i)(\overline{\chi}\chi),$$

$$\mathcal{O}_{\ell\chi2}^{P,ji} = (\overline{\ell_j}\gamma_5\ell_i)(\overline{\chi}\gamma_5\chi),$$

$$\mathcal{O}_{\ell\chi2}^{V,ji} = (\overline{\ell_j}\gamma^\mu\ell_i)(\overline{\chi}\gamma_\mu\gamma_5\chi),$$

$$\mathcal{O}_{\ell\chi1}^{V,ji} = (\overline{\ell_j}\gamma^\mu\ell_i)(\overline{\chi}\gamma_\mu\chi),$$ **(X)**

$$\mathcal{O}_{\ell\chi1}^{A,ji} = (\overline{\ell_j}\gamma^\mu\gamma_5\ell_i)(\overline{\chi}\gamma_\mu\chi),$$ **(X)**

$$\mathcal{O}_{\ell\chi2}^{A,ji} = (\overline{\ell_j}\gamma^\mu\gamma_5\ell_i)(\overline{\chi}\gamma_\mu\gamma_5\chi),$$

$$\mathcal{O}_{\ell\chi1}^{T,ji} = (\overline{\ell_j}\sigma^{\mu\nu}\ell_i)(\overline{\chi}\sigma_{\mu\nu}\chi),$$ **(X)**

$$\mathcal{O}_{\ell\chi2}^{T,ji} = (\overline{\ell_j}\sigma^{\mu\nu}\ell_i)(\overline{\chi}\sigma_{\mu\nu}\gamma_5\chi),$$ **(X)**

(X) means these operators vanish for **real (Majorana) scalar** and **vector (fermion) DM**

$$A\overleftrightarrow{D_\mu}B \equiv A(D_\mu B) - (D_\mu A)B$$

Dark Sector Effective Field Theory (DSEFT)

Vector DM-Case A:

$$\mathcal{O}^{S,ji}_{\ell X} = (\overline{\ell_j}\ell_i)(X^\dagger_\mu X^\mu),$$

$$\mathcal{O}^{T,ji}_{\ell X1} = \frac{i}{2}(\overline{\ell_j}\sigma^{\mu\nu}\ell_i)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu),$$ **(X)**

$$\mathcal{O}^{V,ji}_{\ell X1} = \frac{1}{2}[\overline{\ell_j}\gamma_{(\mu}i\overleftrightarrow{D}_{\nu)}\ell_i](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu),$$

$$\mathcal{O}^{V,ji}_{\ell X2} = (\overline{\ell_j}\gamma_\mu\ell_i)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu),$$

$$\mathcal{O}^{V,ji}_{\ell X3} = (\overline{\ell_j}\gamma_\mu\ell_i)(X^\dagger_\rho\overleftrightarrow{\partial}_\nu X_\sigma)\epsilon^{\mu\nu\rho\sigma},$$

$$\mathcal{O}^{V,ji}_{\ell X4} = (\overline{\ell_j}\gamma^\mu\ell_i)(X^\dagger_\nu i\overleftrightarrow{\partial}_\mu X^\nu),$$ **(X)**

$$\mathcal{O}^{V,ji}_{\ell X5} = (\overline{\ell_j}\gamma_\mu\ell_i)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu),$$ **(X)**

$$\mathcal{O}^{V,ji}_{\ell X6} = (\overline{\ell_j}\gamma_\mu\ell_i)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma},$$ **(X)**

$$\mathcal{O}^{P,ji}_{\ell X} = (\overline{\ell_j}i\gamma_5\ell_i)(X^\dagger_\mu X^\mu),$$

$$\mathcal{O}^{T,ji}_{\ell X2} = \frac{1}{2}(\overline{\ell_j}\sigma^{\mu\nu}\gamma_5\ell_i)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu),$$ **(X)**

$$\mathcal{O}^{A,ji}_{\ell X1} = \frac{1}{2}[\overline{\ell_j}\gamma_{(\mu}\gamma_5 i\overleftrightarrow{D}_{\nu)}\ell](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu),$$

$$\mathcal{O}^{A,ji}_{\ell X2} = (\overline{\ell_j}\gamma_\mu\gamma_5\ell_i)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu),$$

$$\mathcal{O}^{A,ji}_{\ell X3} = (\overline{\ell_j}\gamma_\mu\gamma_5\ell_i)(X^\dagger_\rho\overleftrightarrow{\partial}_\nu X_\sigma)\epsilon^{\mu\nu\rho\sigma},$$

$$\mathcal{O}^{A,ji}_{\ell X4} = (\overline{\ell_j}\gamma^\mu\gamma_5\ell_i)(X^\dagger_\nu i\overleftrightarrow{\partial}_\mu X^\nu),$$ **(X)**

$$\mathcal{O}^{A,ji}_{\ell X5} = (\overline{\ell_j}\gamma_\mu\gamma_5\ell_i)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu),$$ **(X)**

$$\mathcal{O}^{A,ji}_{\ell X6} = (\overline{\ell_j}\gamma_\mu\gamma_5\ell_i)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma},$$ **(X)**

Vector DM-Case B:

$$\tilde{\mathcal{O}}^{S,ji}_{\ell X1} = (\overline{\ell_j}\ell_i)X^\dagger_{\mu\nu}X^{\mu\nu},$$

$$\tilde{\mathcal{O}}^{S,ji}_{\ell X2} = (\overline{\ell_j}\ell_i)X^\dagger_{\mu\nu}\tilde{X}^{\mu\nu},$$

$$\tilde{\mathcal{O}}^{T,ji}_{\ell X1} = \frac{i}{2}(\overline{\ell_j}\sigma^{\mu\nu}\ell_i)(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu),$$ **(X)**

$$\tilde{\mathcal{O}}^{P,ji}_{\ell X1} = (\overline{\ell_j}i\gamma_5\ell_i)X^\dagger_{\mu\nu}X^{\mu\nu},$$

$$\tilde{\mathcal{O}}^{P,ji}_{\ell X2} = (\overline{\ell_j}i\gamma_5\ell_i)X^\dagger_{\mu\nu}\tilde{X}^{\mu\nu},$$

$$\tilde{\mathcal{O}}^{T,ji}_{\ell X2} = \frac{1}{2}(\overline{\ell_j}\sigma^{\mu\nu}\gamma_5\ell_i)(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu),$$ **(X)**

