

具有伽利略不变性的含热质量源热-流耦合 MRT 格子 Boltzmann 模型

摘要

针对含质量注入/抽吸及其携热效应的复杂热流问题，本文发展了一种具有伽利略不变性的热-流耦合多松弛时间格子 Boltzmann 模型。首先，基于含质量源的连续性方程和动量方程，在 MRT 框架下构造了满足伽利略不变性的质量源项矩空间形式，并通过 Chapman-Enskog 多尺度分析证明该模型能够正确恢复含质量源的 Navier-Stokes 方程。在一维质量源验证中，数值误差随网格加密近似满足 N_{LB}^{-2} 衰减，并随质量源强度近似满足 q_0^2 关系，表明模型具有二阶空间精度。进一步地，在单组分、常物性、低马赫数弱可压假设下，本文从质量守恒、动量守恒和总能量守恒方程出发，推导了含热质量源的守恒型温度控制方程，并建立了与流场源项一致耦合的温度分布函数 MRT 模型。为保证温度方程在存在体力、空间变热源及背景输运时仍能正确恢复，本文在温度源项一阶矩中引入温度-体力耦合项和速度-源项耦合项修正。数值结果表明，所提出模型能够有效消除参考系平移引起的非物理振荡和各向异性误差条纹，在移动质量源算例中全局相对误差保持在 10^{-6} 量级以内，相比未修正模型降低约两个数量级；在高波数质量源稳定性测试中，可承受的质量源扰动幅值超过传统 SRT 模型的 3 倍；在全局压力调控算例中，平均压力的相对误差低于 10^{-13} 量级。对于含热质量源的二维制造解算例， u_x 、 u_y 和 T 的观测收敛阶分别约为 2.08、1.87 和 2.10，密度扰动收敛阶约为 2.48，验证了模型对质量源、动量交换和热源项耦合效应的近二阶恢复能力。本文工作为相变传热、反应流、注入/抽吸流动及压力调控等含质量与能量交换问题提供了一种统一、稳定且物理一致的格子 Boltzmann 建模方法

关键词

格子 Boltzmann 方法；多松弛时间模型；质量源；热质量源；伽利略不变性；热-流耦合；二阶精度

Abstract

A Galilean-invariant thermal-flow coupled multiple-relaxation-time lattice Boltzmann model is developed for complex flows involving mass injection/suction and the associated heat transport. First, based on the continuity and momentum equations with a mass source, a Galilean-invariant mass-source formulation is constructed in moment space within the MRT framework. Chapman-Enskog multiscale analysis demonstrates that the proposed formulation correctly recovers the Navier-Stokes equations with mass source terms. In the one-dimensional benchmark test, the numerical error decreases approximately as N_{LB}^{-2} under grid refinement and scales approximately with q_0^2 with respect to the source strength, confirming the second-order spatial accuracy of the model. The formulation is then extended to thermally coupled mass-source flows. Under the assumptions of a single-component fluid, constant properties, and low-Mach-number weak compressibility, a conservative temperature equation with a heated mass source is derived from the conservation laws of mass, momentum, and total energy. A corresponding MRT lattice Boltzmann model for the temperature field is established and consistently coupled with the flow-field source formulation. To ensure the correct recovery of the target temperature equation in the presence of body forces, spatially varying heat sources, and background advection, two correction terms are introduced into the first-order moments of the thermal source term, namely the temperature-force coupling term and the velocity-source coupling term. Numerical results show that the proposed model effectively eliminates non-physical oscillations and anisotropic error stripes induced by reference-frame translation. In moving mass-source tests, the global relative error remains at or below the order of 10^{-6} , corresponding to an approximately two-order-of-magnitude reduction compared with the uncorrected model. In high-wavenumber mass-source stability tests, the tolerable amplitude of mass-source perturbations is more than three times that of the conventional SRT model. In the global pressure-regulation test, the relative error of the average pressure remains below the order of 10^{-13} . For the two-dimensional manufactured solution with heated mass sources, the observed convergence orders of u_x , u_y , and T are approximately 2.08, 1.87, and 2.10, respectively, while the perturbation-based density

error gives an observed order of approximately 2.48. These results demonstrate the nearly second-order recovery capability of the proposed model for coupled mass-source, momentum-exchange, and heat-source effects. The present work provides a unified, stable, and physically consistent lattice Boltzmann framework for thermally coupled flows with mass and energy exchange, such as phase-change heat transfer, reactive flows, injection/suction flows, and pressure-regulation problems.

Keywords

Lattice Boltzmann method; multiple-relaxation-time model; mass source; heated mass source; Galilean invariance; thermal-flow coupling; second-order accuracy

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