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

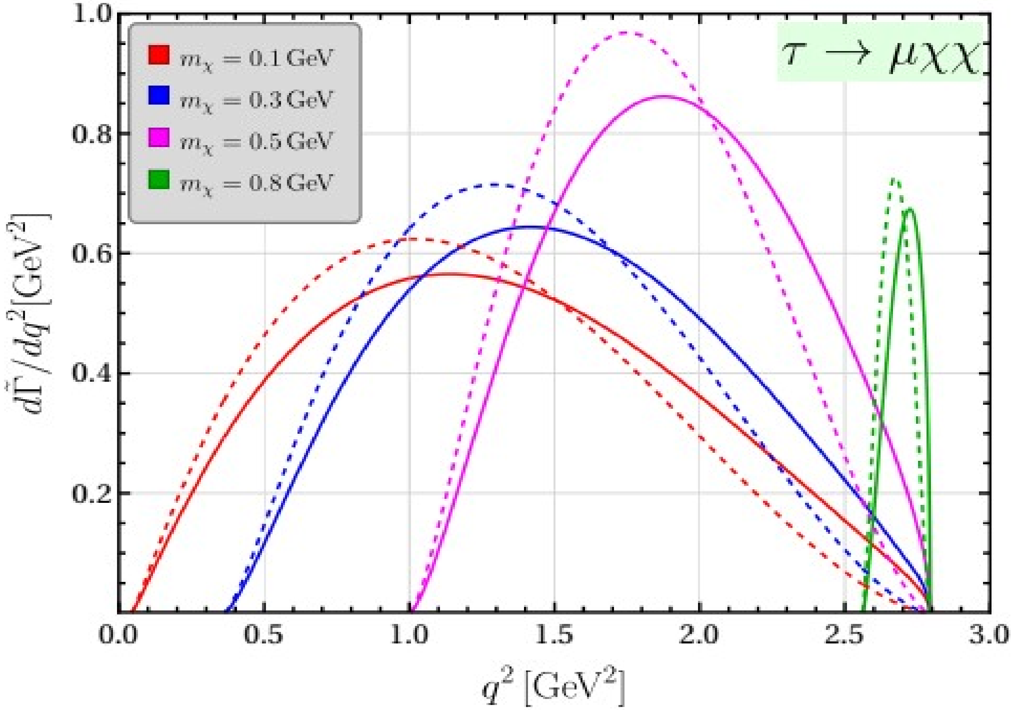
q^2 distributions

Process: $\ell_i(p) \to \ell_j(k) + \mathrm{DM}(k_1) + \mathrm{DM}(k_2)$

$q^2 = (p-k)^2 = (k_1+k_2)^2 =$ **Invariant squared mass** of the invisible particles.

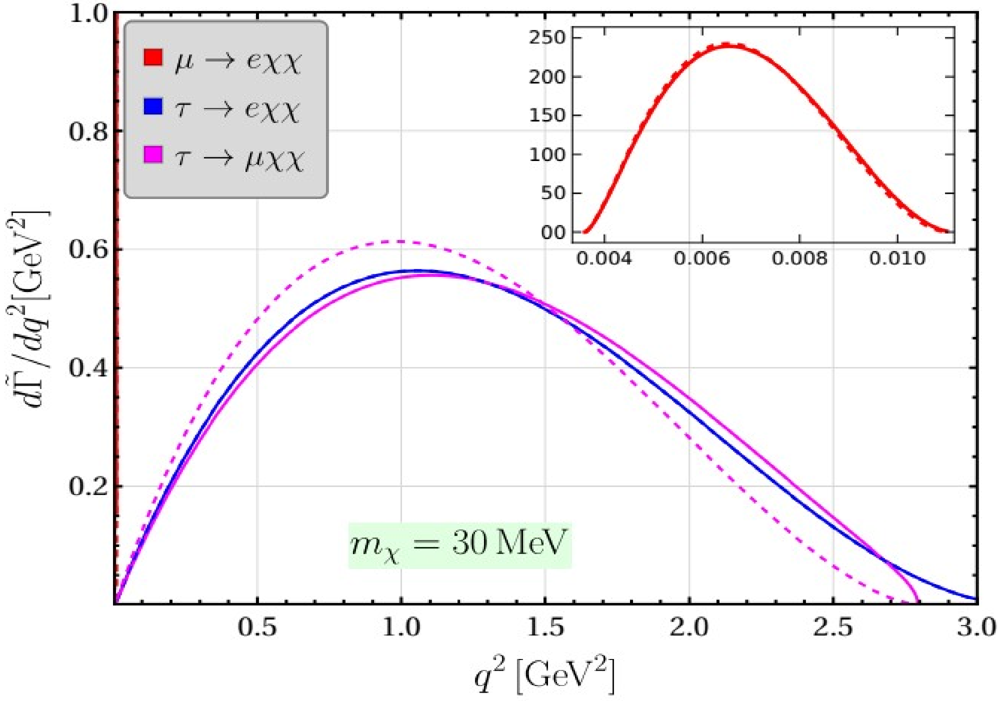
Normalized differential decay rate: $$\frac{d\tilde{\Gamma}}{dq^2} = \frac{1}{\Gamma^{\mathrm{tot}}}\left(\frac{d\Gamma}{dq^2}\right)$$

Distinction between $\mathcal{O}^{S,ji}_{\ell\chi 1}$ & $\mathcal{O}^{P,ji}_{\ell\chi 1}$



Maximal distinction at $m_{\mathrm{DM}} = m_{max}/2$

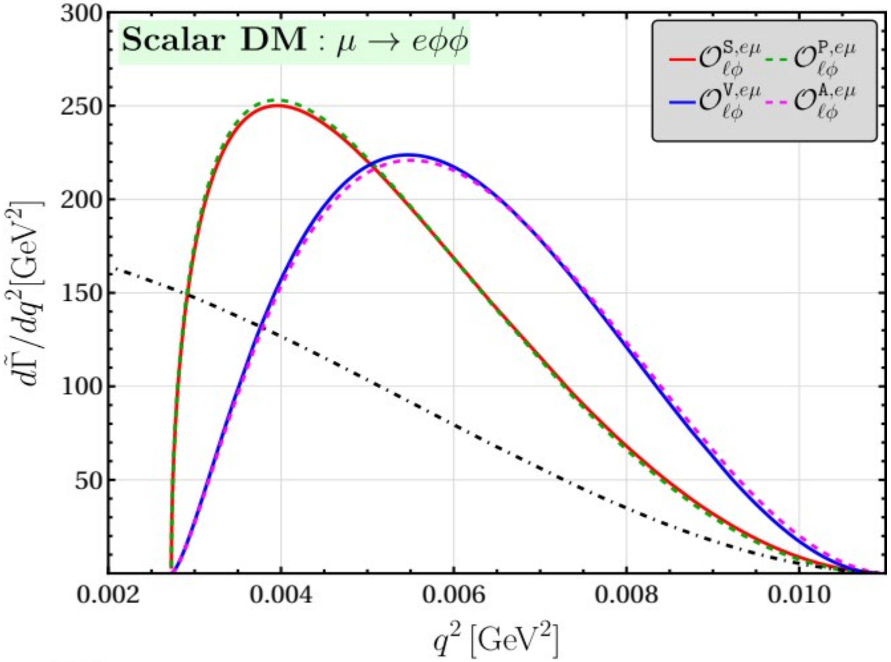
$m_{\mathrm{max}} = (m_i - m_j)/2$

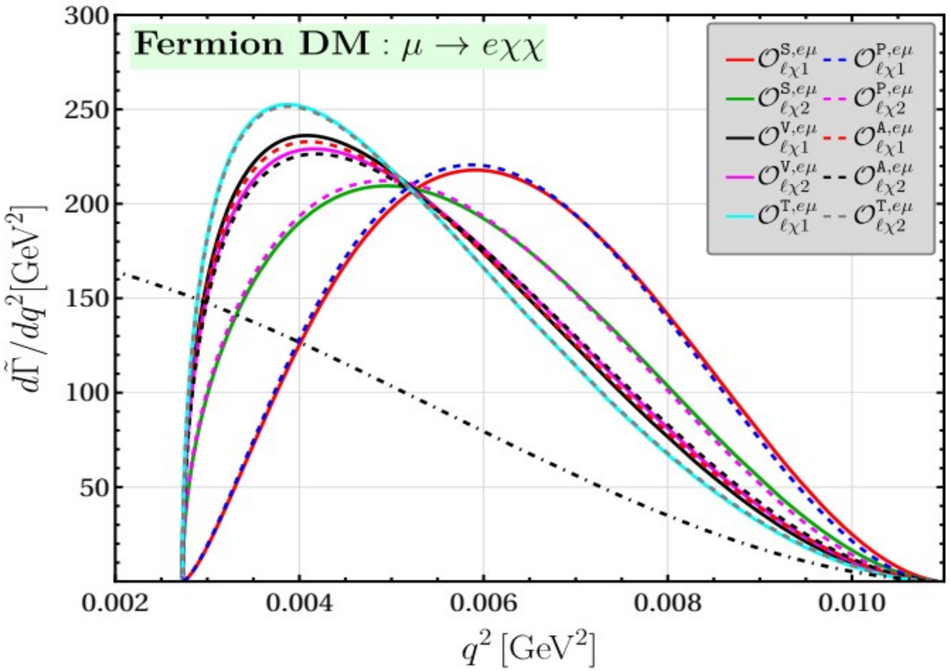


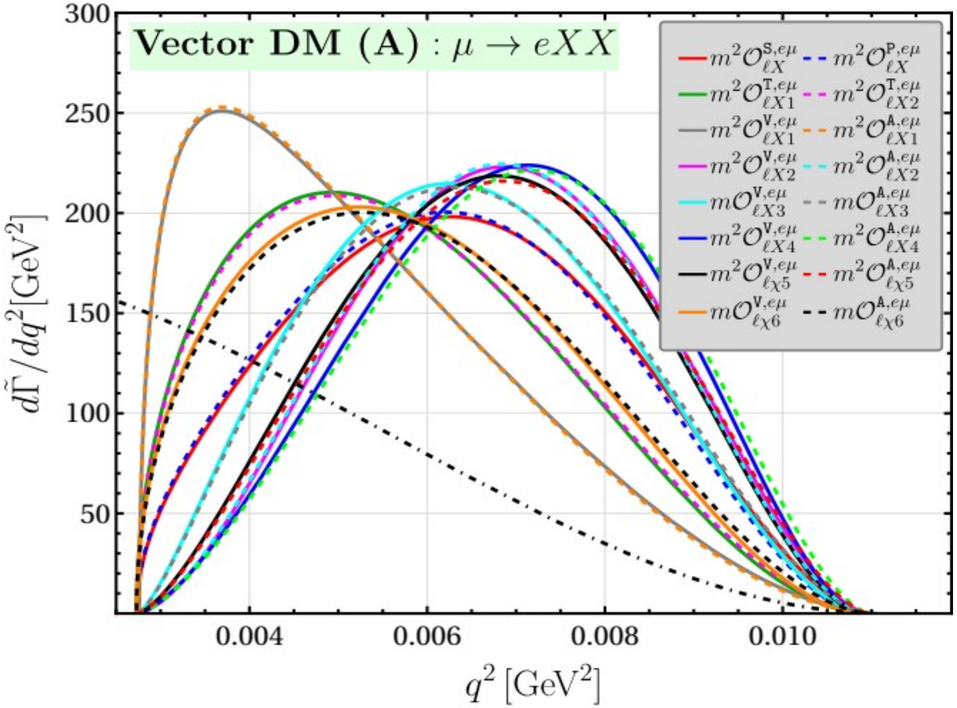
$\tau \to \mu\chi\chi$ provides maximal distinction

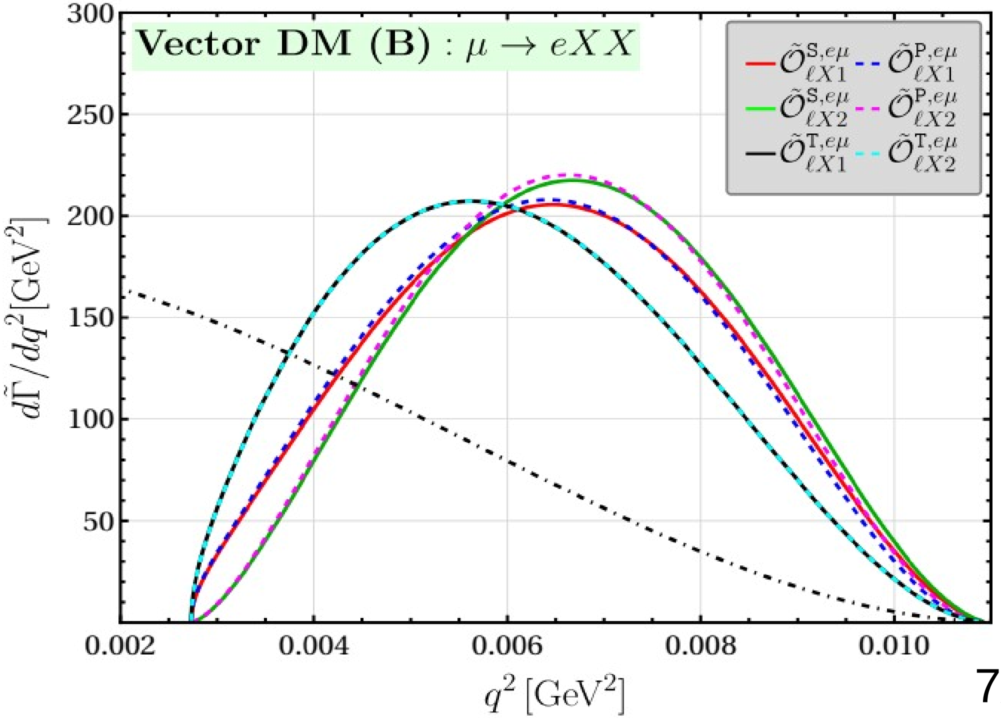
q^2 distributions

Process: $\mu \to e + \mathrm{DM} + \mathrm{DM}$



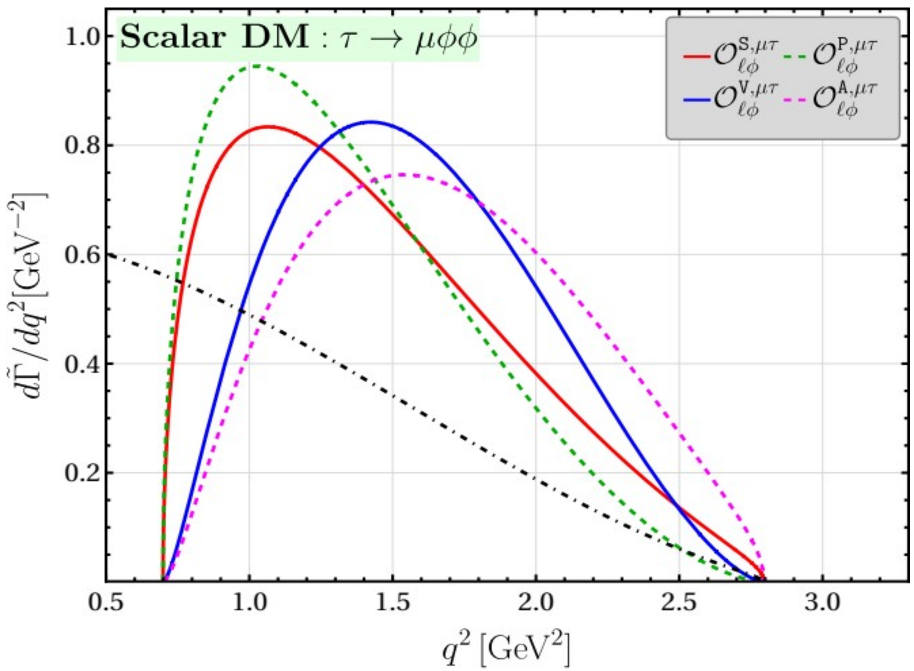


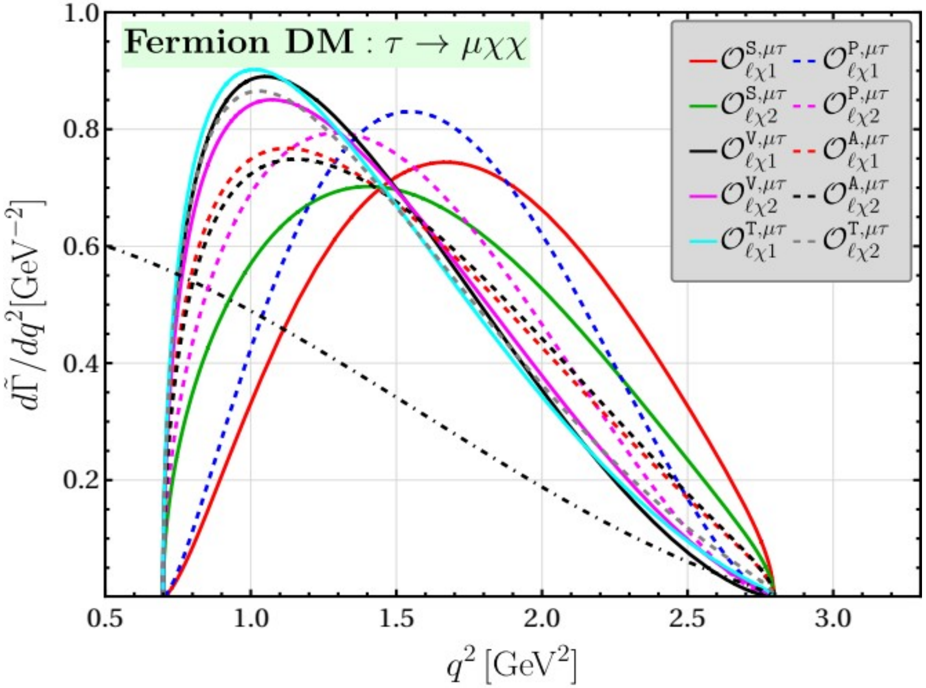


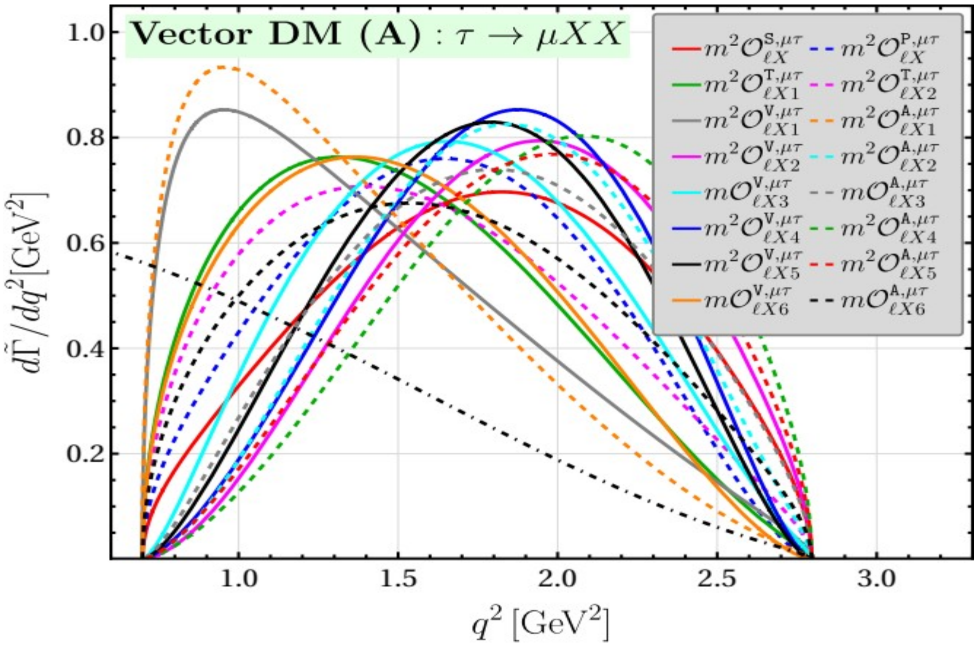


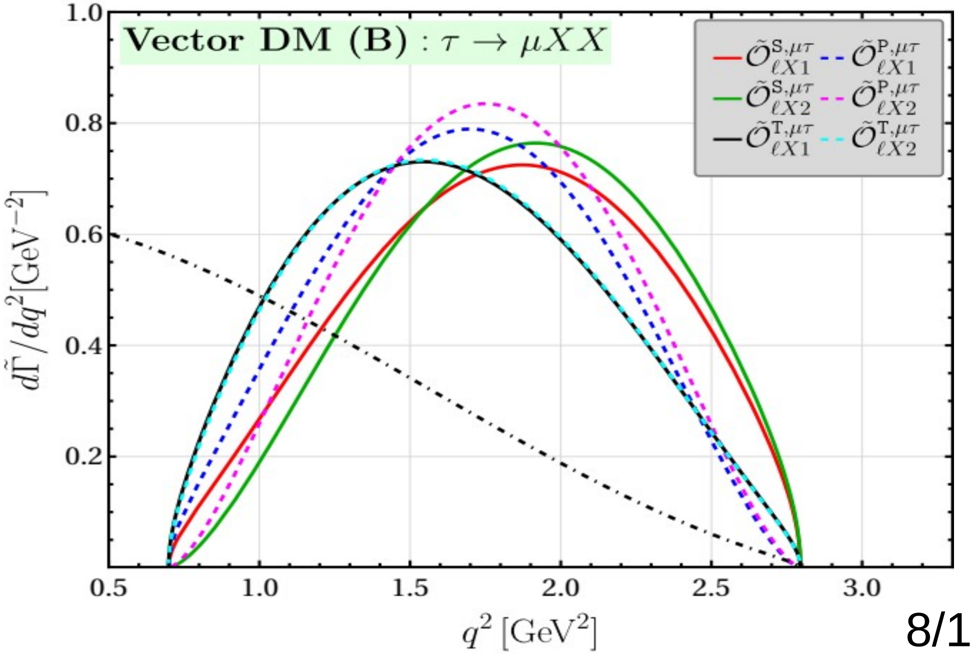
q² distributions

Process: $\tau \to \mu + \mathrm{DM} + \mathrm{DM}$



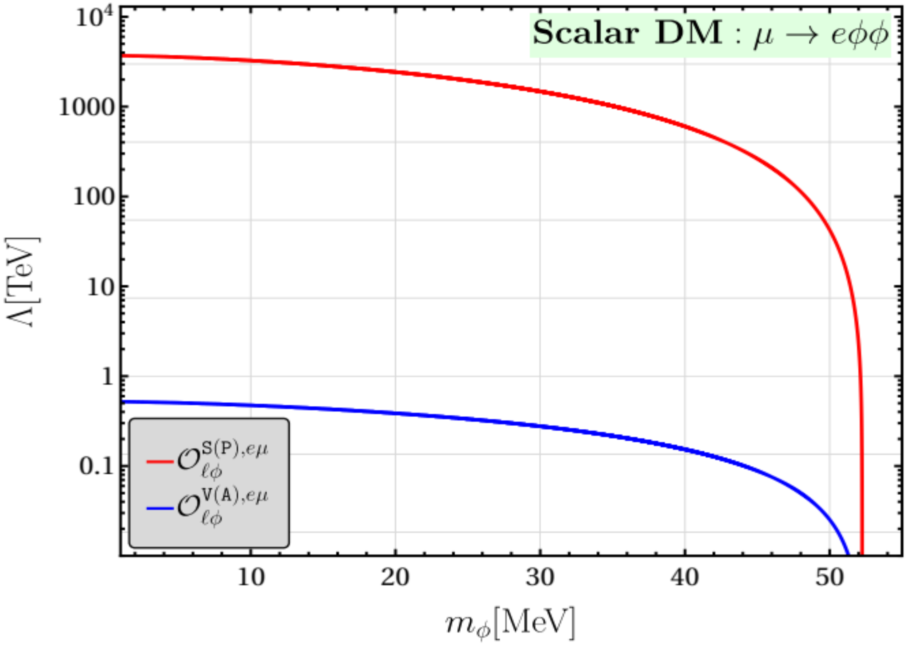


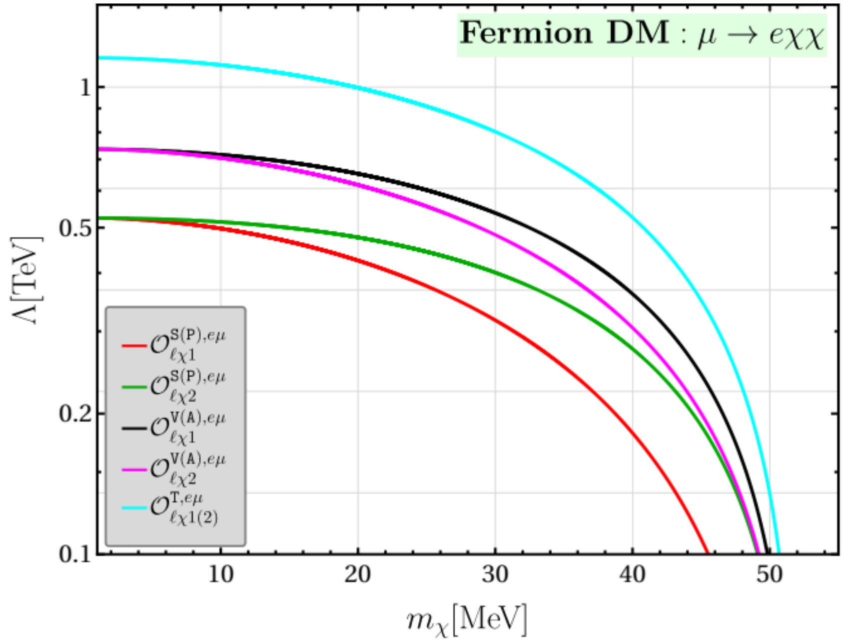


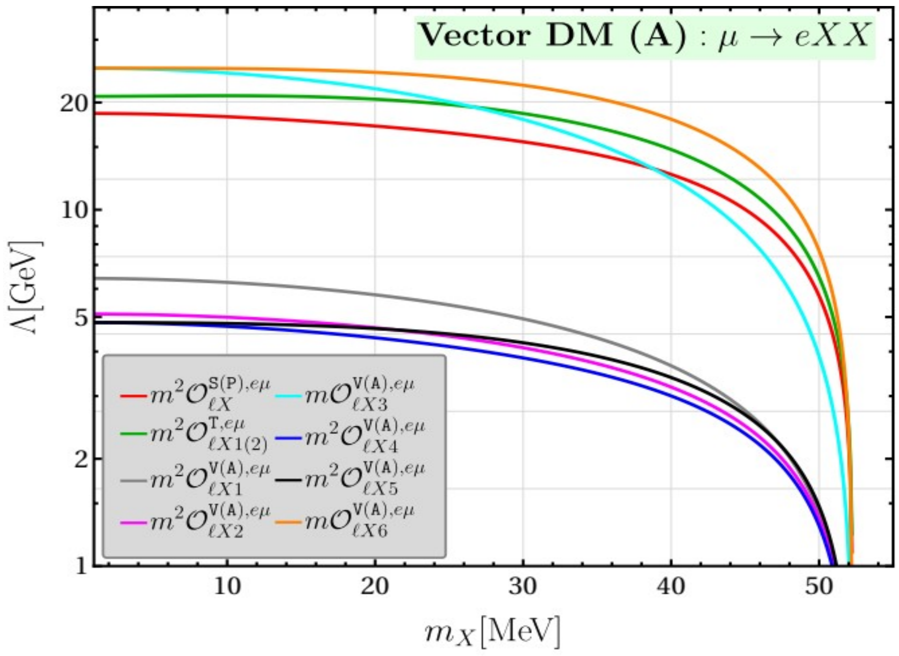


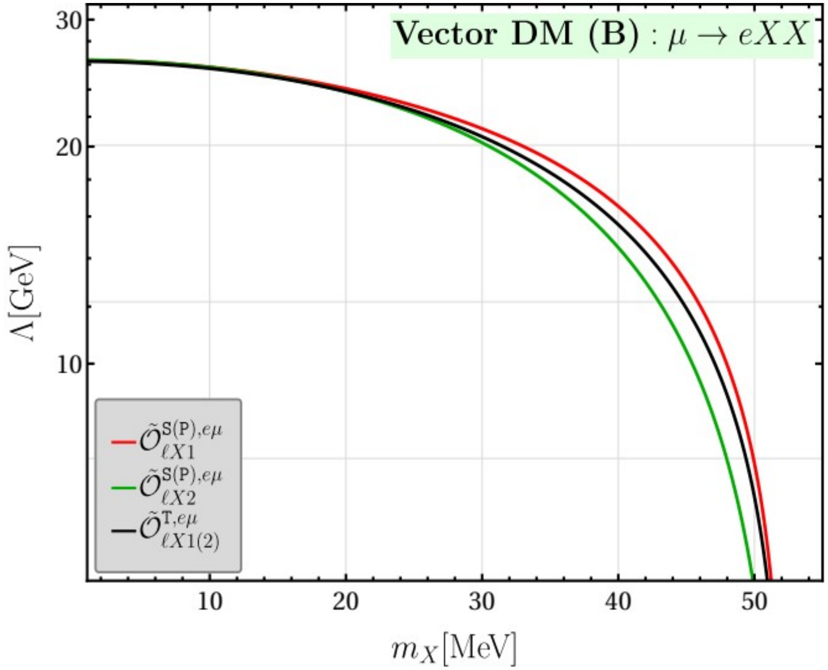
Constrains

Process: $\mu \to e + \mathrm{DM} + \mathrm{DM}$



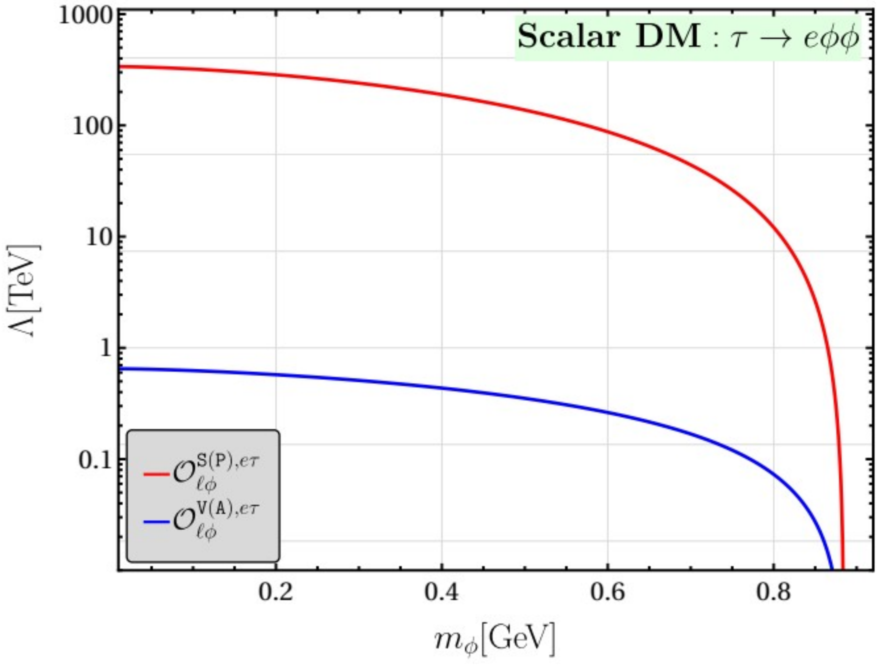


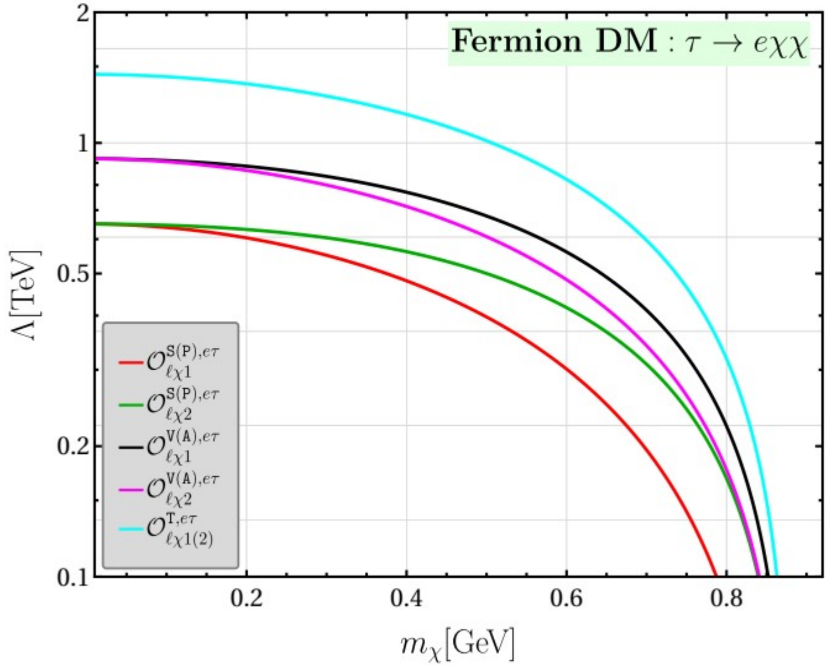


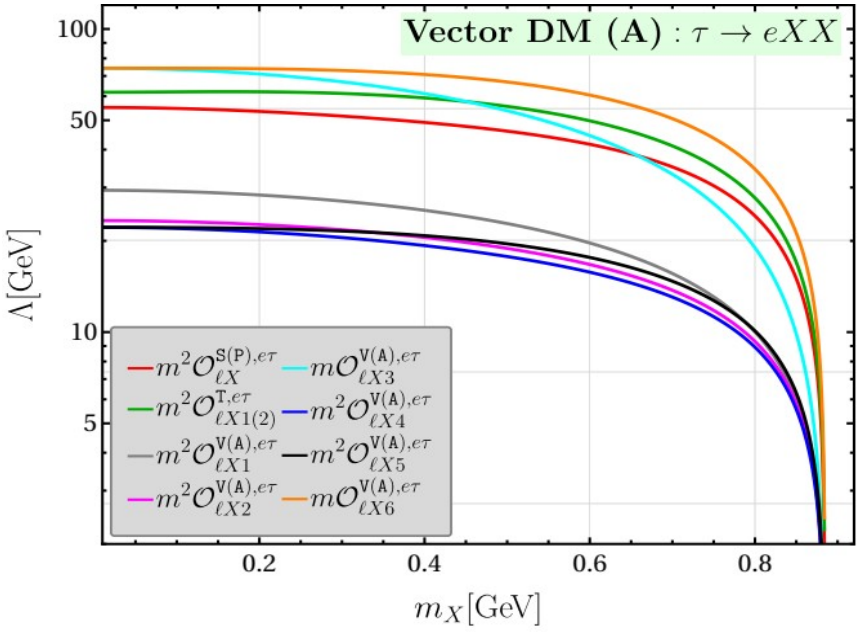


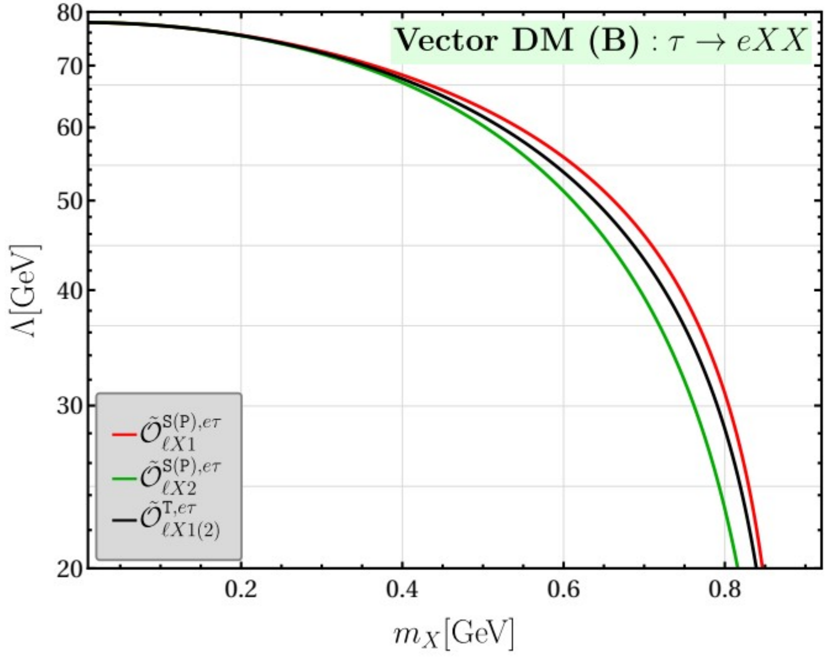
Constrains

Process: $\tau \to e + \mathrm{DM} + \mathrm{DM}$



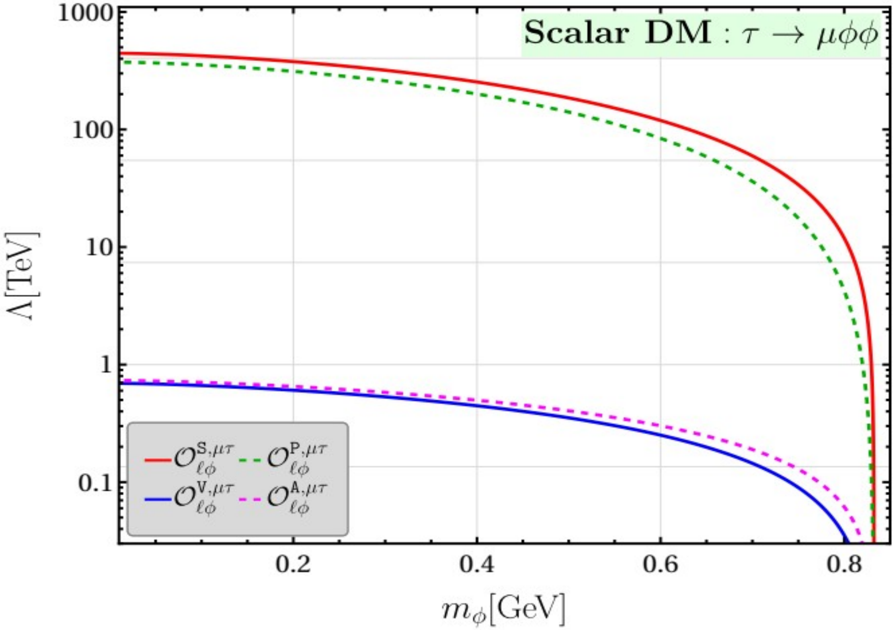


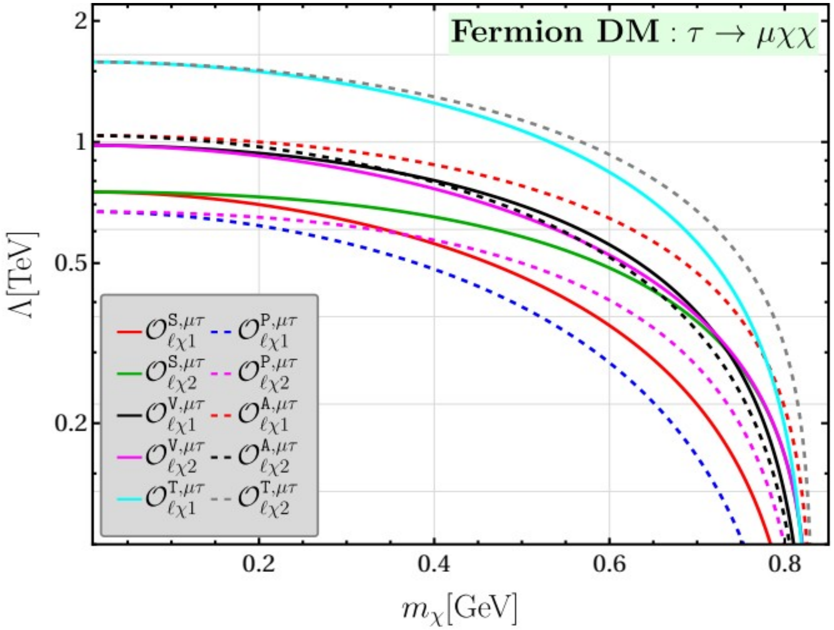


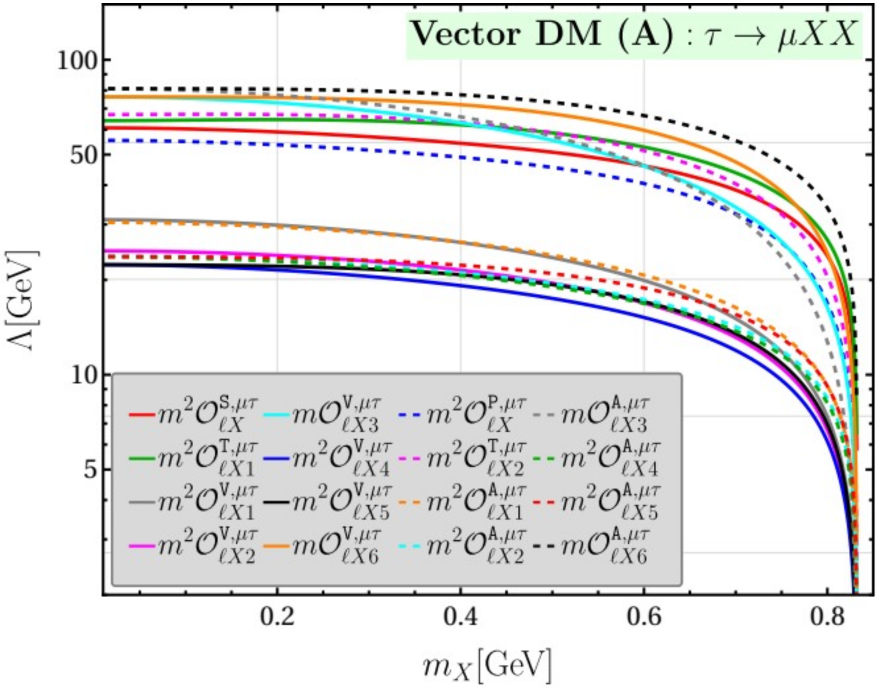


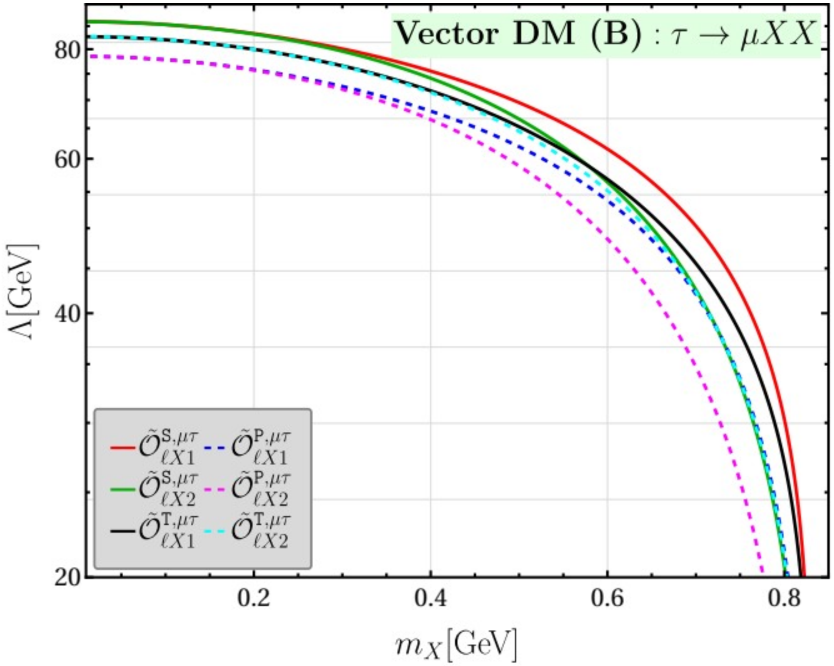
Constrains

Process: $\tau \to \mu + \mathrm{DM} + \mathrm{DM}$

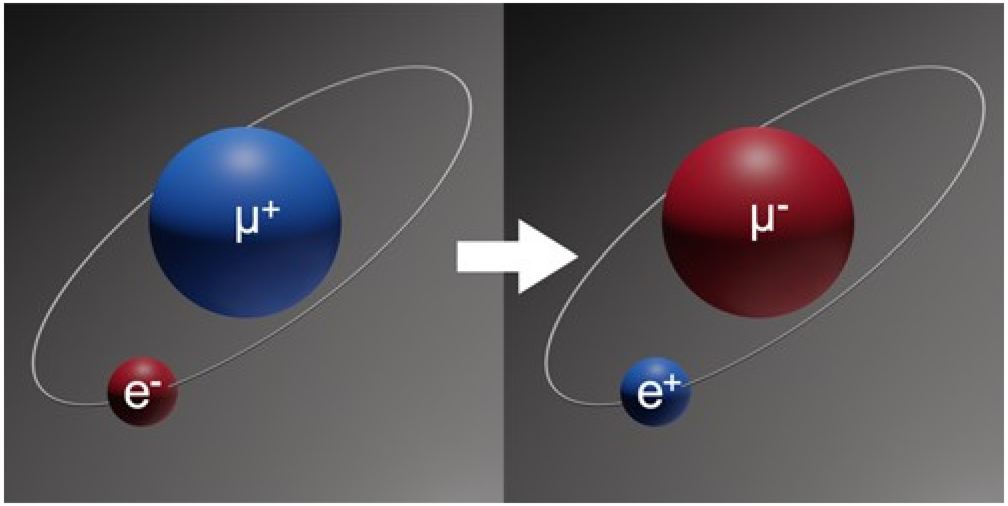








Muonium invisible decay



SM Prediction:

$$\mathcal{B}(M_\mu^O \to \nu_e \bar{\nu}_\mu) \equiv \frac{\Gamma(M_\mu^O \to \nu_e \bar{\nu}_\mu)}{\Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\nu)}$$

$$\simeq 6.6 \times 10^{-12}$$

MuLan collaboration estimation:

$$\mathcal{B}(M_\mu^O \to \text{inv.}) < 5.7 \times 10^{-6} \quad @\,90\%\,\text{C.L.}.$$

(Gninenko, Krasnikov, Matveev)

(Phys.Rev.D 87 (2013) 015016)

Ortho-muonium

(Spin triplet)

$$\langle 0|\bar{\mu}\gamma^\alpha e|M_\mu^O\rangle = i f_V M_M \epsilon_M^\alpha$$

$$\langle 0|\bar{\mu}\sigma^{\alpha\beta} e|M_\mu^O\rangle = i f_T(\epsilon_M^\alpha p^\beta - \epsilon_M^\beta p^\alpha)$$

Para-muonium

(Spin singlet)

$$\langle 0|\bar{\mu}\gamma_5 e|M_\mu^P\rangle = -i f_P M_M,$$

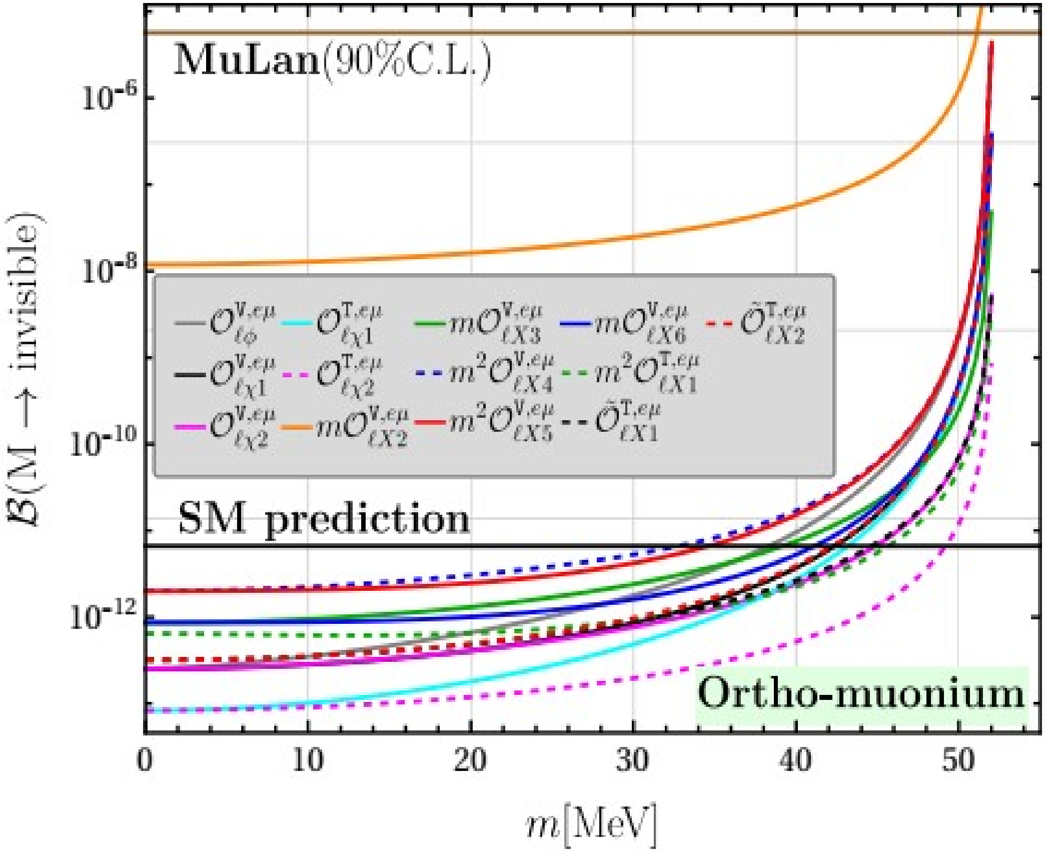
$$\langle 0|\bar{\mu}\gamma^\alpha\gamma_5 e|M_\mu^P\rangle = i f_A p^\alpha$$

Decay constants at non-relativisic limit are equal.

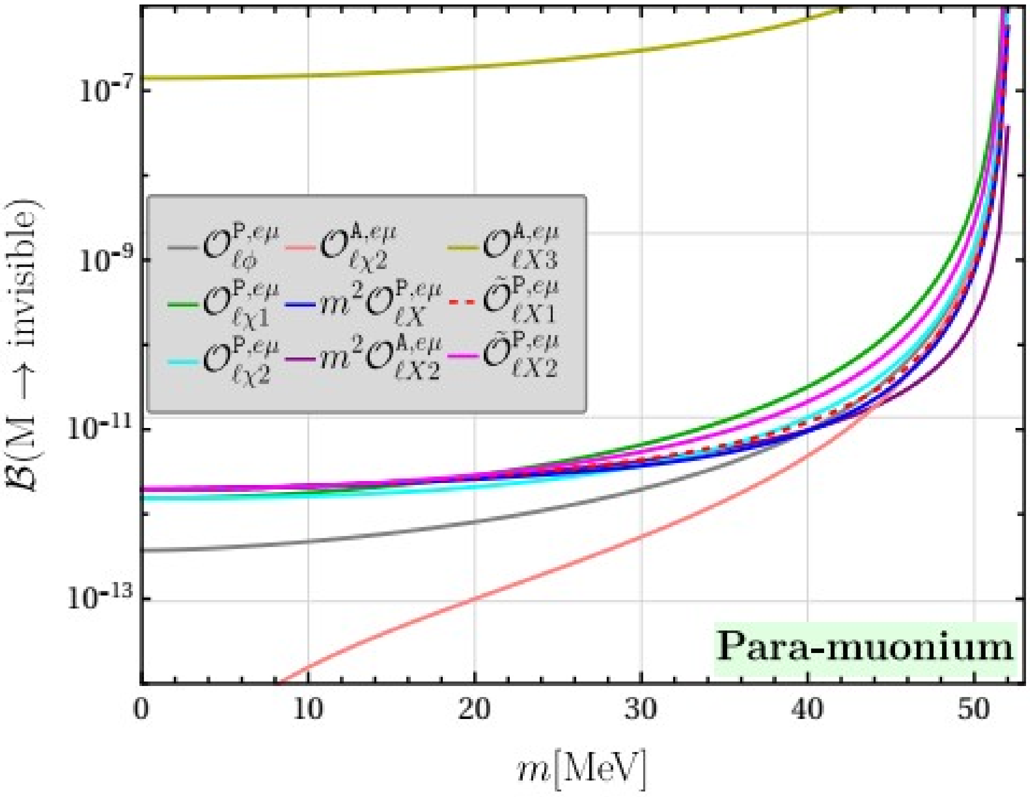
$$f_P = f_V = f_T \equiv f_M = 4\frac{|\phi(0)|^2}{M_M}$$

$$|\phi(0)|^2 = \frac{(m_{\text{red}}\alpha)^3}{\pi}, \quad m_{\text{red}} = \frac{m_e m_\mu}{m_e + m_\mu}$$

Branching ratios



The mass range below the black solid line for each relevant operator is challenging to observe in future experiments.



Smoking gun signature as no SM prediction.

Summary

- The DSEFT framework is an adequate way to explain cLFV by incorporating particle anti-particle pair.

- The q^2 distributions of three body decays are pivotal to distinguish different DSEFT operators and Lorentz structure of the SM leptonic current.

- Due to the dimensionality, (pseudo)scalar operator for scalar DM possess the most stringent limits on the effective scale.

- Invisible decay from the para-muonium can be regarded as a smoking gun signature of new physics.

Thank you!

Wilson coefficients for vector DM case A

$$\frac{d\Gamma^{A}_{\ell_i\to\ell_j XX}}{ds} = \frac{\sqrt{\kappa_f\lambda(s,m_i^2,m_j^2)}}{3072\pi^3 m^4 m_i^3 s}\left\{3s\left(s^2-4m^2 s+12m^4\right)((m_i+m_j)^2-s)|C^{\mathtt{S},ji}_{\ell X}|^2\right\}$$

$$C^{\mathtt{S,P}}_{\ell X} \equiv \frac{m_X^2}{\Lambda^3_{\text{eff}}}$$

$$C^{\mathtt{V,A}}_{\ell X1,2,4,5} \equiv \frac{m_X^2}{\Lambda^4_{\text{eff}}}$$

$$C^{\mathtt{T}}_{\ell X1,2} \equiv \frac{m_X^2}{\Lambda^3_{\text{eff}}}$$

$$C^{\mathtt{V,A}}_{\ell X3,6} \equiv \frac{m_X}{\Lambda^3_{\text{eff}}}$$